Corrected Value of offsetScaledLogVariance for 802.1AS Corrigendum

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Introduction - 1

- The default value of the offsetScaledLogVariance attribute is specified in 8.6.2.4, item (b) of IEEE 802.1AS-2011.
- The value is supposed to correspond to the value of PTPDEV at observation interval equal to the default Sync message transmission interval
- The default Sync message transmission interval is 0.125 s, as specified in 11.5.2.3 of 802.1AS (i.e., initialLogSyncInterval is -3)
- The wander generation requirement is given in Annex B, B.1.3.2; it is specified by the TDEV mask of Figure B.1 and Table B.1.
    - The PTPDEV mask that corresponds to this TDEV mask is given in Table B.3 and Figure B.3
    - Note that PTPDEV is equal to Allan Deviation multiplied by $\tau / \sqrt{3}$ (just as TDEV is equal to Modified Allan Deviation multiplied by $\tau / \sqrt{3}$)
        - Here, τ is the observation interval (= $n\tau_0$, where τ_0 is the sampling interval)

Introduction - 2

- From Table B.3 and Figure B.3 of 802.1AS, the value of PTPDEV at 0.125 s observation interval is 6.08τ ns, where τ is the observation interval in s
- The, PTPDEV at τ = 0.125 s is

$$(6.08 \times 10^{-9})(0.125\ \text{s}) = 7.6 \times 10^{-10}\ \text{s}$$

- The method for computing offsetScaledLogVariance from PTPDEV is described in IEEE 1588 – 2008, subclause 7.6.3
  - An example is given in NOTE 1 of 7.6.3.3
- The specified default value of offsetScaledLogVariance in IEEE 802.1AS – 2011, subclause 8.6.2.4(b) is 16640 (i.e., 4100_{16})
- Computation of offsetScaledLogVariance corresponding to the above value of PTPDEV, 7.6×10^{-10} s yields the value 17258 ($436A_{16}$)
  - i.e., the current value in 802.1AS is in error

Introduction - 2

- The error apparently is due to the fact that, in the development of IEEE 802.1AS, the TDEV mask requirement (and therefore the ADEV and PTPDEV masks) were originally smaller by a factor of two
- As a result of measurement data for an inexpensive oscillator presented in [1], the TDEV mask (and therefore the ADEV and PTPDEV masks) were increased by a factor of 2
  - However, the default value of offsetScaledLogVariance was not recalculated
- The correction of the default value of offsetScaledLogVariance will be an item for the planned 802.1AS corrigendum
- The purpose of the current presentation is to document the calculation of the corrected value, from the correct value of PTPDEV at 0.125 s observation interval
  - The presentation will also facilitate the review of this calculation

Introduction - 3

- In addition, this contribution reviews the current value of PTPDEV at $\tau = 0.125$ s, as the default value of offsetScaledLogVariance is based on this value of PTPDEV
  - This review includes the relations among TDEV, ADEV, and PTPDEV for Flicker Frequency Modulation (FFM), i.e., the noise type of the TDEV, ADEV, and PTPDEV masks of Annex B

Review of PTPDEV value at τ = 0.125 s - 1

- The definitions of ADEV (see [2] and [3]) and PTPDEV (see [4]) are
  - ADEV

$$\sigma_y(\tau) = \sqrt{\frac{1}{2n^2\tau_0^2(N-2n)}\sum_{i=1}^{N-2n}\left(x_{i+2n} - 2x_{i+n} + x_i\right)^2} \quad , \quad n = 1, 2, \ldots \left\lfloor \frac{N-1}{2} \right\rfloor$$

  - PTPDEV

$$\sigma_{PTP}(\tau) = \frac{\tau}{\sqrt{3}}\sigma_y(\tau) = \sqrt{\frac{1}{6(N-2n)}\sum_{i=1}^{N-2n}\left(x_{i+2n} - 2x_{i+n} + x_i\right)^2} \quad , \quad n = 1, 2, \ldots \left\lfloor \frac{N-1}{2} \right\rfloor$$

where

$\tau = n\tau_0$ = observation interval

τ_0 = sampling interval

N = total number of samples [$(N-1)\tau_0$ = measurement interval]

$\lfloor y \rfloor$ denotes the floor function, i.e., the greatest integer less than or equal to y

x_i = measured phase (time) error at the i^{th} sampling time

Review of PTPDEV value at τ = 0.125 s - 2

- The definitions of MDEV and TDEV are (see [2] and [3])
  - MDEV

$$\operatorname{mod}\sigma_y(\tau) = \sqrt{\frac{1}{2n^4\tau_0^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{n+j-1}(x_{i+2n}-2x_{i+n}+x_i)\right]^2}\ , \quad n = 1, 2, \ldots \left\lfloor \frac{N}{3} \right\rfloor$$

  - TDEV

$$\sigma_x(\tau) = \sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{n+j-1}(x_{i+2n}-2x_{i+n}+x_i)\right]^2}\ , \quad n = 1, 2, \ldots \left\lfloor \frac{N}{3} \right\rfloor$$

where

$\tau = n\tau_0$ = observation interval

τ_0 = sampling interval

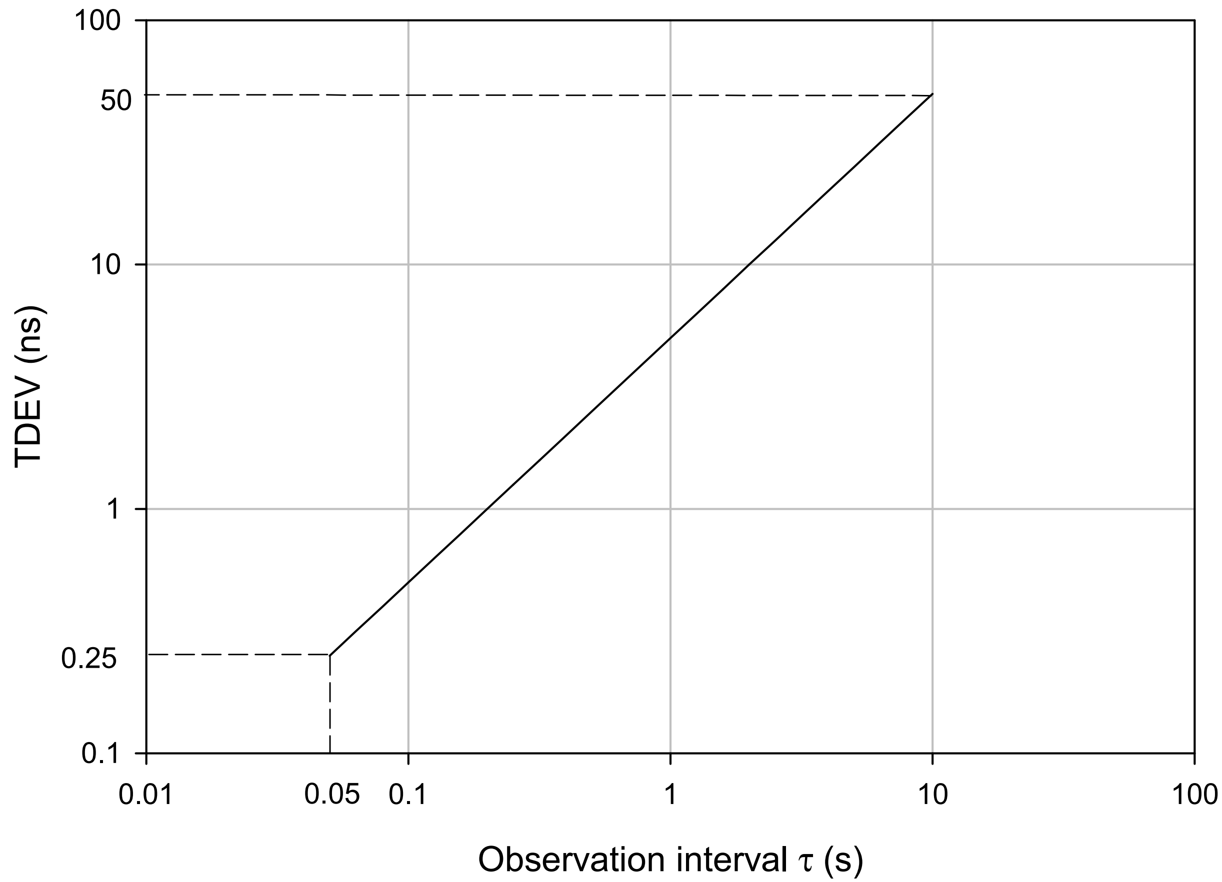
N = total number of samples [$(N–1)\tau_0$ = measurement interval]

$\lfloor y \rfloor$ denotes the floor function, i.e., the greatest integer less than or equal to y

x_i = measured phase (time) error at the i^{th} sampling time

Review of PTPDEV value at τ = 0.125 s - 3

- Wander generation TDEV requirement in Annex B, B.1.3.2 of 802.1AS



Review of PTPDEV value at τ = 0.125 s - 4

- Wander generation TDEV requirement in Annex B, B.1.3.2 of 802.1AS

Table B.1 – Wander generation TDEV requirement for LocalClock entity

TDEV limit	Observation interval τ
No requirement	$\tau < 0.05$ s
5.0τ ns	$0.05 \le \tau \le 10$ s
No requirement	$\tau > 10$ s

- TDEV is proportional to τ
  - The noise type is Flicker Frequency Modulation (FFM) (see [2] and [3]), and has power spectral density proportional to $1/f^3$
- The above mask is based on the measurements described in [1]
- We need to determine PTPDEV for τ = 0.125 s
  - Therefore, we need to relate TDEV and PTPDEV (or, equivalently, TDEV and ADEV)
- The relation between TDEV and ADEV depends on power spectral density, and is different for different noise types

Review of PTPDEV value at τ = 0.125 s - 5

- The relations between TDEV and ADEV for various power-law noises, i.e., noises with power spectral density proportional to $1/f^{\alpha}$, with α = 0, 1, 2, 3, and 4, are given in [3]
- For FFM

$$S_x(f) = \frac{A}{f^3}$$

where A is a constant

- Allan Variance (AVAR, square of ADEV) and Time Variance (TVAR, square of TDEV) for FFM are given by (see Tables 5.4 and 5.6) of [3]

$$\sigma_y^2(\tau) = (2\ln 2)A$$

$$\sigma_x^2(\tau) = \frac{0.675 \cdot 2\ln 2}{3} A\tau^2$$

- Then TVAR and AVAR are related by (for FFM)

$$\sigma_x^2(\tau) = 0.225\tau^2 \cdot \sigma_y^2(\tau)$$

Review of PTPDEV value at τ = 0.125 s - 6

- Then TDEV and ADEV are related by (for FFM)

$$\sigma_x(\tau) = \sqrt{0.225}\tau \cdot \sigma_y(\tau) = 0.47434\tau \cdot \sigma_y(\tau)$$

- Then ADEV corresponding to TDEV of Annex B, B.1.3.2, for observation intervals $0.05 \text{ s} \le \tau \le 10 \text{ s}$ is

$$\sigma_y(\tau) = \frac{\sigma_x(\tau)}{\sqrt{0.225}\tau} = \frac{5\tau \text{ ns}}{\sqrt{0.225}\tau} = 10.54 \text{ ns/s} = 1.054 \times 10^{-8}$$

- The above expression agrees with Table B.2 of Annex B
- Then PTPDEV corresponding to TDEV of Annex B, B.1.3.2, for observation intervals $0.05 \text{ s} \le \tau \le 10 \text{ s}$ is

$$\sigma_{PTP}(\tau) = \frac{\tau}{\sqrt{3}}\sigma_y(\tau) = \frac{1.054 \times 10^{-8}\tau}{\sqrt{3}} \text{s} = 6.08 \times 10^{-9}\tau \,\text{s} = 6.08\tau \text{ ns}$$

Review of PTPDEV value at τ = 0.125 s - 7

- Then PTPDEV corresponding to TDEV of Annex B, B.1.3.2, for observation intervals $0.05\ \mathrm{s} \le \tau \le 10\ \mathrm{s}$ is

$$\sigma_{PTP}(\tau) = \frac{\tau}{\sqrt{3}}\sigma_y(\tau) = \frac{1.054\times 10^{-8}\tau}{\sqrt{3}}\,\mathrm{s} = 6.08\times 10^{-9}\tau\,\mathrm{s}$$
$$= 6.08\tau\ \mathrm{ns}$$

- The above expression agrees with Table B.3 of Annex B
  - This expression is used on slide 3 to obtain PTPDEV at τ = 0.125 s

$$\sigma_{PTP}(0.125\,\mathrm{s}) = (6.08\times 10^{-9})(0.125\,\mathrm{s}) = 7.6\times 10^{-10}\ \mathrm{s}$$

Calculation of offsetScaledLogVariance – 1

- IEEE 1588 – 2008 describes the representation of PTP variance (i.e., the square of PTPDEV) in subclause 7.6.3.3. The procedure for obtaining the representation, which is the value of offsetScaledLogVariance, is (the following is preproduced from IEEE 1588 – 2008):

7.6.3.3 Variance representation

PTP variances shall be represented as follows:

a) An estimate of the variance σ^2_{PTP} specified in 7.6.3.2 is computed in units of seconds squared.

b) The logarithm to the base 2 of this estimate is computed. The computation of the logarithm need not be more precise than the precision of the estimate of the variance.

c) The logarithm is multiplied by 2^8 to produce a scaled value.

Calculation of offsetScaledLogVariance - 2

d) This scaled value is modified per the hysteresis specification of this subclause to produce the reported value.

e) The reported value is represented as a 2's complement Integer16. The value 8000_{16} is added to the reported value represented in this form, and any overflow is ignored. The result, i.e., the offset scaled reported value, is cast as a UInteger16.

f) This offset scaled reported value, represented as UInteger16, shall be the value of the log variances specified in 7.6.3.1.

NOTE 1— For example, suppose the PTP variance value is $1.414 \times 2^{-73} = 1.497 \times 10^{-22}$ s^2. Therefore, $\log_2(1.414 \times 2^{-73}) = -73 + 0.5 = -72.5$. If this were expressed as an Integer16, it would truncate to −72. To retain some precision, the value is scaled by 2^8 to yield a scaledLogVariance of −18560, i.e. $B780_{16}$, which retains 8 bits more precision. To this is added 8000_{16} to yield the offset scaled reported value 3780_{16}.

Calculation of offsetScaledLogVariance - 3

NOTE 2— The smallest variance that can be represented is 2^{-128} or ~3 × 10^{-39} s^2, which results in an offsetScaledLogVariance of 0000_{16}. The maximum variance that can be represented is ~$2^{+127.99609}$, which results in an offsetScaledLogVariance of $FFFF_{16}$.

- NOTE 3— This representation ensures that the ordering of variances algorithm of 7.6.3.4 produces identical results in all implementations. This result cannot be guaranteed with a floating-point representation.

Calculation of offsetScaledLogVariance - 4

- To calculate offsetScaledLogVariance that corresponds to PTPDEV of 7.6×10^{-10} s, we follow the above procedure
- First, PTP variance is the square of PTPDEV, or
  - $\sigma^2_{PTP} = (7.6 \times 10^{-10}\ \mathrm{s})^2 = 5.776 \times 10^{-19}\ \mathrm{s}^2$
- The logarithm to base 2 of the above is
  - $\log_2(\sigma^2_{PTP}) = [\log_{10}(5.776 \times 10^{-19})] / \log_{10}2$

$$= -60.58656306$$

- Multiplying the above logarithm to base 2 by 2^8 produces
  - $(2^8)(-60.58656306) = -15510.1601435$
- The above result is truncated to

-15510

Calculation of offsetScaledLogVariance - 5

- Next, the above result must be written in 2s complement form. To do this, we first write the result as a signed integer in base 16, and then convert to 2s complement form

$$-15510 = -(3 \times 4096 + 12 \times 256 + 9 \times 16 + 6)$$

$$= -3C96_{16}$$

- The result is written in 2s complement form by complementing (i.e., replacing 0 by 1 and 1 by 0 in the binary representation) the absolute value of the above and adding 1. The result is

$$C369_{16} + 1_{16} = C36A_{16}$$

Calculation of offsetScaledLogVariance - 6

- We next add 8000_{16} to the above 2s complement representation, and ignore any overflow. The result is

$$C36A_{16} + 8000_{16} = 1436A_{16}$$

- Ignoring the overflow produces the result for offsetScaledLogVariance

$$436A_{16} = 17258_{10}$$

This value is proposed for the corrected value for 802.1AS – 2011, subclause 8624(b), for the planned corrigendum

References

[1] Lee Cosart and Geoffrey M. Garner, Wander TDEV Measurements for Inexpensive Oscillator, IEEE 802.1 AVB TG presentation, November 2, 2009.

[2] ITU-T Recommendation G.810*, Definitions and terminology for synchronization networks*, ITU-T, Geneva, August, 196.

[3] Stefano Bregni, *Synchronization of Digital Telecommunications Networks*, Wiley, 2002.

- [4] IEEE Std 1588TM – 2008, *IEEE Standard for a Precision Clock Synchronization Protocol for Networked Measurement and Control Systems*, IEEE, 24 July 2008.