Analysis of the Accumulation of Constant Time Error in an IEC/IEEE 60802 Network

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Outline

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Introduction - 1

- Recent discussions in the IEC/IEEE 60802 joint project have concerned whether the accumulation of constant time error (cTE) should be linear or square-root-of-sum-of-squares (i.e., RMS)
- On the one hand, it has been argued that if cTE is uniformly distributed, it is extremely unlikely that cTE will be at one extreme or another for all PTP instances and links in a hypothetical reference model (HRM)
  - With this argument, linear accumulation is overly conservative, and RMS accumulation should be used
- On the other hand, it has been argued that, since cTE is fixed and constant, if a particular connection happens to have cTE at one extreme for all PTP Instances and links, it will remain this way permanently, or at least until the network is re-initialized
  - In the case where the cTE is permanently fixed, it is possible that cTE at most or all PTP Instances is similar and at one extreme if, for example, the same physical devices are used in most or all PTP Instances in the connection
  - With this argument, linear accumulation should be used

Introduction - 2

- This presentation analyzes the accumulation of two types of cTE:
  - cTE that is constant and permanently fixed
  - cTE that is constant and fixed for the duration of the interval that the network is up and operating, i.e., the cTE can change when the network is re-initialized
- Two HRM cases are considered
  - 100 hops (IEC/IEEE 60802 HRM)
  - 20 hops (ITU-T G.8271.1 HRM, for comparison)
- Delay variation due to very long term effects, e.g., di-urnal temperature variation effects in links or aging of oscillators, is considered to be part of dTE and not considered here

Analysis - 1

- There are three categories of cTE to consider
  1) cTE associated with the physical medium (i.e., link)
     - cTE in this category is fixed and constant for all time
     - Denote this by $x_{l,1}$
  2) cTE associated with the PTP Instance (i.e., node) that is fixed, and constant for all time
     - Denote this by $x_{n,2}$
  3) cTE associated with the PTP Instance (i.e., node) that is fixed while the node is up and operating, but changes when the node (or port if this cTE is associated only with the port) re-initializes
     - Denote this by $x_{n,3}$
- The category nomenclature is used here only for convenience; it is not standardized terminology, and can be replaced by other terminology if desired

Analysis - 2

- For simplicity, assume all the above components of cTE have zero mean and are uniformly distributed
  1) $x_{l,1}$ is uniformly distributed over [- $D_{l,1}$, $D_{l,1}$]
  2) $x_{n,2}$ is uniformly distributed over [- $D_{n,2}$, $D_{n,2}$]
  3) $x_{n,3}$ is uniformly distributed over [- $D_{n,3}$, $D_{n,3}$]
- In saying that the cTE components are uniformly distributed, we mean
  - For components 1 and 2, a random cTE sample is permanently associated with each link and PTP Instance, respectively; this sample is taken from the respective uniform probability distribution
  - For component 3, a random cTE sample is taken from the respective uniform probability distribution whenever the PTP Instance is re-initialized, and this value (sample) is associated with that PTP Instance until the next time it is initialized

Analysis - 3

- As indicated above, cTE has zero mean; the cTE probability density functions are

$$p_{x_k}(u) = \frac{1}{2D_k} \quad \text{for} \quad -D_k \le u \le D_k \quad \text{and} \quad k = l,1;\ n,2;\ n,3$$

- The variances of the cTE components are

$$\sigma_k^2 = \int_{-D_k}^{D_k} \frac{u^2}{2D_k}\, du = \frac{D_k^2}{3} \quad \text{for} \quad k = l,1;\ n,2;\ n,3$$

- In the following slides, consider an HRM consisting of N PTP Instances and therefore N-1 links

Analysis - 4

- Consider first cTE categories 1 and 2
- For these categories, cTE is fixed and permanent, for all time (i.e., for as long as the respective links and PTP Instances are in service.
  - This means that, if a particular path happens to have large cTE, the cTE will always be large for that path
  - In addition, category 2 cTE is due to components used in the respective PTP Instances. If the components in different PTP Instances are from the same vendor, the cTE in the different PTP Instances could be similar (or correlated). In addition, there could be bias in the compensation procedures for link and node cTE.
  - It is true that category 1 and 2 cTE can be compensated. The cTE of interest here (i.e., the values of $D_{l,1}$ and $D_{n,2}$) of interest here are the values after any compensation
    - **In other words, the category 1 and 2 cTE of interest is the whatever cTE remains after compensation). This remaining cTE, while it might be small, will not be identically zero**

Analysis - 5

- Given the above, the accumulation of category 1 and 2 cTE should be linear. The worst-case accumulated category 1 and 2 cTE is

$$\max\left|\mathrm{cTE}(\text{accumulated, category 1 and 2})\right| = ND_{n,2} + (N-1)D_{l,1}$$

- Next, consider cTE category 3
- For this category, cTE in each PTP Instance:
  - is independent of cTE in other PTP Instances
  - is different each time the PTP Instance is initialized
- With the above assumptions, the accumulated category 3 cTE is the sum of N random variables that are independent and identically distributed (and, actually, uniformly distributed)
- Then, by the Central Limit Theorem (which is a good approximation for $N = 20$ (and therefore also for $N = 100$)), the distribution of the accumulated cTE is approximately Gaussian (i.e., normal)

Analysis - 6

- The accumulated category 3 cTE has zero mean and variance equal to

$$\sigma^2_{\text{accum, category 3}} = \frac{ND^2_{n,3}}{3}$$

- The maximum absolute value of accumulated category 3 cTE can be taken to be an upper quantile of the Gaussian distribution for the accumulated category 3 cTE that corresponds to a chosen number of standard deviations from the mean. Below are exceedance probabilities for several different numbers of standard deviations from the mean, for a Gaussian distribution

$$\Pr\{|x| > 6\sigma\} = \frac{2}{\sigma\sqrt{2\pi}} \int_{6\sigma}^{\infty} e^{-u^2/2\sigma^2} du = 1.9732 \times 10^{-9}$$

$$\Pr\{|x| > 5\sigma\} = 5.7330 \times 10^{-7}$$

$$\Pr\{|x| > 4\sigma\} = 6.3342 \times 10^{-5}$$

$$\Pr\{|x| > 7\sigma\} = 2.5596 \times 10^{-12}$$

Analysis - 7

- In the quality control area, a 6-sigma criterion is often used, and that will be used here, i.e., we will take the maximum absolute value of accumulated cTE to be 6 standard deviations from the mean
- Then max|accumulated category 3 cTE| is equal to

$$\max|\mathrm{cTE}(\text{accumulated category 3})| = 6D_{n,3}\sqrt{\frac{N}{3}} = 3.464D_{n,3}\sqrt{N}$$

- While the above accumulation of category 3 cTE goes asymptotically like the square root of N, note the presence of the multiplier 3.464
  - For $N \le 12$, $3.464 \ge N$, and the above result gives a larger answer than the result of simply multiplying $D_{n,3}$ by N.
  - For N moderately larger than 12, linear accumulation gives a reasonable conservative approximation.
  - For N much larger than 12, the above equation should be used

Case of 100 Hops - 1

- For $N = 100$, max|cTE| for the accumulated category 1 plus category 2 cTE is

$$\max|\mathrm{cTE}(\text{accumulated, category 1 and 2})| = 100 D_{n,2} + 99 D_{l,1}$$

- In the above
  - $D_{n,2}$ = max|cTE| for a single PTP Instance for category 2, **after any compensation has been performed**
  - $D_{l,1}$ = max|cTE| for a single link (category 1), **after any compensation has been performed**
- **While it is expected that max|cTE| will be small after compensation, it will not be identically zero (i.e., a requirement must be specified)**

Case of 100 Hops - 2

- For $N = 100$, max|cTE| for the accumulated category 3 cTE is

$$\max|\text{cTE(accumulated category 3)}| = 6D_{n,3}\sqrt{\frac{100}{3}} = 34.64D_{n,3}$$

- In the above
  - $D_{n,3}$ = max|cTE| for a single PTP Instance for category 3
- **Note that while the above expression gives max|cTE| for the accumulated category 3 cTE that is less than linear (i.e., less than $100D_{n,3}$), it is larger than the square root of 100, i.e., 10, multiplied by $D_{n,3}$**

Case of 20 Hops

- For $N = 20$, max|cTE| for the accumulated category 1 plus category 2 cTE is

$$\max|\mathrm{cTE}(\text{accumulated, category 1 and 2})| = 20D_{n,2} + 19D_{l,1}$$

- In the above
  - $D_{n,2}$ = max|cTE| for a single PTP Instance for category 2, **after any compensation has been performed**
  - $D_{l,1}$ = max|cTE| for a single link (category 1), **after any compensation has been performed**
- **While it is expected that max|cTE| will be small after compensation, it will not be identically zero (i.e., a requirement must be specified)**

Case of 20 Hops - 2

- For $N = 20$, max|cTE| for the accumulated category 3 cTE is

$$\max|\mathrm{cTE}(\text{accumulated category 3})| = 6D_{n,3}\sqrt{\frac{20}{3}} = 15.49D_{n,3}$$

- In the above
  - $D_{n,3}$ = max|cTE| for a single PTP Instance for category 3
- **Note that while the above expression gives max|cTE| for the accumulated category 3 cTE that is less than linear (i.e., less than $20D_{n,3}$), it is larger than the square root of 20, i.e., 4.47, multiplied by $D_{n,3}$. In fact, the multiplier of 15.49 is sufficiently close to 20 that using 20 as the multiplier gives a reasonable conservative approximation.**
  - **The N = 20 HRM is sufficiently short that linear accumulation of category 3 cTE gives a reasonable conservative approximation**

Case of 20 Hops - 3

- In fact, the multiplier of 20 corresponds to (20/15.49)(6) = 7.7469 standard deviations, or max|cTE| with an exceedance probability of 9.416×10^{-15}

Conclusion

- The accumulation of maximum absolute value of category 1 and category 2 cTE is linear
    - If cTE is compensated, then the max|cTE| for a single PTP Instance or a single link must be specified after compensation
- The accumulation of maximum absolute value of category 3 cTE is asymptotically RMS, for the number of hops much greater than 12
    - However, note the additional constant multiplier of 3.464 (i.e., that multiplies the RMS expression); the resulting accumulated max|cTE| for category 3 exceeds $D_{n,3}N^{1/2}$ by this factor
    - For an HRM of 20 hops, category 3 max|cTE| accumulation is approximately linear (this is a slightly conservative approximation)

Thank you