Undergraduate Research Opportunity Programme in Science

THE LENGTH OF VERMEER'S STUDIO

Chew Mei Ru Madeleine

Lee Yiwei Christina

Supervisor: Associate Professor Helmer Aslaksen

Department of Mathematics

National University of Singapore

Academic Year 2008/2009

Acknowledgement

We thank our project supervisor A/P Helmer Aslaksen for his guidance, invaluable advice and probing questions, which have contributed significantly to our more in-depth appreciation of perspective geometry in art. We are grateful for his patience and sense of humour, which have inspired us to complete this project with passion and academic rigour.

<u>CONTENTS</u>

Abstract

The purpose of this project is to verify that the artist Johannes Vermeer painted *The Music Lesson* and *A Lady Standing at a Virginal* in the same room. The main objective is to elucidate the methods used by Tomas Garcia-Salgado by discussing the numerical values mentioned in his paper and comparing the calculations with the findings in Aditya Liviandi's thesis.

1 Introduction

Johannes Vermeer's alleged use of the camera obscura in the creation of his artwork has been a controversial topic discussed for many years. Philip Steadman, in *Vermeer's Camera*, provides evidence that Vermeer used the camera obscura to create his paintings. Steadman uses the inverse perspective method to reconstruct the studio using the reflection of the mirror in the *The Music Lesson* (ML) [1]. Steadman also argues that Vermeer painted at least six paintings in the same room. Based on the findings of Steadman, Tomas Garcia-Salgado attempts to use his Modular Perspective method [2] to confirm and extend the findings of Steadman, that the room depicted in Vermeer's paintings are the same. Unfortunately, many of the statements made by Garcia-Salgado are not explained fully. In his thesis *Reconstruction of Vermeer's The Music Room*, Aditya Liviandi attempts to reconstruct Vermeer's studio using a method that is different from both Steadman and Garcia-Salgado [3]. This paper aims to elucidate the methods used by Garcia-Salgado in calculating the numerical values mentioned in his paper *Modular Perspective and Vermeer's Room*, and compare the findings of Garcia-Salgado with that of Liviandi.

2 Projective Geometry in Paintings

To understand the geometry behind Vermeer's works, we first need to understand the concepts of projective geometry used in paintings. Projective geometry refers to the field of Mathematics which assigns points on a two-dimensional image to points in three-dimensional real-world space and vice versa [4]. A painting can be regarded as a two-dimensional image. It has been speculated that Vermeer's paintings were created using a camera. Hence, if Vermeer had used a camera to create his painting, the observer point (*O*) is the position of the camera lens when a photograph of the scene is captured. Otherwise, the observer point is the eye of the painter when the painting is being created.

The depiction of objects in a painting is constructed by linear perspective, which is a mathematical representation of three-dimensional objects on a two-dimensional surface as viewed by the observer [5]. Using this method, any set of parallel lines in three-dimensional space, which are depicted in a painting and are not parallel to the plane of the painting, will in its two-dimensional representation converge at a point. This point is called a vanishing point (*vp*). When the set of parallel lines in three-dimensional space are orthogonal to the plane of the painting, the lines are called orthogonals, and the point at which they converge is called the central vanishing point (*cvp*).

In Vermeer's *The Music Lesson* [6] as shown in Fig. 1, there are three vanishing points formed by three sets of parallel lines along the edges and diagonals of the floor tiles. The point at which a set of lines along the tile edges converges to the left of the painting is called the left vanishing point (*lvp*). Similarly, the point at which a set of lines along the tile edges converges to the right of the painting is called the right vanishing point (*rvp*). We denote the left and right vanishing points as side vanishing points. It appears that the set of parallel lines along the tile diagonals converge orthogonally into the painting. If this is true, the left and right vanishing points will be equidistant from the point that this set of lines converges to, and this point of convergence will be the central vanishing point (*cvp*). However, it will be shown later in this paper that this vanishing point is not the central vanishing point. To elucidate the concepts in his papers, we first assume, as Garcia-Salgado did in his paper *Modular Perspective and Vermeer's Room* [2], that the vanishing point formed by the lines along the tile diagonals is the central vanishing point in this paper. Additionally, since the sets of parallel lines lie on the horizontal floor in three-dimensional space, all three aforementioned vanishing points lie on the horizon.

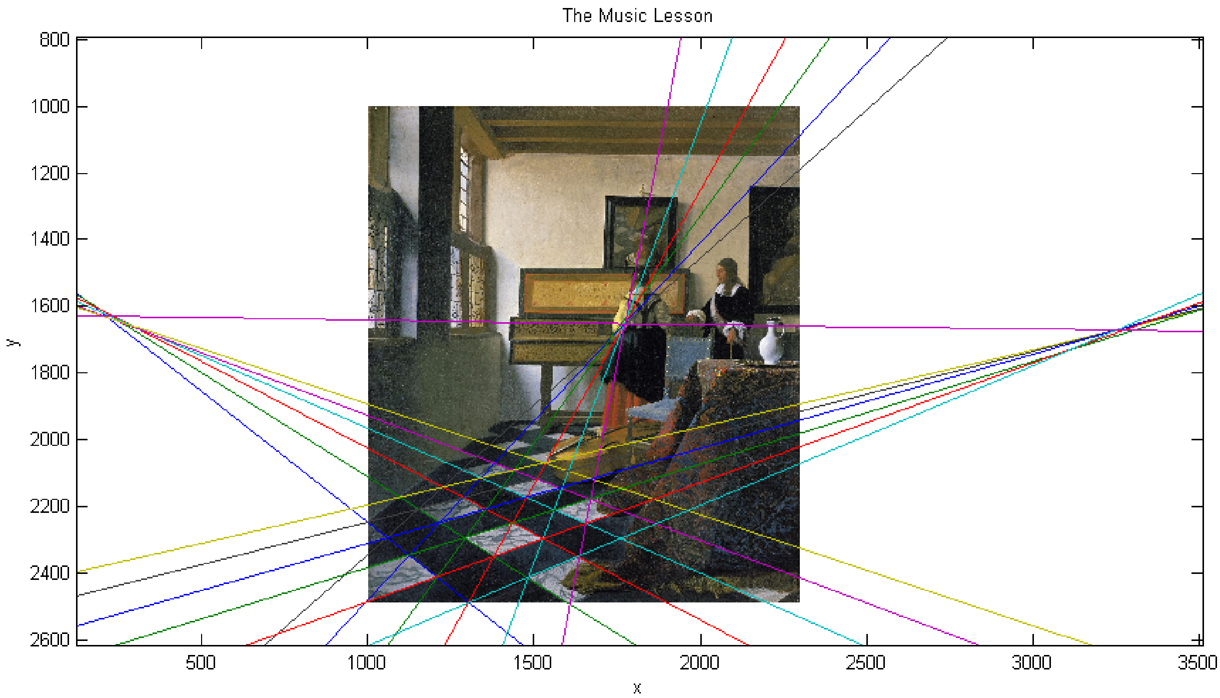

Fig. 1 Left, central and right vanishing points indicated on *The Music Lesson*

Assuming that the set of parallel lines along the tile diagonals converge orthogonally into *The Music Lesson* at the central vanishing point, the angle subtended by lines drawn from either of the side vanishing points and the central vanishing point to the observer would be 45° as depicted in Fig. 2. In the figure, the positions of the central vanishing point (*cvp*) and the right vanishing point (*rvp*) on the top view of the painting are labelled as points *A* and *B* respectively.

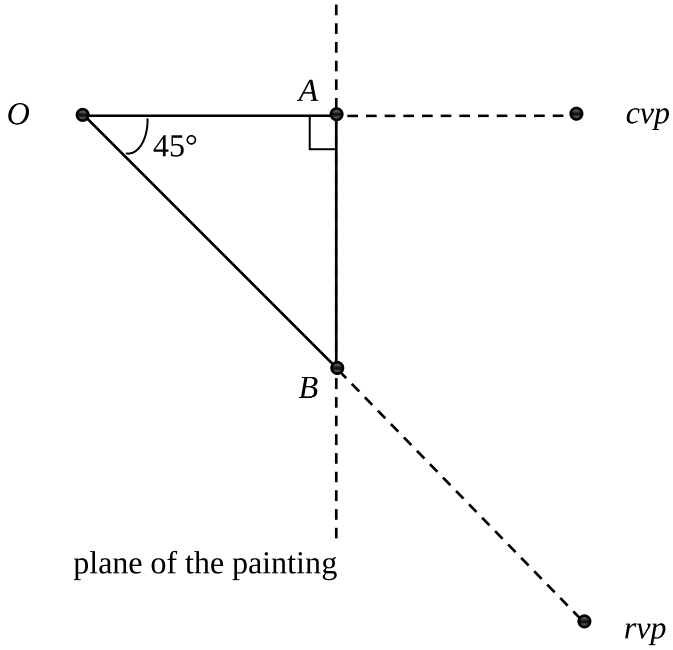

Fig. 2 Top view of *The Music Lesson* depicting the positions of the observer, central vanishing point, right vanishing point and the horizontal positions of the two vanishing points on the plane of the painting

Since the triangle formed by joining the points *O*, *A* and *B* is an isosceles right triangle, the length of *OA* is equal to the length of *AB*. This implies that the perpendicular distance of the observer from the plane of the painting is equal to the distance between the positions of the central and right vanishing points on the painting. This is also true when we replace the right vanishing point with the left vanishing point. Therefore, we have the formula: perpendicular distance of the observer from the plane of the painting = *cvp*-*lvp* = *cvp*-*rvp*, where *A*-*B* indicates the distance between any two points *A* and *B*.

3 The Possible Use of the Camera Obscura in the Works of Johannes Vermeer

The camera obscura is a device which includes a lens or a pinhole, with which an image of a scene viewed by an observer can be projected onto a screen. The projected image on the screen can then be traced. The camera obscura is the precursor to the photographic camera.

Many people have speculated that the Dutch artist Johannes Vermeer (1632-75) used the camera obscura to help him create his paintings [6]. The speculations on Vermeer's possible use of a camera obscura are based on the general observations about his paintings.

Firstly, Vermeer's paintings have a "photographic perspective" as described by the American etcher and lithographer James Pennell [6]. Pennell highlighted the disproportionately large figure of the soldier in the foreground of *Soldier and the Laughing Girl*. The perspective of this painting seems like one taken with a camera, which is an unusual perspective for a 17th century painting. Pennell was the first to suggest that Vermeer used the camera obscura.

Secondly, the maps in some of Vermeer's paintings are precisely copied from the original maps, a piece of evidence presented by the historian James Welu [6]. The maps hung on the walls in Vermeer's paintings are real maps which still exist today, and the camera obscura was used in the 18th and 19th centuries for copying existing prints such as maps.

Thirdly, Vermeer's treatment of highlights on reflective surfaces suggests that he uses the camera obscura. Metal and ceramics in Vermeer's paintings show small circles of white or yellow pigment, which are suggested to be the "circles of confusion" seen when we view bright highlights through a low quality or out-of-focus lens [6].

Finally, the British artist Lawrence Gowing described in his monograph on Vermeer that the pattern of light and shade of the subject in Vermeer's paintings are transcribed with little of the underlying drawing, which most artists will use to build up a representation [6]. Gowing argues that Vermeer's pattern of light and shade of the subject is attributed to a technique which is based on prolonged observation of patterns of light which fall on a camera screen. The speculations remained without any actual evidence that Vermeer had indeed used the camera obscura in creating his paintings.

Besides the speculations made based on the general observations of Vermeer's paintings, the availability of the camera obscura to Vermeer also suggests Vermeer's possible use of the camera obscura in his paintings [6]. Holland was a centre for the manufacture of high quality optical instruments in the 17th century. Furthermore, there were books like della Porta's *Magia Naturalis* (1558), which described the camera obscura and its possible use in painting, circulating in Holland in the 1600s. The camera obscura was also used by astronomers, such as Johannes Kepler, who made detailed studies of sunspots in the early 1600s. Therefore, the camera obscura was known in Holland during Vermeer's lifetime. However, again, there is no documentary evidence that Vermeer owned a camera obscura device or was familiar with it.

In *Vermeer's Camera* [1], Philip Steadman, Professor of Architectural and Urban Morphology at the Open University in United Kingdom, presents an analysis of the perspective geometry of Vermeer's paintings. The analysis provides evidence for Vermeer's use of the camera obscura.

Steadman argues that most of Vermeer's paintings were painted in the same room and Vermeer constructed his perspective views with a high precision that the shape and dimensions of the room can be measured to a high degree of accuracy. Steadman was able to determine the observer point for at least six of Vermeer's paintings. The scene visible in the painting must be contained within a visual pyramid where the apex is the observer point. We shall explore the concept of the visual pyramid in Section 4 of this paper. The visual pyramid can be extended through the observer point to the back wall. The intersection of the visual pyramid and the back wall, determined from the mirror reflection in *The Music Lesson*, forms a rectangular area on the back wall. For at least six of Vermeer's paintings, the sizes of the rectangular areas are almost exactly the same as the dimensions of the actual paintings.

Therefore, Steadman proposes that Vermeer used the camera obscura with the lens at the observer point to project the scene onto the back wall and traced the image. As a result, the projected image would be of the same size as the actual painting.

4 Key Terms and Concepts in the Papers of Tomas Garcia-Salgado

Having explored the foundational concepts of projective geometry in paintings or pictures as well as the background of Vermeer's works, we now turn to the works of Tomas Garcia-Salgado, a researcher in the Faculty of Architecture of the Autonomous National University of Mexico. In his paper, *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado attempts to prove that Vermeer painted *A Lady Standing at a Virginal* and other of his paintings in the same room as that for *The Music Lesson*. In this paper, we will focus on Garcia-Salgado's proof for the paintings *A Lady Standing at a Virginal* and *The Music Lesson*, which constitute Section 1, Section 2 and Section 3 of his paper [2]. Before we explore these three sections, we will introduce some of the key terms used in his paper.

The visual angle of a viewed object is the angle subtended by two visual rays drawn from two extreme points of the object to the observer. There are two visual angles for the picture – the horizontal visual angle and the vertical visual angle. When the viewed object is the picture, the two extreme points for the horizontal visual angle are any pair of points on each of the left and right edges of the picture that lie on the same horizontal line on the picture. Likewise, the two extreme points for the vertical visual angle are any pair of points on each of the top and bottom edges of the picture that lie on the same vertical line on the picture. From Section 2 of this paper, we know that the perpendicular distance of the observer from the plane of a picture is equal to the distance between a side vanishing point and the central vanishing point as measured on the plane of the picture. Using this equation, we may obtain the distance between the observer and a picture. Subsequently, the visual angles for the picture can be obtained when the picture is placed at this measured distance in front of the observer.

The visual pyramid is the set of all visual rays of an observer viewing a scene. The apex of the visual pyramid is the observer. In this paper, the base of the visual pyramid is a four-sided polygon which is determined by the horizontal and vertical visual angles of the picture.

In Chapter 6 of *Vermeer's Camera* [1], Steadman calculates the position of the back wall using the mirror reflection in *The Music Lesson* (ML). Steadman hypothesizes that Vermeer painted at least six of his paintings in the same room, implying that the back wall position of the paintings are the same as the back wall position of ML. Steadman then proceeds to verify his hypothesis. The verification begins with an extension of the visual lines in the visual pyramid through the observer point to the back wall. The intersection of the visual pyramid with the back wall forms a rectangular area on the back wall. The plane view of this construction for six pictures, including ML is shown in Fig. 3 below, which is extracted from Fig. 49 in Chapter 6 of Steadman's book, *Vermeer's Camera* [1]. For all six pictures, the sizes of the rectangles constructed by Steadman are almost exactly the same size as the actual dimensions of the corresponding paintings. From this construction and comparison, Steadman verifies that his hypothesis - the six paintings are painted in the same room as ML. Hence, Steadman suggests that Vermeer used a camera obscura to project the scene of the room onto the back wall and traced the projected image.

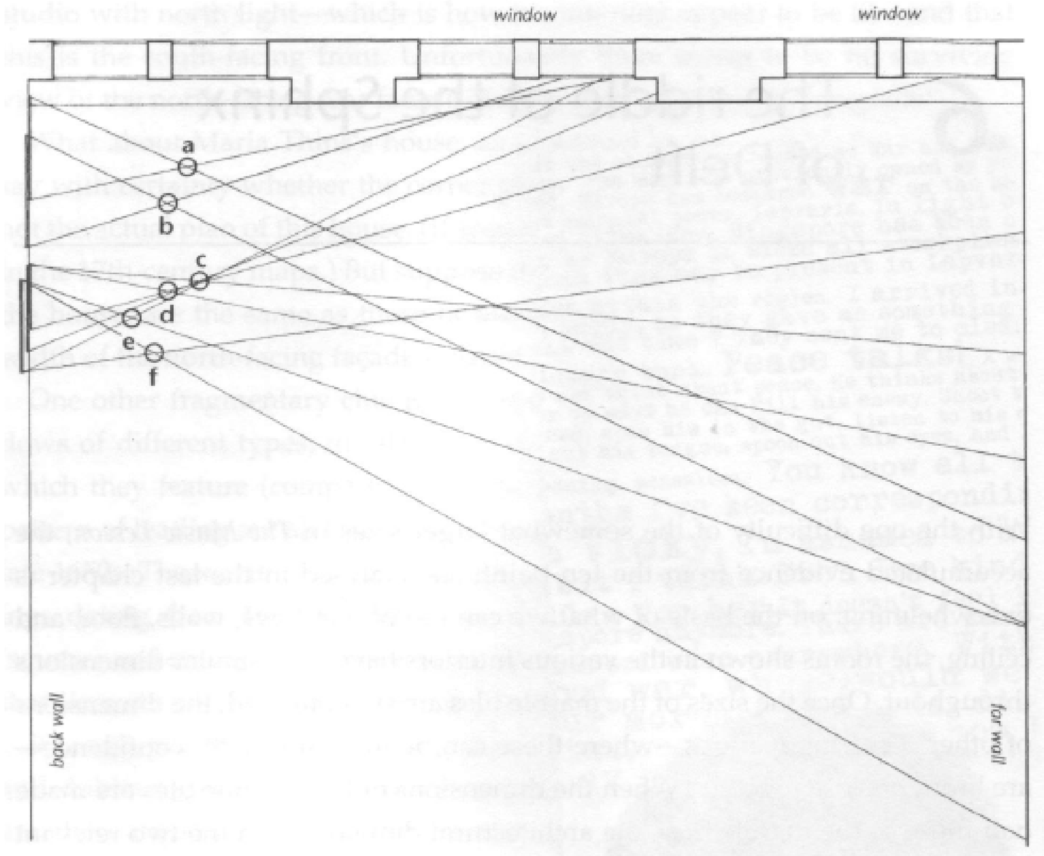

Fig. 3 Reconstructed plan of the room, showing the intersection of the back wall with the visual pyramids of six of Vermeer's paintings: (a) *The Girl with a Wineglass*, (b) *The Glass of Wine*, (c) *Lady Writing a Letter, with her Maid*, (d) *A Lady Standing at a Virginal*, (e) *The Music Lesson*, and (f) *The Concert*

Garcia-Salgado makes references to the results obtained by Steadman explored above. Garcia-Salgado assumes that the rooms in which Vermeer painted share the same back wall as ML. To verify this, he finds the projected images of the paintings on the hypothetical back wall by intersecting the visual pyramids corresponding to Vermeer's paintings with the back wall. In *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado further calculates the dimensions of these projected images and finds them to be acceptably close to the actual dimensions of the paintings.

There is no concrete evidence for us to ascertain that the actual back wall of the room is the back wall position on which the images of the pictures were projected. It is possible for the reflected object assumed as the back wall in the mirror of ML to be an additional vertical object placed at that position rather than the actual back wall of the room. For example, the vertical object could be one of the walls of the camera obscura cubicle used by Vermeer. However, to shed light on Garcia-Salgado's papers, we will assume in this paper as he did that the vertical object where the images of the pictures are projected on is the actual back wall of the room.

We will now discuss the method of Modular Perspective used by Garcia-Salgado. Garcia-Salgado introduces the term "module" to refer to an object at a certain depth in the picture, which can be regarded as a unit of measurement for lengths of other objects at the same depth [2]. In his papers, the module is taken to be the tile diagonal length at the depth at which the length of an object is measured. For example, to measure the height of the man in ML, the module is taken to be the tile diagonal at the depth of the man.

An arbitrary image of a painting can be shifted along the visual pyramid. The length of the tile diagonal at the bottom edge of a painting is denoted by n. We introduce the term Perspective Plane (*PP*) to refer to the picture which is shifted along the line of sight and can be positioned at any arbitrary position within the visual pyramid. As *PP* is shifted within the visual pyramid, it is scaled proportionately to fit exactly into the initial visual pyramid at all positions along the line of sight and the visual pyramid does not change. The size of *PP* increases as the image is shifted further away from the observer and vice versa. The absolute length of *n* changes by the same scaling factor by which the image size changes. Consequently, the absolute distance between the central and side vanishing points also changes by the same scaling factor by which the image size changes. As a result, the number of modules between the central and side vanishing points remains the same when *PP* is shifted along the line of sight in the visual pyramid.

In contrast with Garcia-Salgado's notation, we denote the Perspective Plane in the visual pyramid that touches the floor as the Perspective Plane on the floor (*PPf*), whereas Garcia-Salgado denotes it by *PP* [2]. This is done to differentiate *PPf*, which touches the floor in three-dimensional real-world space, from the previously defined Perspective Plane (*PP*), which can be arbitrarily positioned along the line of sight in the visual pyramid. The perpendicular distance between *PPf* and the observer point, *O*, is denoted by *O-PPf*. It is noteworthy to highlight that the bottom edge of *PPf* touches the floor of the room. As a result, the tile diagonal at the bottom edge of *PPf* is at the same depth in the visual pyramid as *PPf*. Hence, the length of the tile diagonal at the bottom edge of *PPf* as seen from the observer position is equal to the actual length of the tile diagonal in real-world space when viewed at a distance *O-PPf*.

Garcia-Salgado assumes that Vermeer's paintings were created by projecting images of the scenes onto a back wall [2]. The pictures projected on the back wall can be obtained by shifting *PP* within the visual pyramid to the back wall position assumed by Garcia-Salgado. In addition, he introduces the term Perspective Plane of the back wall (*PPbw*) to refer to the picture that is projected onto the back wall [2].We will use the same notation, *PPbw*, for the Perspective Plane of the back wall in this paper.

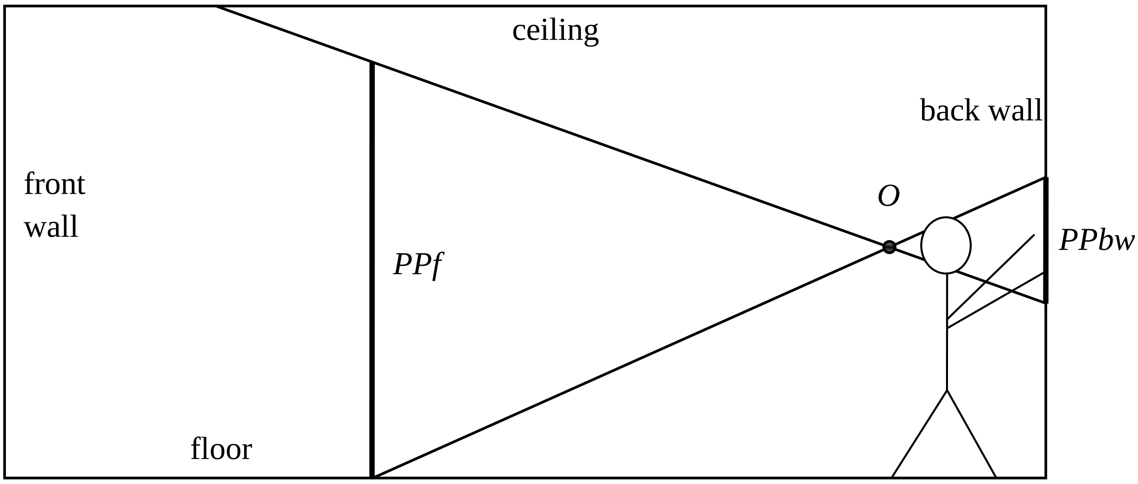

Fig. 4 Possible side view of a visual pyramid while Vermeer was painting in a room, with the observer point (*O*) as the position of the camera lens, in the case where Vermeer used the camera obscura

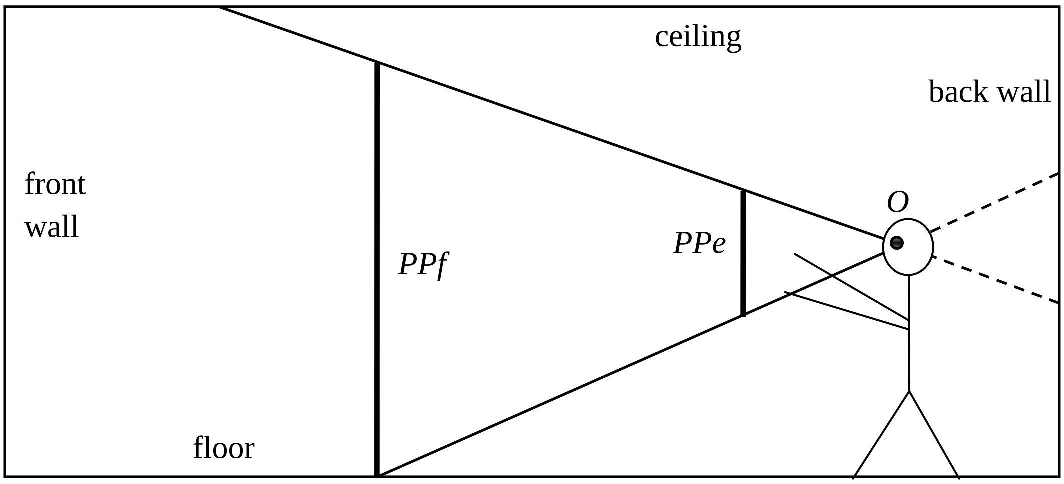

Fig. 5 Possible side view of a visual pyramid while Vermeer was painting in a room, with the observer point (*O*) as the position of his eye, in the case where Vermeer did not use the camera obscura

Fig. 4 and Fig. 5 depict how the side view of the visual pyramid may appear while Vermeer was painting in a room, in the cases where a camera obscura was used and was not used respectively. In Fig. 5, the Perspective Plane on the easel is denoted as *PPe*. The distance between the observer and the back wall is the same as the distance between the observer and the easel. In other words, *O-PPbw* = *O-PPe*. Since the distances are the same, we will follow Garcia-Salgado's convention of using *PPbw* to refer to the picture painted by Vermeer.

To find the perpendicular distance between the observer point and *PP* in the visual pyramid in terms of tile diagonals, we may make use of the fact that the central vanishing point on *PP*, a side vanishing point on *PP* and the observer point form a right isosceles triangle as illustrated in Fig. 6 below. For example, we consider the case where the *PP* of ML is shifted to the back wall in the visual pyramid. This means that we want to find the distance between the observer point and *PPbw* for ML. We first measure the distance between the central vanishing point, *A*, and the right vanishing point, *B*, on *PPbw*. From Fig. 6, *O-PP* = *A-B*, implying that the length *A-B* in terms of tile diagonals equates to the length *O-PP* in terms of tile diagonals, where *PP* is *PPbw* in this case. The tile diagonal at the bottom edge of *PPbw* is at the same depth as *PPbw* in the visual pyramid. Hence, we choose this tile diagonal to be the module in our measurement of *A-B* in terms of tile diagonals. By measuring the tile diagonal length of this tile and dividing the length *A-B* by the measured tile diagonal length, we find *A-B* in terms of tile diagonals. The perpendicular distance between the observer point and the *PPbw* of ML, *O-PPbw*, in terms of tile diagonals would be equal to the derived value of *A-B* in terms of tile diagonals. This concept will be used repeatedly in this paper.

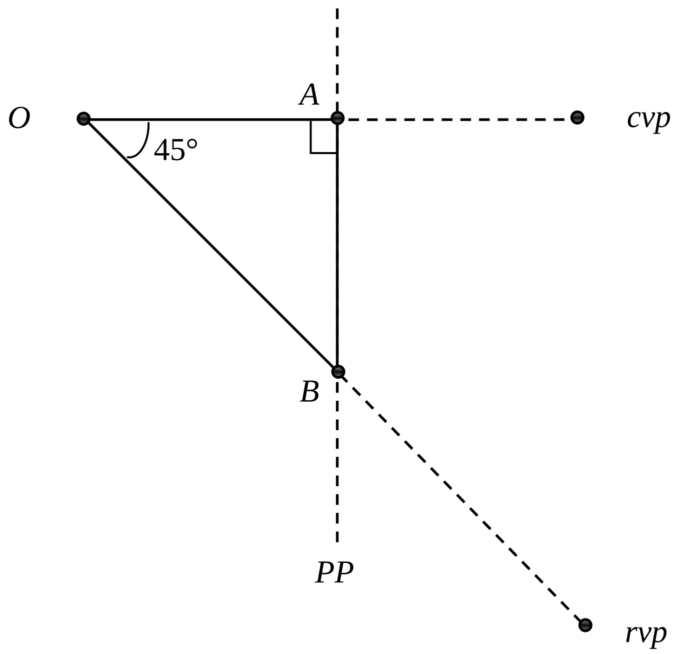

Fig. 6 Top view of the *PP* of a painting depicting the positions of the observer, central vanishing point, right vanishing point and the horizontal positions of the two vanishing points on *PP*

5 The Papers of Tomas Garcia-Salgado

Now that we have an understanding of the key terms and concepts used by Garcia-Salgado, we proceed to delve into Section 1, Section 2 and Section 3 of *Modular Perspective and Vermeer's Room* [2] by Garcia-Salgado. In the course of this, we will also examine some of the numerical values and calculations in *Some Perspective Considerations on Vermeer's The Music Lesson* [7] by Garcia-Salgado.

We will throw light on the numerical values that Garcia-Salgado calculates for distances related to *The Music Lesson* (ML) in Section 5.1 of this paper. Similarly, the numerical values related to *A Lady Standing at a Virginal* (LSV) will be explicated in Section 5.2 of this paper. In Section 5.3, the derivation of difference in the observer positions of ML and LSV and a side vanishing point of LSV will be made clear. Thereafter, we will move on to Section 5.4 to discuss the significance of trimming the Perspective Plane of the back wall (*PPbw*). Finally, Section 5.5, Section 5.6 and Section 5.7 will be dedicated to explore Garcia-Salgado's verification of the rooms depicted in ML and LSV to be the same.

5.1 *The Music Lesson*

In this section, we will focus on *The Music Lesson* (ML). We will explore Garcia-Salgado's derivation of the actual length of the tile diagonal depicted in ML in real-world space. Subsequently, we will explore how Garcia-Salgado calculates other distances depicted in ML in real-world space, such as the distance between the observer point and the Perspective Plane on the floor (*PPf*) of ML. Finally, we will look into the expression of the distances in terms of modules, where the modules used are the tile diagonals at the bottom edge of the *PPf* of ML and the Perspective Plane of the back wall (*PPbw*) of ML.

5.1.1 Length of the Module on the *PPf* of *The Music Lesson*

In *Some Perspective Considerations on Vermeer's The Music Lesson* [7] and *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado uses the term "module" to refer to the tile diagonal at the bottom edge of the Perspective Plane (*PP*) of *The Music Lesson* (ML), as viewed from the observer point. We denote the actual length of the tile diagonal, which is not affected by perspective and measured at the bottom edge of the arbitrary plane *PP* of ML, as m. Similarly, we denote the length of the tile diagonal at the bottom edge of the Perspective Plane on the floor (*PPf*) of ML and at the bottom edge of the Perspective Plane of the back wall (*PPbw*) of ML, as viewed by the observer, as m_0 and m_0' respectively. We also denote the actual length of the tile diagonal to be m_a and the actual length of the tile diagonal on the painting of ML to be m_a'. The notations introduced here will be used for the entire paper.

In Section 2 of *Some Perspective Considerations on Vermeer's The Music Lesson* [7], Garcia-Salgado makes the assumption that the actual height of the man depicted in ML is 180 cm. Taking measurements on the picture of ML, the ratio of the height of the man to the tile diagonal length at the same depth at which the man is depicted to be standing in the picture can be found to be 5.50 cm : 1.20 cm. This implies that the height of the man is approximately 4.6 times the length of a tile diagonal or $4.6m_a$, where m_a is the actual length of a tile diagonal in ML. Garcia-Salgado estimates this value to be $4.6m_a$ as well. Dividing 180 cm by 4.6, Garcia-Salgado finds the value of m_a to be 39.13 cm. Hence, the actual length of a tile diagonal in three-dimensional real-world space is estimated to be 39.13 cm.

It should be noted at this point that the measured values of 5.50 cm and 1.20 cm in this section should only be able to result in an estimated value of the actual tile diagonal length up to three significant figures. The ratio of the height of the man to the tile diagonal length at the same depth as the man is obtained by measurement on a picture of ML. This ratio consists of the measurements 5.50 cm and 1.20 cm, which are measured using a ruler with a precision of ±0.1 cm. The two measured values are accurate up to three significant figures. However, the estimated actual length of a tile diagonal is given to be 39.13 cm by Garcia-Salgado. This derived value has an accuracy of up to four significant figures, which cannot be derived from values of up to three significant figures. Hence, it is possible that the value of 39.13 cm was derived by Garcia-Salgado with the use of measurements with a precision of at least four significant figures.

5.1.2 Distances of the *PPf* and the *PPbw* of *The Music Lesson* From the Observer Point

In Section 2 of *Some Perspective Considerations on Vermeer's The Music Lesson* [7], Garcia-Salgado assumes that the left vanishing point and right vanishing point are equally spaced from the central vanishing point. It is further stated that he estimates the absolute distance between the central vanishing point and each of the side vanishing points on the Perspective Plane of the back wall (*PPbw*) of *The Music Lesson* (ML) to be 76 cm. As the side vanishing points are equidistant from the central vanishing point, the distance between the central vanishing point and side vanishing point on *PPbw* is also the distance between the observer point and *PPbw*.

Garcia-Salgado states that the distance between the observer point and the Perspective Plane on the floor (*PPf*) for ML is 196 cm in Section 2 of *Some Perspective Considerations on Vermeer's The Music Lesson* [7]. Garcia-Salgado could have calculated this value by using the ratio of the actual length of the tile diagonal at the bottom edge of *PPbw* to the distance between the central vanishing point (*cvp*) and a side vanishing point (*rvp* or *lvp*) on the *PPbw* of ML to calculate the distance between the observer point and *PPf*. The calculation steps Garcia-Salgado may have used to find this value are presented below.

Width of picture of ML = 1296 pixels = 64.5 cm [8]

Ratio of pixels to cm on the picture of ML = 1296 pixels : 64.5 cm

Actual length of tile diagonal at bottom edge of the picture of ML, m_a'

= 303.5 pixels

= 303.5 x (64.5/1296)

= 15.10 cm

cvp-rvp on picture of ML

= 76 cm (estimated by Garcia-Salgado)

Ratio of m_a' to *cvp-rvp* on picture of ML

= 15.10 cm : 76 cm

Actual length of tile diagonal in real-world space

= m_a = 39.13 cm

Since *O-PPf* = *cvp-rvp*, where *cvp-rvp* is measured in terms of the actual length of the tile diagonal at *PPf*,

$$O\text{-}PPf = \frac{76}{15.10} \times 39.13cm = 196.9cm$$

The derived value of *O-PPf*, 196.9 cm, is close to Garcia-Salgado's value of 196 cm.

5.1.3 Relative Positions of the *PPf*, the *PPbw* and the Front Wall of *The Music Lesson* in Terms of Modules

In this section, we will express the distances calculated previously in Section 5.1.2 of this paper in terms of modules. For the distances on the Perspective Plane of the back wall (*PPbw*) of ML viewed from the observer point, the module is taken to be the length of the tile diagonal on the bottom edge of *PPbw* of *The Music Lesson* (ML), m_0'. For the distances on the Perspective Plane on the floor (*PPf*) of ML viewed from the observer point, the module is taken to be the length of the tile diagonal on the bottom edge of *PPf* of ML, m_0.

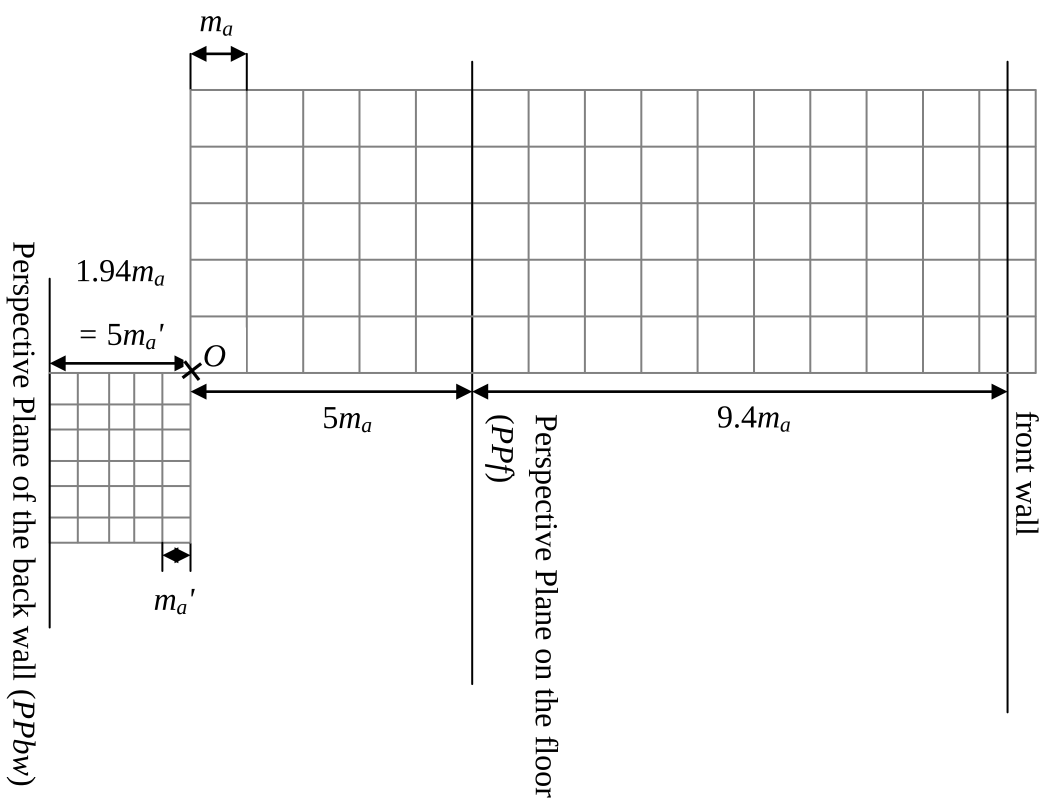

Fig. 7 Side view of the relative positions of *PPf*, *PPbw* and front wall from the observer point (*O*) in *The Music Lesson*

Fig. 7 above shows the side view of the relative positions of *PPf* and *PPbw* from the observer point (*O*) in ML. In Fig. 7, the modules determined by tiles in *PPf*, are represented by big grid units, m_a, and the modules determined by tiles in *PPbw* are represented by small grid units, m_a', where m_a and m_a' are the actual lengths of the tile diagonals in real-world space and on the painting of ML respectively. Fig. 7 is drawn based on Fig. 1a in Section 1 of *Modular Perspective and Vermeer's Room* [2] by Garcia-Salgado.

In Fig. 1a of *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado states that the distance between the observer and the *PPf* of ML is $5m_a$. $5m_a$ is also shown in Fig. 7 above. We can compute $5m_a$ as follows: In Section 5.1.2 of this paper, the distance between the observer point and the *PPf* of ML is calculated to be 196 cm. The actual length of the tile diagonal in *PPf* is the tile diagonal length in real-world space, m_a, which is 39.13 cm as computed in Section 5.1.1 of this paper. By dividing 196 cm by the actual length of the tile diagonal, 39.13 cm, the distance between the observer and *PPf* is found in terms of modules to be $5m_a$.

The number of modules between the central and side vanishing points remain the same regardless of the position of the Perspective Plane (*PP*) as it is shifted along the line of sight within the visual pyramid. Since the distance between the observer and *PPf* is $5m_a$ and the module for *PPbw* is represented by the tile diagonals of actual length m_a', the distance between the observer and *PPbw* is $5m_a'$ as shown in Fig. 7 above. From Section 5.1.2 of this paper, the distance between the observer and the picture is 76 cm. Therefore $5m_a'$ is 76 cm.

In Fig. 1a of *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado states that the distance between the observer point and *PPbw*, in terms of the module for *PPf*, is $1.94m_a$. Garcia-Salgado may have obtained the value of $1.94m_a$ in the following manner. The ratio of the actual length of the tile diagonal in real-world space to the actual length of the tile diagonal on the painting of ML is equal to the ratio of the distance between the observer point and *PPf* to the distance between the observer point and *PPbw*, which were obtained in Section 5.1.2. Using this ratio, we can express the distance between the observer and *PPbw*, $5m_a'$, in terms of m_a and this distance is found to be $1.94m_a$. Hence $5m_a'$ is equal to $1.94m_a$ as shown in Fig. 7. We shall illustrate the method of obtaining $1.94m_a$ described above in the computations below.

Distance between observer point and *PPbw* = 76 cm = $5m_a'$

Actual length of tile diagonal in real-world space = m_a

Actual length of tile diagonal on the painting = m_a'

Distance between observer point and PPf = 196 cm

Ratio of actual length of tile diagonal represented in *PPf* to actual length of tile diagonal represented in *PPbw*

$$= \frac{m_a}{m'_a} = \frac{196}{76}$$

$$\Rightarrow m'_a = \frac{76m_a}{196}$$

$$\Rightarrow 5m'_a = 5 \times \frac{76m_a}{196} = 1.94m_a$$

In Fig. 1a of *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado states that the distance between the *PP* of ML and the front wall is 9.4*m*. We propose that 9.4*m* can be estimated as follows:

We shall consider the *PP* of ML, where the module *m* is taken to be the tile diagonal length at the bottom edge of the *PP* of ML. In ML, we can count eight full tile diagonals along a visible column of ML. Therefore, in terms of the modules of *PP*, the total length of the eight full tile diagonals along a visible column of ML is 8*m*.

The tiles at the front wall in ML seem to be half tile diagonals by observation. Therefore, we shall assume that the visible length of the tile at the front wall in ML to be 0.5*m*. The tile at the bottom edge of ML is incomplete. By taking the fraction of the visible tile diagonal length over the full tile diagonal length for the tile at the bottom of the ML, the visible length of the tile diagonal at the bottom edge of ML is estimated to be 0.9*m* as shown in Fig. 8 below.

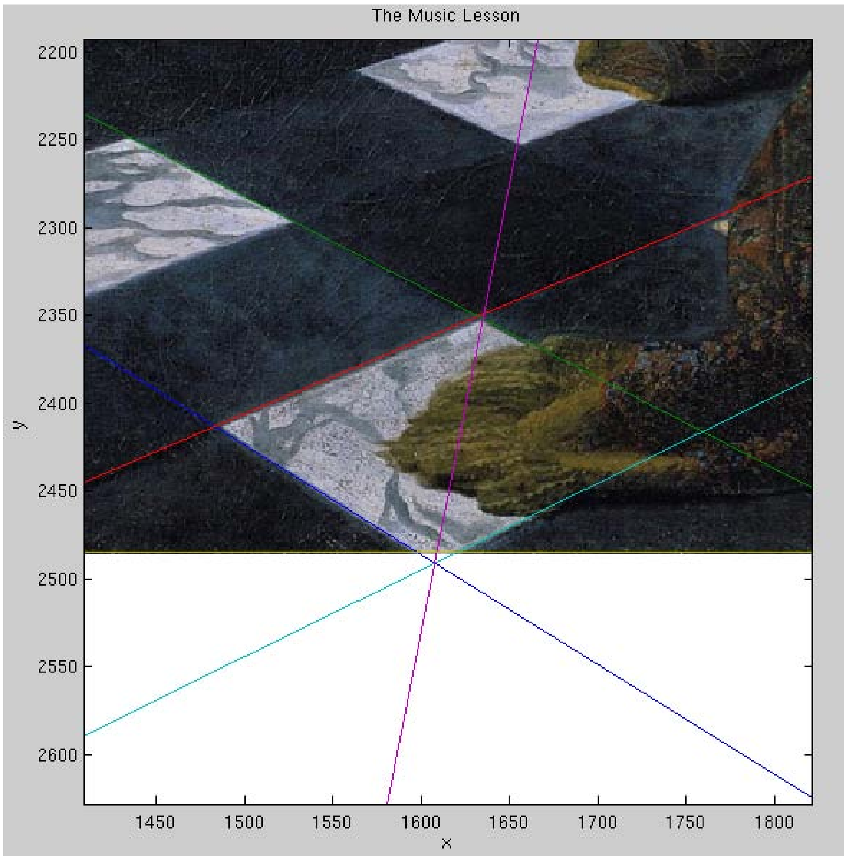

Fig. 8 View of fraction of visible tile diagonal for the tile at the bottom edge of *PP* in *The Music Lesson*

Therefore, adding up the total length of the eight full tile diagonals along a visible column of ML, 8*m*, the assumed visible length of the tile at the front wall in ML, 0.5*m*, and the estimated visible length of the tile at the bottom edge of ML, 0.9*m*, we obtain 9.4*m* as the total visible depth of the room. The total visible depth of the room is also the distance between the *PP* of ML and the front wall.

To find the distance between the *PPf* of ML and the front wall of ML, we express the visible depth of the room in terms of the actual tile diagonals, m_a. Therefore, the distance between *PPf* and the front wall is 9.4m_a as expressed in Fig. 7 above.

5.1.4 Distance between the Observer and the Front Wall and the Total Depth of the Room in *The Music Lesson*

In Section 3 of *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado calculates the distance between the observer point and the front wall in *The Music* Lesson (ML). The distance between the observer and the Perspective Plane on the floor (*PPf*) of ML is $5m_a$ and the distance between *PPf* and the front wall is $9.4m_a$. Therefore, adding up these two distances, we obtain the distance of the observer point from the front wall of ML as $14.4m_a$.

In Section 3 of *Some Perspective Considerations on Vermeer's The Music Lesson* [7], Garcia-Salgado states that the total depth of the room in ML is 640 cm. The value of 640 cm can be calculated in the following manner. The distance of the observer from the front wall is $14.4m_a$ as calculated in the previous paragraph. The distance between the observer and the Perspective Plane of the back wall (*PPbw*) of ML is $1.94m_a$ as calculated in Section 5.1.3. *PPbw* refers to the picture of ML which is projected onto the assumed back wall position. Hence, the distance of the observer from the back wall is the distance between the observer and *PPbw* of ML, which is $1.94m_a$. By summing the distance between the observer and the front wall, $14.4m_a$, and the distance between the observer and the back wall, $1.94m_a$, we obtain the total depth of the room, the distance between the back wall and the front wall, as $16.34m_a$. The tile diagonal length in real-world space, m_a, is 39.13 cm as calculated in Section 5.1.1. Multiplying 16.34 with the absolute length of m_a, 39.13 cm, the total depth of the room is calculated to be 640 cm.

In conclusion, for Section 5.1 of this paper, we have considered ML and discussed the computations to obtain the absolute distance of the tile diagonal and the following distances in terms of modules: the distance between the observer point and the *PPf* of ML, the distance between the observer point and the *PPbw* of ML, the visible depth of the room, the distance of the observer point from the front wall and the total depth of the room.

5.2 *A Lady Standing at a Virginal*

In this section, we will consider *A Lady Standing at a Virginal* (LSV). Similar to Section 5.1, we will explore how Garcia-Salgado obtains distances in terms of modules, where the modules used are the tile diagonals on the bottom edge of the Perspective Plane on the floor (*PPf*) of LSV and the Perspective Plane of the back wall (*PPbw*) of LSV.

Similar to *The Music Lesson* (ML), in *A Lady Standing at the Virginal* (LSV), Garcia-Salgado uses the term "module" to refer to the tile diagonal at the bottom edge of the Perspective Plane (*PP*) of LSV, as viewed from the observer point. We denote the actual arbitrary length, which is not affected by perspective and measured on the bottom edge of the *PP* of LSV, as l. We also denote the length of the tile diagonal length at the bottom edge of the *PPf* of LSV and at the bottom edge of the *PPbw* of LSV, as viewed by the observer, as l_0 and l_0' respectively. We also denote the actual length of the tile diagonal to be l_a and the actual length of the tile diagonal on the painting of LSV to be l_a'. The notations introduced here will be used for the entire paper.

5.2.1 Relative Positions of the *PPf*, the *PPbw* and the Front Wall of *A Lady Standing at a Virginal* in terms of Modules

Fig. 9 below shows the side view of the relative positions of the Perspective Plane on the floor (*PPf*), the Perspective Plane of the back wall (*PPbw*) and front wall from the observer point (*O*). Fig. 9 is drawn based on Fig. 1b in *Modular Perspective and Vermeer's Room* [2] by Garcia-Salgado.

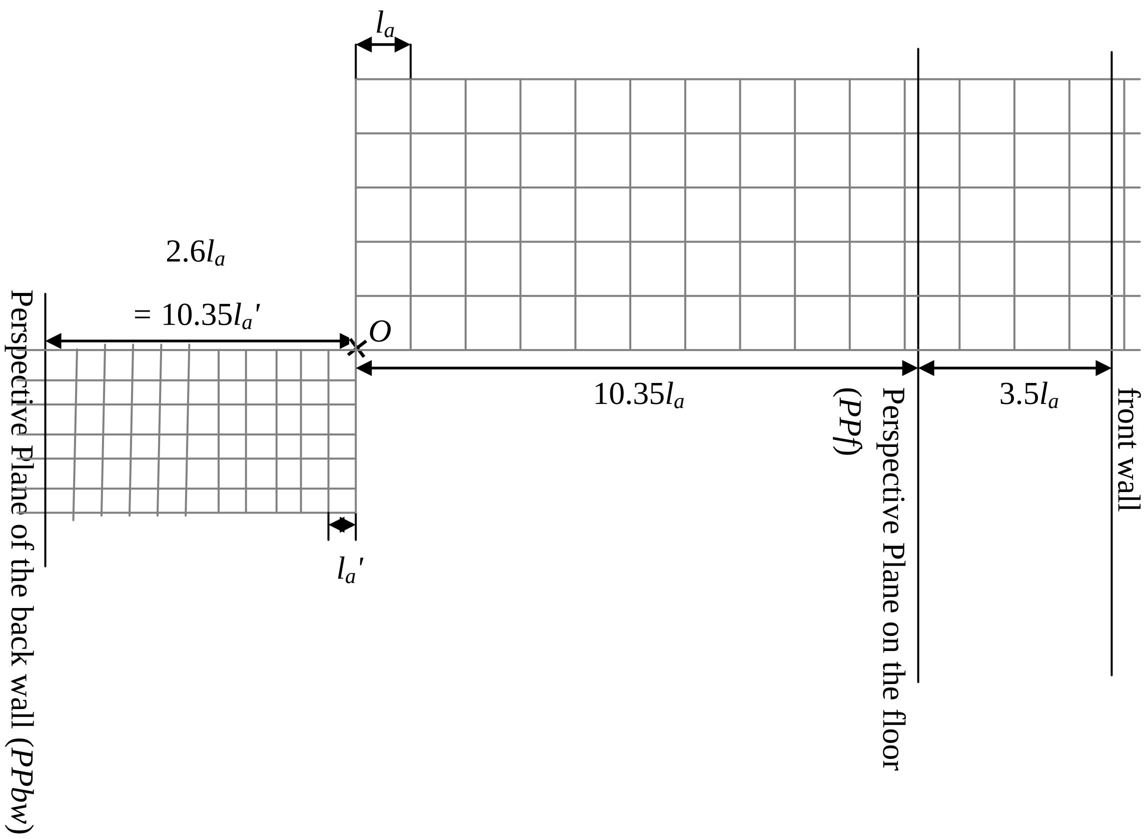

Fig. 9 Side view of the relative positions of the *PPf*, the *PPbw* and front wall from the observer point (*O*) in *A Lady Standing at a Virginal*

In Fig. 9, the modules determined by tiles in the *PPf* of *A Lady Standing at a Virginal* (LSV) are represented by big grid units l_a, and the modules determined by tiles in the *PPbw* of LSV are represented by small grid units l_a', where l_a and l_a' are the actual lengths of the tile diagonals in real-world space and on the painting of ML respectively.

The distances stated on Fig. 9 are taken from Fig. 1b of *Modular Perspective and Vermeer's Room* [2] by Garcia-Salgado. We suggest that the distances are obtained using the methods outlined in Sections 5.1 of this paper, which will be explained in the following paragraphs.

Garcia-Salgado most probably uses an object in LSV as a reference, assumes the actual length of the object in real-world space and estimates its length in terms of the actual length of a tile diagonal in LSV, l_a. Dividing the actual length of the object by its estimated length in l_a, the actual length of the tile diagonal in LSV is obtained. We will denote the actual length of the tile diagonal of LSV as q cm. In other words, $l_a = q$ cm.

The distance between the central vanishing point and a side vanishing point can be measured on LSV. The side vanishing points are assumed to be equidistant from the central vanishing point. Therefore, the distance between the observer point and the *PPbw* of LSV is also the distance between the central and side vanishing points. We denote the absolute distance between the observer point and the *PPbw* of LSV as r cm.

The length of the tile diagonal on the bottom edge of the picture of LSV can be measured and we denote this absolute length s cm.

The calculations required to obtain the values stated on Fig. 9 are shown below.

Width of picture of LSV = 1149 pixels = 45.2 cm [9]

Ratio of pixels to cm on the picture of LSV = 1149 pixels : 45.2 cm

Actual length of tile diagonal at bottom edge of the picture of LSV
= s cm (after conversion from pixels to cm using above stated ratio)

cvp-rvp on picture of LSV
= r cm (possibly estimated by Garcia-Salgado)

Ratio of l_a' to *cvp-rvp* on picture of LSV

= *s* cm : *r* cm

Actual length of the tile diagonal at *PPf*

= $l_a = q$ cm

Since *O-PPf* = *cvp-rvp*, where *cvp-rvp* is measured in terms of actual tile diagonals at *PPf*,

O-PPf = $\frac{r}{s} \times q = t$

O-PPf in terms of actual tile diagonals at *PPf* = $\frac{t}{q} = 10.35 l_a$

Distance between observer point and *PPbw* in terms of actual tile diagonals at *PPbw* = $10.35 l_a'$ because the distance between in terms of modules remains the same regardless of the position of the Perspective Plane (*PP*) of LSV along the line of sight in the visual pyramid of LSV.

Ratio of tile diagonal length on *PPbw* to tile diagonal length on *PPf*

= $l_a' : l_a$

= *s* cm : *q* cm

$$\frac{l_a}{l_a'} = \frac{q}{s} \Rightarrow l_a' = \frac{l_a s}{q}$$

$$\Rightarrow 10.35 l_a' = 10.35 \times \frac{l_a s}{q} = 2.6 l_a$$

Along the left most column of LSV, three full tile diagonals can be counted as the visible depth of the room. The tile diagonal at the bottom edge of LSV is observed to be about half of a full tile diagonal. Therefore, we assume that the tile diagonal at the bottom edge of LSV is $0.5l$.

Distance between *PP* and the front wall

= total visible depth of room

$= 3l + 0.5l$

$= 3.5l$

$\Rightarrow$ Distance between the *PPf* and front wall $= 3.5l_a$

5.2.2 Distance between the Observer and the Front Wall in *A Lady Standing at a Virginal*

In Section 3 of *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado calculates the distance between the observer point and the front wall in *A Lady Standing at a Virginal* (LSV). The calculation is the same as that in *The Music Lesson* (ML) as discussed in Section 5.1.4 of this paper. For LSV, the distance between the observer and the Perspective Plane on the floor (*PPf*) of LSV is $10.35l_a$ and the distance between *PPf* and the front wall is $3.5l_a$. Therefore, summing these distances, we obtain the distance of the observer point from the front wall of LSV as $13.85l_a$.

To summarise Sections 5.2, we have considered LSV and discussed the calculations to obtain the following distances in terms of modules stated on Fig 5.2: the distance between the observer point and the *PPf* of LSV, the distance between the observer point and the Perspective Plane of the back wall (*PPbw*), the visible depth of the room. We also discussed the calculation of the distance of the observer from the front wall.

5.3 Observer Points of *The Music Lesson* and *A Lady Standing at the Virginal* and Side Vanishing Point of *A Lady Standing at the Virginal*

In Sections 5.1 and 5.2 of this paper, we calculated distances from *The Music Lesson* (ML) and *A Lady Standing at the Virginal* (LSV). In this section, we will explore how Garcia-Salgado compares the distance between the observer points and the front wall for both ML and LSV, and calculates the position of the side vanishing point of LSV using the calculated distances of ML.

5.3.1 Comparing the Distance between the Observer Point and the Front Wall for *The Music Lesson* and *A Lady Standing at a Virginal*

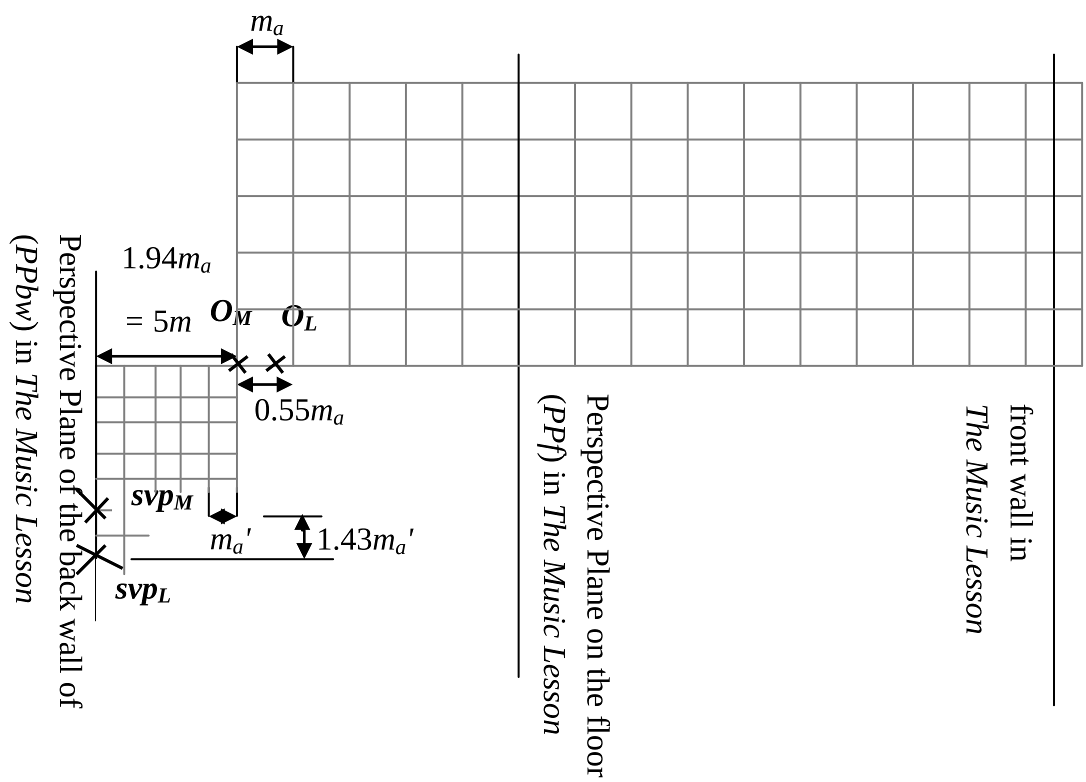

Fig. 10 Observer point of *The Music Lesson* (O_M), observer point of *A Lady Standing at a Virginal* (O_L), side vanishing point of *The Music Lesson* (svp_M) and side vanishing point of *A Lady Standing at a Virginal* (svp_L)

Fig. 10 is drawn based on Fig. 1a in *Modular Perspective and Vermeer's Room* [2] by Garcia-Salgado. The Perspective Plane on the floor (*PPf*), Perspective Plane of the back wall (*PPbw*) and the front wall for *The Music Lesson* (ML) are shown in Fig. 10. Also illustrated in the figure are the observer points of ML and *A Lady Standing at a Virginal* (LSV), together with the side vanishing points of ML and LSV, of which Garcia-Salgado has calculated the positions. We will discuss the derivation of the positions of the aforementioned items in this section.

In Section 2 of *Modular Perspective and Vermeer's Room* [2], Garcia-Salgado calculates the difference in the distances between the observer point and the front wall of ML and LSV. The distance between the observer point and front wall for ML is $14.4m_a$ as discussed in Section 5.1.4, and the distance between the observer point and the front wall for LSV is $13.85l_a$ as discussed in Section 5.1.6. We propose that Garcia-Salgado assumes the size of the floor tiles of the rooms depicted in both ML and LSV to be the same. This means that the actual lengths of tile diagonals depicted in both ML and LSV are equivalent or, in other words, m_a is equal to l_a. Hence, the distance between the observer point and the front wall of ML is still denoted as $14.4m_a$ while that for LSV can be expressed as $13.85m_a$. The difference between the two distances is $0.55m_a$.

Previously in Section 5.1.3, we obtained the relation between the tile diagonal length at the bottom edge of the *PPf* of ML, m_a, and the tile diagonal length at the bottom edge of the *PPbw* of ML, m_a', where $1.94m_a = 5m_a'$. This relation is also illustrated in the small grid in Fig. 10. This gives us the following relation.

$$1.94m_a = 5m'_a \Rightarrow m_a = \frac{5}{1.94}m'_a = 2.6m'_a$$

Garcia-Salgado calculates the difference between the distance between the observer point and the front wall of ML and that of LSV to be $0.55m_a$ as discussed earlier in this section. Garcia-Salgado then reflects this difference on the *PPbw* as the amount of deviation of the side vanishing point of LSV from that of ML. He does so by using the relation between m_a and m_a' discussed in the previous paragraph. Multiplying $0.55m_a$ by the factor 2.6, the difference between the distance between the observer point and the front wall of ML and the distance between the observer point and the front wall of LSV, on *PPbw* is $1.43m_a'$, as shown in Fig. 10. The numerical calculations for this value are shown as follows.

$$m_a = 2.6m_a' \Rightarrow 0.55m_a = (0.55 \times 2.6)\, m_a' = 1.43m_a'$$

5.3.2 Finding the Side Vanishing Point in *A Lady Standing at the Virginal*

In Section 5.3.1 of this paper, the difference between the distance between the observer point and the front wall for *The Music Lesson* (ML) and that for *A Lady Standing at a Virginal* (LSV), on the Perspective Plane of the back wall (*PPbw*), is calculated to be $1.43m_a'$. In Section 5.1.3 of this paper, the distance between the observer point and the *PPbw* of ML was calculated to be $5m_a'$. The distance between the central vanishing point and side vanishing point on the *PPbw* of ML is equal to the distance between the observer point and the *PPbw* of ML, $5m_a'$. Therefore, by adding $1.43m_a'$ to $5m_a'$, Garcia-Salgado obtained $6.43m_a'$ as the distance between the central vanishing point and side vanishing point for LSV. We show the calculations below:

Distance between observer points of ML and LSV on *PPbw* = $1.43m_a'$

Distance between central and side vanishing points on the *PPbw* of ML = $5m_a'$

Distance between central and side vanishing points of LSV

$= 5m_a' + 1.43m_a' = 6.43m_a'$

5.4 Trimming the *PPbw*

In this section, we will explore the effects of trimming the Perspective Plane of the back wall (*PPbw*). By trimming, we mean that the picture is cropped all around the periphery of the picture. As an example, we will consider the *PPbw* of *The Music Lesson* (ML). As calculated in Section 5.1.3, the distance between the observer point and the *PPbw* of ML is $5.0m_a'$, where m_a' is the actual length of the tile diagonal measured at the bottom edge of the *PPbw* of ML. The absolute distance between the observer point and the *PPbw* of ML is 76 cm. The number of visible tiles seen in the picture is 9.4 tiles. Throughout the process of trimming, the *PPbw* of ML is fixed to be at its initial position of distance *O-PPbw* from the observer point within the visual pyramid. Hence, the absolute distance between the observer point and the *PPbw* of ML remains at the value of 76 cm.

An example of trimming is illustrated in Fig. 11a and Fig. 11b below, where the former represents the *PPbw* of ML before trimming and the latter represents the *PPbw* after trimming. After trimming, the observer would view the tile diagonal at the bottom edge of the *PPbw* of ML to be shorter than the initial tile diagonal at the bottom edge. We denote the new length of the tile diagonal at the bottom edge of the *PPbw* of ML in Fig. 11b as m_1'. The absolute distance between the observer point and the *PPbw* of ML remains unchanged because the *PPbw* remains at the same depth within the visual pyramid. However, the tile diagonal chosen as the module has changed. In the visual pyramid, the depth of the new tile diagonal chosen as the module is greater than that of the initial tile diagonal chosen as the module. The distance between the central vanishing point and the right vanishing point, expressed in terms of the length of the new tile diagonal, would be the distance between the observer point and the plane positioned at the same depth as the new tile diagonal. This distance is equivalent to the distance between the central and right vanishing points in terms of the new module. Since the new tile diagonal is shorter than the initial one as seen by the observer, this distance increases in terms of the number of modules. Furthermore, when the *PPbw* of ML is trimmed, the number of visible tiles depicted in the *PPbw* of ML decreases. The number of visible tiles decreases at the same rate at which the distance between the observer point and the plane at the same depth as the new tile diagonal increases, in terms of the number of modules.

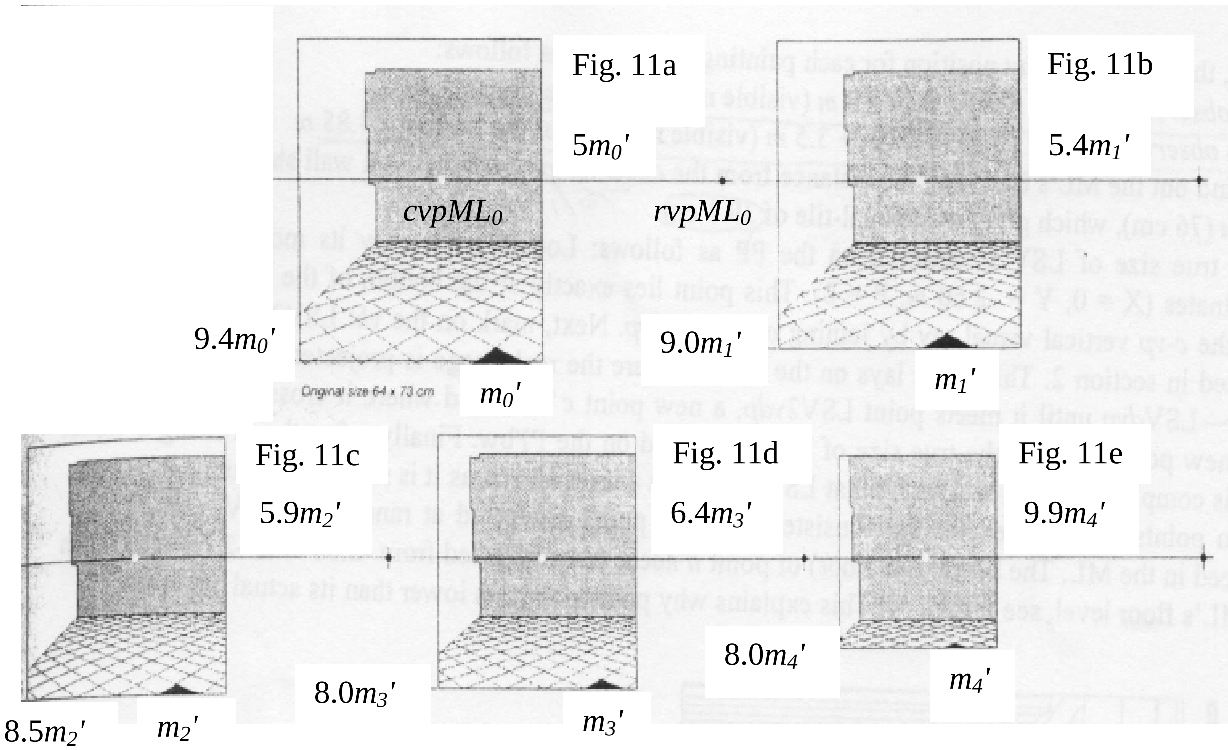

Fig. 11a-11e Trimming the Perspective Plane of the back wall (*PPbw*)

of *The Music Lesson*

From Fig. 11a to Fig. 11b shown above, the distance between the central vanishing point and the right vanishing point increases in terms of number of modules, from $5.0m_0'$ to

5.4*m_1'*. At the same time, the number of visible tiles decreases from 9.4*m_0'* to 9.0*m_1'*. In terms of the number of modules, the distance between the observer point and the front wall in the *PPbw* of ML remains constant at 14.4 modules. It is noted that Garcia-Salgado makes the assumption that the side vanishing points are at equal distances from the central vanishing point, and thus only depicts one side vanishing point for each *PPbw* in the figures. Three more stages of trimming are shown in Fig. 11c, Fig. 11d and Fig. 11e.

It may seem contentious that the absolute distance between the central and right vanishing points remains constant while the value of this distance changes with each trimming of the *PPbw*. However, if we distinguish the significance of these two lengths, we will see that there is no contention. When trimming occurs, the *PPbw* does not change in depth. This implies that the distance between the *PPbw* and the observer remains constant. In Fig. 11a, the four edges of the initial *PPbw* will touch the edges of the visually pyramid defined by itself. This indicates that the depth of the tile diagonal at the bottom of the initial *PPbw* is at the same depth as the *PPbw* in the visual pyramid. When we trim the *PPbw* and measure a new tile diagonal to find the length of the new module, the edges of the new *PPbw* will not touch the edges of the initial visual pyramid. Consequently, the tile at which the new measurement is taken will not be at the same depth as the initial tile diagonal taken as the module. In fact, this new tile diagonal would be at a greater depth or distance away from the observer than the new *PPbw*. Hence, when we use the new tile diagonal length to compute the distance between the *PPbw* and the observer in terms of tiles, we are in reality finding the distance between the plane, which has the same depth as the new tile diagonal, and the observer, rather than the *PPbw* which has not changed in depth. As a result, the absolute distance between the *PPbw* and the observer and the same distance in terms of number of tiles represent different lengths – the distance between the observer and the *PPbw*, and the distance between the observer and the plane at the same depth as the new tile diagonal measured as the new module after trimming. When we consider these two as separate distances, the contention does not exist.

5.5 The Music Lesson and A Lady Standing at a Virginal

In Section 3 of *Modular Perspective and Vermeer's Room*, Garcia-Salgado attempts to verify whether Vermeer's paintings *The Music Lesson* (ML) and *A Lady Standing at a Virginal* (LSV) were painted by the artist in the same studio. In this section, we will explicate the reasoning behind Fig. 3a, Fig. 3b and Fig. 3c in Section 3 of his paper. [2]

5.5.1 Overlaying ML and LSV at the Central Vanishing Point

To begin the verification, Garcia-Salgado depicts ML and LSV such that the central vanishing points (*cvp*) of both pictures are overlapped in Fig. 3a of his paper, as shown in Fig. 12 below. [2] This means that the lines of sight (*O-cvp*) of ML and LSV are overlaid such that their individual *O* positions need not be at the same point in three-dimensional real-world space. Viewing the outlines of these two images along the overlaid *O-cvp* lines, ML and LSV would be overlaid at their individual *cvp* positions as shown in Fig. 12.

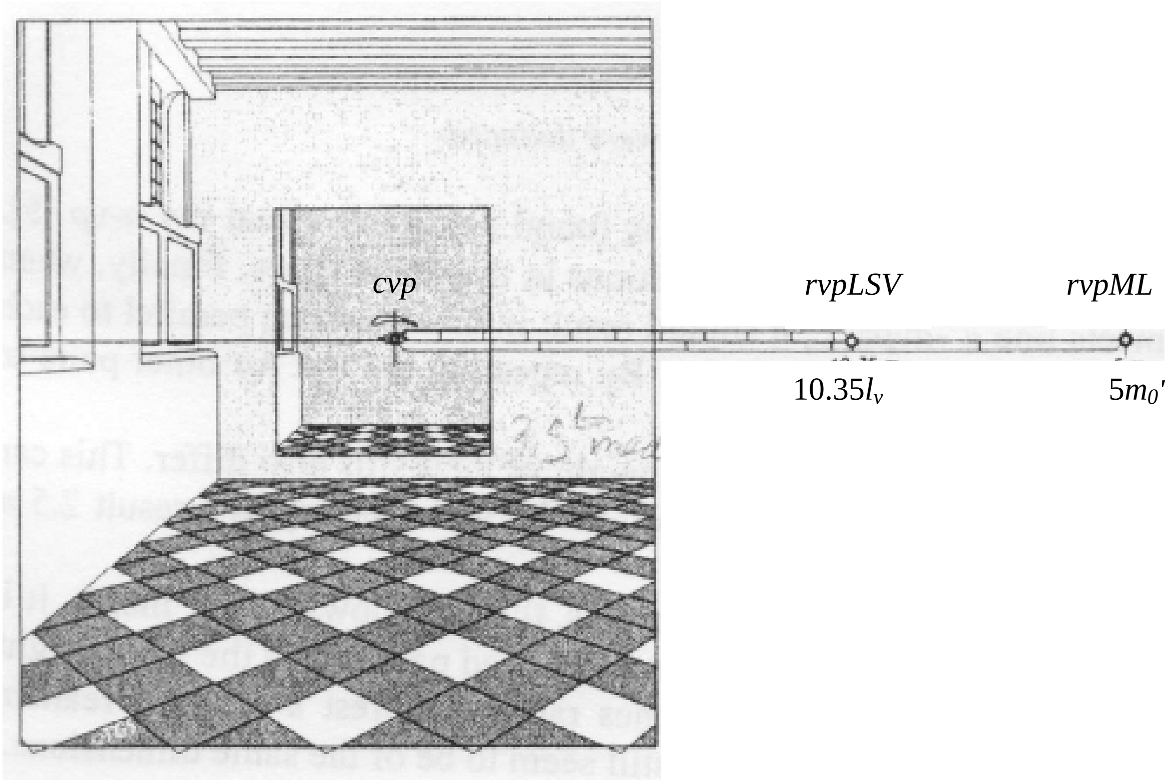

Fig. 12 **View of the *PPbw* of *The Music Lesson* and the *PP* of *A Lady Standing at a Virginal* along their overlapping lines of sight**

In the figure, the image of LSV is arbitrarily set to be smaller than that of ML and the length of the tile diagonal at the bottom edge of LSV, as viewed by the observer, is denoted as the variable l_v. We call this image of LSV the Perspective Plane (*PP*) of LSV. Garcia-Salgado's purpose for setting the *PP* of LSV to be based on an arbitrary module length l_v will be apparent when we proceed to Fig. 13. Based on a measured value indicated in Fig. 10, the *PP* of LSV is 10.35l_v away from the observer [2]. Since *O-PP* = c*vp-rvp*, the distance between *cvp* and *rvpLSV* for the *PP* of LSV is 10.35l_v, as indicated in Fig. 12 by the shorter scale. The length of the tile diagonal at the bottom edge of ML, as viewed from the observer point, is taken to be m_0'. Similarly, based on a measured value in Fig. 9, the plane of ML is 5m_0' away from the observer when viewed from the observer point, resulting in the distance between *cvp* and *rvpML* for ML to be 5m_0', as indicated in Fig. 12 by the longer scale. This plane of ML is the Perspective Plane of the back wall (*PPbw*) of ML because the distance between *cvp* and *rvpML* is $5m_0'$, when viewed from the observer point.

5.5.2 Deriving the Dimensions of the LSV Painting

After depicting the *PP* of LSV and the *PPbw* of ML in Fig. 3a of his paper, Garcia-Salgado depicts the two images with an additional third image in Fig. 3b of his paper, as illustrated in Fig. 13 below. [2] In Fig. 13, the *PP* of LSV and the *PPbw* of ML together with their corresponding scales and *rvp* positions are depicted as previously shown in Fig. 12. The dimensions of the middle-sized plane, which will later be found as the *PPbw* of LSV, will be derived in this section.

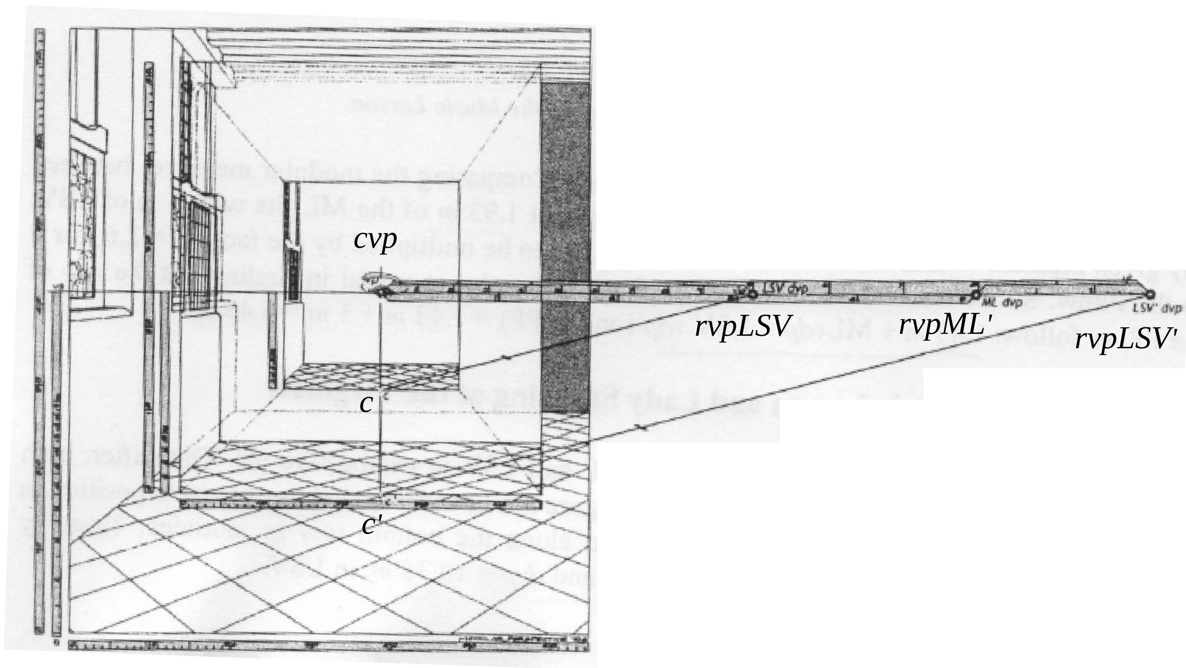

Fig. 13 View of *The Music Lesson* and *A Lady Standing at a Virginal* as depicted in Fig. 12 with the addition of the *PPbw* of *A Lady Standing at a Virginal*

The scale of the *PPbw* of ML, which is in terms of m_0', is used to find the *rvpLSV'* of the *PPbw* of LSV at $6.43m_0'$. Since the *PPbw* of LSV occurs 10.35 tile diagonals away from the observer, $6.43m_0'$ is equivalent to $10.35l_0'$, where l_0' is the length of a tile diagonal at the bottom edge of the *PPbw* of LSV as viewed from the observer point. This implies that l_0' = (6.43/10.35) m_0'. Using MATLAB, a software for numerical computation, the width of the *PPbw* of LSV is measured to be approximately 5.00 tile diagonals or $5.00l_0'$, when the length of the tile diagonal is measured at the bottom edge of the *PPbw* of LSV as viewed from the observer point. Likewise, the width of the *PPbw* of ML is measured to be approximately 4.27 tile diagonals or $4.27m_0'$, when the length of the tile diagonal is measured at the bottom of the *PPbw* of ML as viewed from the observer point. We know that the actual dimensions of ML are given by 73.3 cm by 64.5 cm [8]. This gives

$$64.5 \text{ cm} = 4.27m_0' = 4.27 \times (10.35/6.43) \times l_0'$$

$$\Rightarrow l_0' = \frac{64.5 \times 6.43}{4.27 \times 10.35} cm$$

$\Rightarrow$ width of *PPbw* of LSV

$$= 5.00 \times \frac{64.5 \times 6.43}{4.27 \times 10.35} \text{ cm}$$

$$= 46.92 \text{ cm}$$

We know that the ratio of the height to the width of the *PPbw* of LSV is 1381 : 1200 [9]. Using this ratio, we may convert the actual width of the LSV to the actual height of the LSV, and it is found to be 54.00 cm. Hence, we derive the actual dimensions of the LSV painting to be 54.00 cm by 46.92 cm. This is close to the actual dimensions of the LSV given to be 51.7 cm by 45.2 cm, which implies that the error of this method in deriving the dimensions of the *PPbw* of LSV is up to 4.5%.

The derivation of the dimensions of the *PPbw* of LSV can also be understood geometrically. For the *PP* of LSV, a line *c-rvpLSV* is drawn from *rvpLSV* to the centrally bottommost position, *c*, of that plane. A line parallel to the aforementioned line is extended from the *rvpLSV′* of the *PPbw* of LSV until it reaches a point that has the same horizontal position as the *cvp*, and this point is denoted by *c′*.

The vertical position of *c′* is the vertical position of the base of the *PPbw* of LSV. This is true because a transformation of the *PP* of LSV to the *PPbw* of LSV is analogous to shifting the *PP* of LSV along the line of sight within the visual pyramid of the LSV, such that the *PP* of LSV is scaled proportionately to fit exactly into the initial visual pyramid at all positions along the line of sight. The *PP* of LSV would be transformed into the *PPbw* of LSV at the position where the size of the plane that fits exactly into the visual pyramid is the same as that of the *PPbw* of LSV. This implies that any length between two points in the initial *PP* of LSV would be scaled proportionately to map to the corresponding length in the transformed plane. Hence, if *cvp*-*rvpLSV* is transformed to become *cvp*-*rvpLSV′* by a certain scale, *cvp*-*c* would be mapped to *cvp*-*c′* by the same scale. This would mean that the *PP* of LSV is also mapped to the *PPbw* of LSV by the same scale. Hence, the vertical position of the base of the *PP* of LSV, *c*, would be mapped to the vertical position of the base of the *PPbw* of LSV, denoted by *c′*. From Fig. 13, the ratio of *cvp*-*rvpLSV* to *cvp*-*rvpLSV′* is measured to be 4.4 : 9.1. Therefore, the ratio of *cvp*-*c* to *cvp*-c′ and consequently that of the dimensions of the *PP* of LSV to the dimensions of the *PPbw* of LSV would be 4.4 : 9.1.

Finally, to derive the dimensions of the *PPbw* of LSV, we make use of the positions of *cvp*, *c′* and the four corners of the *PP* of LSV. When the *PP* of LSV is shifted along the line of sight within the visual pyramid of the LSV, the direction of each line joining a corner to *cvp* as viewed along the line of sight would remain the same. This means that the lines joining the four corners of the *PP* of LSV may be extended to reach the corresponding four corners of the *PPbw* of LSV. To construct the base of the *PPbw* of LSV, a horizontal line is constructed along *c′* until either side of the line intersects the lines extended from the two lower corners of the *PP* of LSV. The two resulting intersection points would be the lower left and right corners of the *PPbw* of LSV. At each of these two lower corners of the *PPbw* of LSV, a vertical line is constructed until it intersects a line extended from an upper corner of the *PP* of LSV. In this case, the resulting two intersection points would be the upper left and right corners of the *PPbw* of LSV. Joining the four derived corners of the *PPbw* of LSV, we determine the *PPbw* of LSV, indicated in Fig. 13 by the middle-sized frame.

With depictions of the *PPbw* of LSV and the *PPbw* of ML in Fig. 13, the actual dimensions of the *PPbw* of the LSV can be derived. It is known that the actual dimensions of the *PPbw* of ML are 73.3 cm by 64.5 cm [8]. The ratio of the height of the *PPbw* of LSV to the height of the *PPbw* of ML in Fig. 13 is measured to be 4.95 : 6.95. Similarly, the ratio of the width of the *PPbw* of LSV to the width of the *PPbw* of ML in Fig. 13 is measured to be 4.35 : 6.15. Using these two ratios, the actual dimensions of the *PPbw* of ML can be converted to the actual dimensions of LSV, which are calculated to be 52.2 cm by 45.6 cm. This is reasonably close to the actual dimensions of 51.7 cm by 45.2 cm [9] up to an error of 1.0%.

Garcia-Salgado finds the actual dimensions of LSV in this method to be approximately 51 cm by 45 cm [2], which is also reasonably close to the actual dimensions up to an error of -1.4%. This means that this method used by Garcia-Salgado in deriving the dimensions of the PPbw of LSV is most probably accurate. We can infer from this result that Garcia-Salgado proved that the LSV was painted in the same room as the ML, because the accuracy of this method relied on the assumption that the paintings were painted in the same room.

At this juncture, it is noted that there are three possible sets of dimensions of ML as mentioned in Appendix B, two of which are found from online sources and one from Garcia-

Salgado's paper [7]. In this paper, we only use one set of dimensions. This is because the second set of dimensions of ML is extracted from a relatively less credible source and we cannot ascertain that the third set of dimensions, which is stated by Garcia-Salgado, are the dimensions of the image of ML shown in his paper [7]. The accuracy of the derived dimensions of LSV may be improved by basing measurements and calculations on the actual dimensions and image of ML that are used by Garcia-Salgado. This is because the two methods stated above require the use of the actual dimensions of ML and the width of ML in terms of the length of the tile diagonal at the bottom edge of ML.

5.5.3 Consistency of the Rooms for ML and LSV

Finally, to verify that the room depicted in *A Lady Standing at a Virginal* (LSV) is the same as that depicted in *The Music Lesson* (ML), Garcia-Salgado attempts a consistency proof by verifying that an arbitrary point chosen in the room for LSV would map to the same point in three-dimensional real-world space for ML.

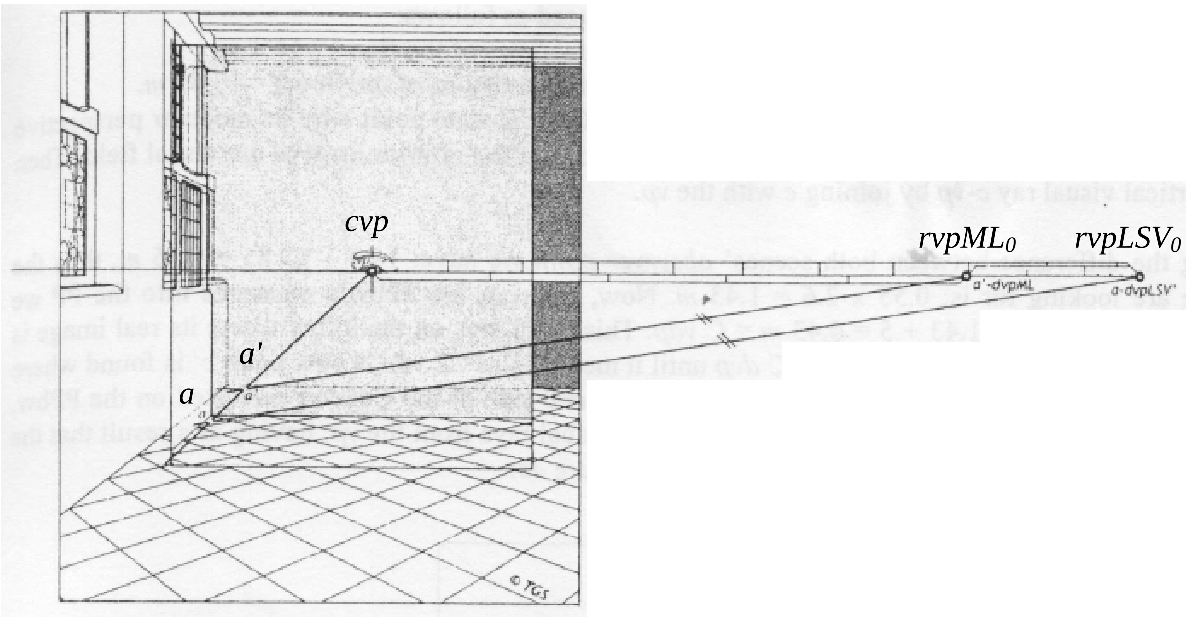

Fig. 14 View of the *PPbw* of ML and the *PPbw* of LSV for a consistency proof

Referring to Fig. 3c in Section 3 of Garcia-Salgado's paper, Fig. 14 depicts the *PPbw* of ML and the *PPbw* of LSV from Fig. 13 [2]. To begin the consistency proof, Garcia-Salgado first chooses an arbitrary point *a* at the intersection of the floor and the front wall in the room for LSV. In Section 5.5.2, it is mentioned that Garcia-Salgado estimates the dimensions of the *PPbw* of LSV to be 51 cm by 45 cm. The dimensions of the *PPbw* of ML used by Garcia-Salgado are 74cm by 63cm [2]. Furthermore, we know from Section 5.3 in this paper that ML and LSV have different observer positions with respect to the front wall, provided the two paintings were painted in the same room. Due to the differences in observer positions and dimensions of the *PPbw* for ML and LSV, each picture has a different visual pyramid. Hence, the bottom edges of the Perspective Plane on the floor (*PPf*) of ML and LSV have different vertical positions. The height of the *PPbw* of ML, 74 cm, is greater than that of the *PPbw* of LSV, 51 cm. This implies that the bottom edge of the *PPbw* of ML is below the bottom edge of the *PPbw* of LSV, as illustrated in Fig. 14. By taking measurements of the position of *cvp* from the base of the *PPbw* of ML and the *PPbw* of LSV, we can find that the position of *cvp* is 2.60 tile diagonals above the bottom edge of the *PPbw* of LSV, where the length of the tile diagonal is measured at the bottom edge of the *PPbw* of LSV. Similarly, by measurement, we can find the position of *cvp* to be 2.75 tile diagonals above the bottom edge of the PPbw of ML, where the tile diagonal is measured at the bottom edge of ML. These values of 2.6 tile diagonals and 2.75 tile diagonals also refer to the height of *cvp* above the floor depicted in LSV and ML respectively, in terms of the tile diagonal length measured at any depth in the visual pyramid. Hence, in the consistency proof, point *a* chosen on LSV is shifted from the original vertical bottom edge position of $Y = -2.60 l_0'$ to the new bottom edge position of $Y = -2.75 l_0'$ so that point *a* lies on the floor of ML. This is done to account for the difference in the visual pyramid of the two pictures due to the different dimensions of the pictures. In Fig. 14, point *a* is the point that is shifted to the vertical position of Y = -2.75l_0', on the floor of ML.

Previously, in Section 5.3, the actual length of the tile diagonal in ML, m_a, and the actual length of the tile diagonal in LSV, l_a, were assumed to be equal. Resultantly, the difference between the observer positions from the front wall for ML and LSV was calculated to be 0.55m_a. By shifting point *a* by 0.55m_0 along the visual ray *a-vp*, the point *a'* is obtained on the intersection of the floor and front wall of ML.

On the scales in Fig. 14, the right vanishing point of ML is denoted as $rvpML_0$ and the right vanishing point of LSV is denoted as $rvpLSV_0$. The lines a-$rvpLSV_0$ and a'-$rvpML_0$ are then drawn. Sliding the line a-$rvpLSV_0$ down to intersect the line a'-$rvpML_0$, Garcia-Salgado finds the lines to be parallel to each other. Hence, he concludes that point *a* on the intersection of the floor and front wall of LSV and point *a'* on the intersection of the floor and front wall of ML map to the same point in three-dimensional real-world space. This is because the sliding is akin to shifting one plane along the line of sight in the visual pyramid to the position of the other plane. Although the visual pyramids for ML and LSV are not the same, the difference in the visual pyramid due to the size of the picture had been accounted for. This was achieved through the shifting of the arbitrary point *a* chosen in LSV to be on the floor of ML. Additionally, Garcia-Salgado suggests that the consistency proof was carried out for other pairs of arbitrary points, and each of the pairs of points were found to match to its corresponding point in three-dimensional real-world space.

For the second part of the consistency proof, Garcia-Salgado compares the heights of the window sills of ML and LSV. To find the height of the window sill in ML, Garcia-Salgado first draws a line at an angle of 45° to the floor of ML. This 45° line forms a right isosceles triangle with the floor and the wall. Hence, at the point where this 45° line intersects the floor, the number of tile diagonals from this point of intersection to the side wall is counted to be the height of the window sill of ML in terms of tile diagonals. Garcia-Salgado counts the height of the window sill of ML to be $2.5m_a$. The same procedure is repeated for LSV and the height of the window sill of LSV is counted to be $2.3l_a = 2.3m_a$ by Garcia-Salgado. The height of the window sills of ML and LSV are estimated to be different. Hence, Garcia-Salgado comments that the type of the window in the rooms for ML and LSV are not the same.

Garcia-Salgado concludes from the matching of the pairs of arbitrary points that the rooms depicted in ML and LSV seem to have the same dimensions and type of floor tiles. However, the windows in ML and LSV do not match, as discussed above. Therefore, Garcia-Salgado concludes that the room depicted in ML and LSV may not be the same but the rooms seem to have similar dimensions.

6 Analysis and Discussion

In this Analysis and Discussion section, we discuss some of the assumptions made by Garcia-Salgado and suggest improvements to his calculations.

6.1 Analysis on Garcia-Salgado's Calculations for *The Music Lesson*

6.1.1 Difference in the Distance between Central Vanishing Point and Side Vanishing Points

In Section 2 of this paper, we have used the assumption made by Garcia-Salgado that the set of parallel lines along the tile diagonals converge orthogonally into *The Music Lesson* (ML) thus resulting in the side vanishing points situated at equal distance to each side of the central vanishing point. To illustrate this, we present Fig. 2 from Section 2 of this paper as Fig. 15 here.

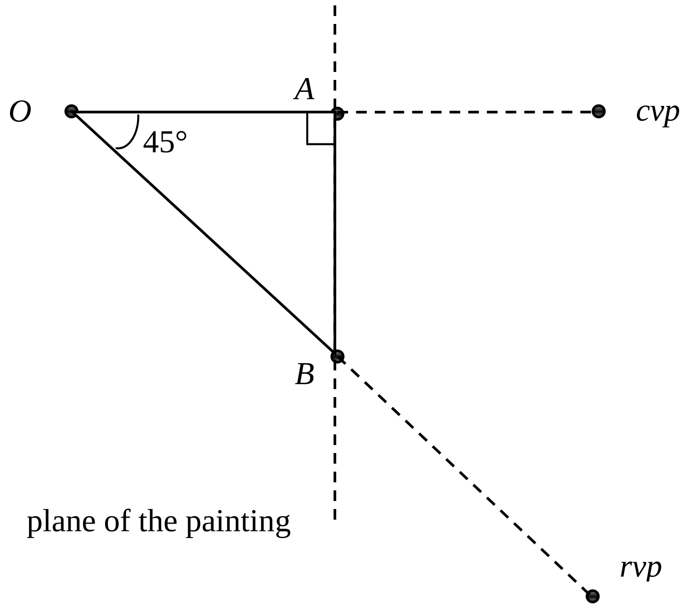

Fig. 15 Top view of *The Music Lesson* depicting the positions of the observer, central vanishing point, right vanishing point and the horizontal positions of the two vanishing points on the plane of the painting

However, in *Reconstruction of Vermeer's The Music Room* [3], Aditya Liviandi calculated the distance between the central vanishing point and right vanishing point and the distance between the central vanishing point and left vanishing point and these two distances are found to be different. In other words, contrary to what Garcia-Salgado mentions in his papers, the right vanishing point and left vanishing point are not situated at equal distance from the central vanishing point. In view of the difference in the results of Garcia-Salgado and Liviandi, we reconstruct the vanishing points on a picture of ML and calculate the distances between them using the MATLAB, as used by Liviandi in *Reconstruction of Vermeer's The Music Room* [3],. We shall now present our findings in this section.

For this section, the coordinate system used for the room in real-world space is such that the *x*-axis is parallel to the width of the picture and x-coordinates becomes increasingly large towards the right side of the picture. The *y*-axis is parallel to the height of the picture and y-coordinates become increasingly large towards the bottom of the picture. The *z*-axis intersects the observer point and is parallel to the line of sight, with z-coordinates increasing towards the picture. Each unit on each of the axes of the coordinate system is a pixel.

First we shall explain how we get the coordinates of the vanishing points on the picture of ML. The central vanishing point is found by using the set of parallel lines along the tile diagonals which seems to converge orthogonally into the picture. We plot each line on ML in MATLAB using manually selected coordinates. The best fit line is then found for each set of coordinates using the MATLAB function *polyfit()*. The equations of all the lines are then found using MATLAB. Finally, we use the MATLAB function *mldivide()* to find the least square solutions to the set of equations, which is the coordinates of the central vanishing point on the picture of ML. The same procedure is used to find the left vanishing point and right vanishing point on the picture of ML. The set of lines along the tile edges that converges to the left of the picture are used to find the left vanishing point. The set of lines along the tile edges that converges to the right of the picture are used to find the right vanishing point.

After the coordinates of the vanishing points are obtained, the best fit line is calculated and this best fit line is the vanishing line of the floor on the picture of ML. We shall use the term “horizon” to refer to this line. The vanishing points projected on the horizon of ML are then computed on MATLAB using perpendicular projection. We present the coordinates of the initial vanishing points on the picture of ML, the equation of the horizon and the projected vanishing points on the picture of ML below:

Initial vanishing points:

	x	**y**
left	227	1631.1
centre	1772.1	1650.7
right	3263.4	1671.4

Equation of horizon: y = 0.01327x + 1627.8

Projected vanishing points:

	x	**y**
left	227.0	1630.8
centre	1772.1	1651.4
right	3263.4	1671.2

The two-dimensional picture is used to find the vanishing points thus there are only *x*-coordinates and *y*-coordinates. The coordinates are in pixels. We will use the coordinates of the projected vanishing points for the rest of this section.

Now that we have the coordinates of the vanishing points, we shall use the distance formula to calculate the distance between the central vanishing point and left vanishing point and the distance between the central vanishing point and right vanishing point. The calculations of the distances are shown below:

Distance between central and left vanishing points

$= \sqrt{(1772.1 - 227.0)^2 + (1651.4 - 1630.8)^2}$

= 1545.2 pixels

Distance between central and right vanishing points

$= \sqrt{(3263.4 - 1772.1)^2 + (1671.2 - 1651.4)^2}$

= 1491.4 pixels

The distance between the central vanishing point and left vanishing point is 1545.2 pixels. The distance between the central vanishing point and right vanishing point is 1491.4 pixels. Therefore, from our results, the left vanishing point and right vanishing point are not at equal distances from the central vanishing point. The difference in the distances between the central vanishing point and each side vanishing point suggests that the set of lines used to find the central vanishing point are not converging orthogonally into the picture of ML. Since this set of lines is orthogonal to the front wall, this implies that the picture plane of ML is not parallel to the front wall. In addition, the vanishing point midway between the left and right vanishing points would not be a central vanishing point. We call this vanishing point the middle vanishing point (*mvp*).

Since the left vanishing point and right vanishing point are not at equal distances to each side of the central vanishing point, the perpendicular distance of the picture plane from the observer is not equal to the distance between the middle vanishing point and side vanishing point. We now proceed to find the perpendicular distance of the picture plane from the observer using the projected vanishing points we obtain earlier. Let k denote this distance. The perpendicular distance of the picture plane from the observer point is $O\text{-}O_{PP}$ in Fig. 16 below:

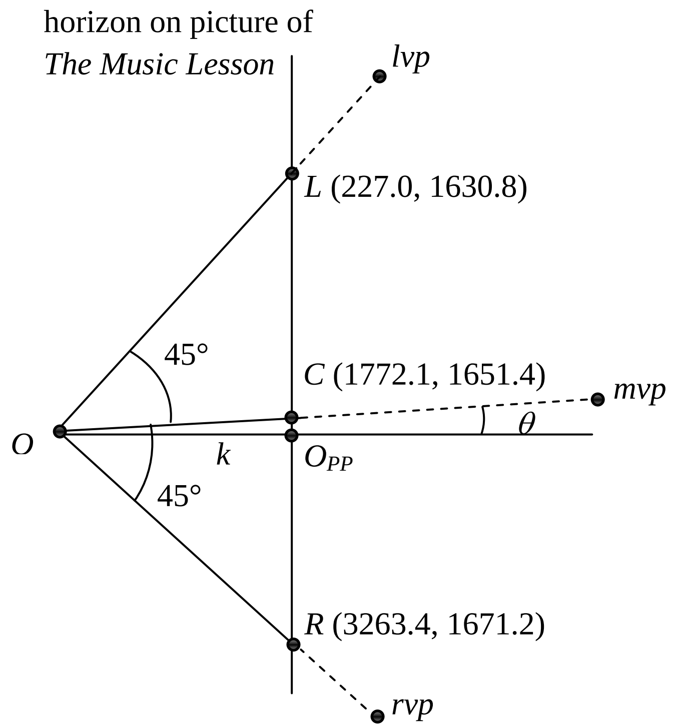

Fig. 16 Top view of *The Music Lesson* depicting the positions of the observer, middle vanishing point, right vanishing point and left vanishing point on ML

Fig. 16 is drawn based on Fig. 14 in *Reconstruction of Vermeer's The Music Room* [3] by Liviandi. In Fig. 16, we present the top view of ML depicting the positions of the observer point, middle vanishing point, right vanishing point and left vanishing point on the painting of ML. Point O represents the observer point, Point C represents the middle vanishing point on the picture of ML, Point L represents the left vanishing point on the picture of ML and Point R represents the right vanishing point on the picture of ML. Point O_{PP} is the intersection of the horizon and the perpendicular line of the picture plane from the observer point. We present the calculations to obtain the perpendicular distance of the picture plane from the observer point, $O\text{-}O_{PP}$.

$\angle LOC = \angle COR = 45°$

We let $\angle O_{PP}OC = \theta$ and denote the distance $O\text{-}O_{PP}$ as k.

LC = Distance between *mvp* and *lvp* = 1545.2 pixels.

CR = Distance between *mvp* and *rvp* = 1491.4 pixels

From Fig. 16, we obtain the following equations:

$k \tan \theta = CO_{PP}$

$k \tan (45° + \theta) = LO_{PP}$

$k \tan (45° - \theta) = O_{PP}R$

From the three equations above, we obtain the following equations:

$1545.2 = LC = LO_{PP} - CO_{PP} = k (\tan (45° + \theta) - \tan \theta)$

$\Rightarrow k (\tan (45° + \theta) - \tan \theta) = 1545.2$ (1)

$1491.4 = CR = CO_{PP} + O_{PP}R = k (\tan \theta + \tan (45° - \theta))$

$\Rightarrow k (\tan \theta + \tan (45° - \theta)) = 1491.4$ (2)

Solving equations (1) and (2) in MATLAB, we obtain the following results:

$k = 1517.4$ pixels and $\theta = 1.015°$

Therefore, the perpendicular distance of the picture plane from the observer point, O-O_{PP} is 1517.4 pixels.

In conclusion, for Section 6.1 of this paper, we have determined the positions of the middle vanishing point, left vanishing point and right vanishing point on the picture of ML and calculated the distances between the vanishing points. Our results show that the distance between the middle vanishing point and the left vanishing point is not equal to the distance between the middle vanishing point and the right vanishing point. As a result, the perpendicular distance of the picture plane from the observer point is not equal to the distance between the middle vanishing point and either of the side vanishing points. We have also calculated the perpendicular distance between the picture plane and the observer point.

6.1.2 Distance between the Observer Point and the Front Wall of *The Music Lesson*

In Section 5.1.4 of this paper, we have discussed how Garcia-Salgado has computed the the distance between the observer and the front wall, in terms of the number of tile diagonals on the Perspective Plane on the floor (*PPf*) of *The Music Lesson* (ML). The distance between the observer and the front wall computed by Garcia-Salgado is also based on the assumption that the side vanishing points are equidistant from the central vanishing point. We have explored in Section 6.1.1 of this paper that the side vanishing points are not equidistant from the central vanishing point. Hence, in this Section, we proceed to find the distance between the observer point and the front wall without using the above-mentioned assumption made by Garcia-Salgado.

Calculating the distance between the observer and the front wall of ML requires the perpendicular distance of the *PPf* of ML which we will calculate here. First, we obtain the tile diagonal length at the bottom edge of the picture of ML using MATLAB. The length obtained is in pixels. We then obtain the ratio of pixels to centimetres on the picture of ML, using the width of the picture of ML in [8]. This ratio is used to convert the tile diagonal length at the bottom edge of the picture from pixels to centimetres. We also use this ratio to convert the perpendicular distance of the picture plane from the observer, from pixels to centimetres. We use the tile diagonal length at the bottom edge of the *PPf* of ML computed by Garcia-Salgado, 39.13 cm. With the tile diagonal lengths at the bottom edge of *PPf* and picture of ML, we have a ratio of tile diagonal length on the *PPf* of ML to tile diagonal length on the picture of ML. With this ratio and the perpendicular distance of the picture plane of ML from the observer point obtained in Section 6.1.1 of this paper, we calculate the perpendicular distance of the *PPf* of ML from the observer point.

The tile diagonal length at the bottom edge of the *PPf* of ML, 39.13 cm, is also the actual length of the tile diagonal in ML as discussed in Section 4. Dividing the perpendicular distance between the observer point and the *PPf* of ML by the actual length of the tile diagonal in the *PPf* of ML, we obtain this distance in terms of m_a, the tile diagonal at the bottom edge of the *PPf* of ML.

The calculations to obtain the perpendicular distance of the *PPf* from the observer point are presented below:

Width of picture of ML = 1296 pixels = 64.5 cm

Ratio of pixels to cm on the picture of ML = 1296 pixels : 64.5 cm

Tile diagonal length at bottom edge of the picture of ML

= 303.5 pixels

$$= 303.5 \times \frac{64.5}{1296}$$

= 15.10 cm

Perpendicular distance of *PPf* of ML from observer point

= 1517.4 pixels

$$= 1517.4 \times \frac{64.5}{1296}$$

= 75.52 cm

Tile diagonal length at bottom edge of the *PPf* of ML = 39.13 cm

Ratio of tile diagonal length on *PPf* of ML to tile diagonal length on picture of ML

= 39.13 : 15.10

Perpendicular distance of *PPf* of ML from observer point

$$= 75.52 \times \frac{39.13}{15.10}$$

= 195.7 cm

Perpendicular distance of *PPf* of ML from observer point in terms of m_a

$$= \frac{195.7}{39.13} = 5.0013 m_a$$

The distance between the *PPf* of ML and the front wall obtained by Garcia-Salgado as discussed in Section 5.1.3 is 9.4m_a. Adding this distance to the perpendicular distance of the picture of ML from the observer point we just calculated, 5.0013m_a, we obtain the distance between the observer point and the front wall of ML to be 14.40m_a.

The distance between the observer point and the front wall of ML we just calculated is the same as that calculated by Garcia-Salgado. We have taken into account that the left vanishing point and right vanishing point are not at equal distances to each side of the central vanishing point. However, as computed in Section 6.1.1 of this paper, the angle bounded by the lines OO_{PP} and OC in Fig. 16, $\theta = 1.015°$ which is quite small. Therefore, although the difference in the distance between the central vanishing point and left vanishing point and the distance between the central vanishing point and right vanishing point affects the accuracy of the results, the effect is only significant to a small extent.

6.2 The Assumed Height of the Man in *The Music Lesson*

In Section 5.1.1, it was mentioned that Garcia-Salgado uses the assumption that the height of the man in *The Music Lesson* (ML) is 180 cm to calculate the actual length of the tile diagonal in ML.

ML was painted in the 1600s. In [10], it is stated that the average height of a Dutchman was 175.26 cm approximately two thousand years ago and the average height decreased over the 1800 years that followed before the average height of Dutchman increased again in the mid 1800s to reach the value of about 180 cm [11] today. Consequently, the average height of the Dutchman in the 1600s would have been estimated to be much lower than 180 cm, in contrast with the assumption made by Garcia-Salgado. Therefore, the actual length of the tile diagonal of ML, estimated based on the assumed height of the man in ML, may be inaccurate. Since the absolute lengths of objects are determined in terms of tile diagonals in Garcia-Salgado's papers, the effect of an error in the height of the man in the picture may be amplified and lead to even greater inaccuracy.

In contrast with Garcia-Salgado, Steadman uses dimensions of relics of objects depicted in Vermeer's paintings, rather than an assumed length, as a reference of lengths in the picture to actual lengths of object in real-world space. As mentioned in Section 2 of this paper, the maps seen in some of Vermeer's paintings are real maps that still exist today. Presently, some of the furniture and instruments depicted in Vermeer's paintings also exist in museums. For instance, relics of the lions' head chair painted in ML are kept in the Prinsenhof Museum in Delft and in the Rijksmueum in Amsterdam [1]. Since the maps and furniture are located in museums, their dimensions can be measured. To find the actual lengths of objects depicted in Vermeer's paintings, Steadman first tries to find the length of a marble floor tile edge that would result in derived dimensions of maps in ML, which are close to the actual dimensions of the maps in real-world space. He finds such a length of a marble floor tile edge to be 29.3 cm.

Using the estimated actual length of a marble tile edge, Steadman calculates the height, width and depth of the lions' head chairs in ML to be 108 cm, 42 cm and 42 cm respectively. These dimensions are close to the true known dimensions of the chair which are 108 cm, 39 cm and 43 cm. Therefore, it seems even more acceptable to use the estimated length of a marble tile edge to calculate the actual dimensions of objects in ML. Steadman also proceeds to use this actual length to find the dimensions of other objects in ML and the dimensions of the room depicted in ML.

Garcia-Salgado calculates the actual length of the tile diagonal in ML based on the assumed height of the man in ML, which is inaccurate from our discussion in the earlier part of this section. Instead of assuming the height of the man in ML, an object depicted in ML with known actual dimensions can be used as a reference to find the actual lengths of other objects depicted in ML.

7 Conclusion

In this paper, Section 2 and Section 3 introduced readers to the background of the Perspective Geometry in paintings and the case for Vermeer's use of the camera obscura in Sections 2 and 3. Thereafter, the key terms and an understanding of the key concepts used in Garcia-Salgado papers were presented in Section 4. Following the foundational knowledge required for understanding Garcia-Salgado's papers, an understanding of Sections 1 to 3 of

Modular Perspective and Vermeer's Room by Garcia-Salgado was proposed in Section 5. Finally, in Section 6, assumptions made by Garcia-Salgado were questioned and suggestions for improvements to the accuracy of the relevant numerical values were suggested.

BIBLIOGRAPHY

[1] Steadman, Philip. Vermeer's Camera: Uncovering the Truth Behind the Masterpieces. New York: Oxford University Press, 2002.

[2] Garcia-Salgado, Tomas. Guest Lecture. The Bridges Conference: Mathematical Connections in Art, Music, and Science. Institute of Education, London Knowledge Lab, London.

[3] Liviandi, Aditya. "Reconstruction of Vermeer's The Music Room." 2008. National University of Singapore. 14 May 2009 <http://www.math.nus.edu.sg/aslaksen/projects/Aditya%20Liviandi%20-%20Vermeer.pdf>.

[4] Hartley, Richard, and Andrew Zisserman. Multiple View Geometry in Computer Vision. United Kingdom: Cambridge University Press, 2003.

[5] Dictionary.com. 10 July 2009 <http://dictionary.reference.com/search?q=linear%20perspective&db=luna>.

[6] "The Mystery in the Mirror." Grand Illusions. 14 May 2009 <http://www.grand-illusions.com/articles/mystery_in_the_mirror/>.

[7] Garcia-Salgado, Tomas. "Some Perspective Considerations on Vermeer's The Music Lesson." perspectivegeometry.com. No. 1 (2003). 14 May 2009 <http://www.perspectivegeometry.com/>.

[8] Vermeer, Johannes. *The Music Lesson.* 1662-1664. Buckingham Palace, London. Essential Vermeer. "Vermeer: The Music Lesson." 14 May 2009 <http://www.essentialvermeer.com/catalogue/music_lesson.html>.

[9] Vermeer, Johannes. *A Lady Standing at a Virginal.* 1670-1673. National Gallery, London. Essential Vermeer. “Vermeer: A Lady Standing at a Virginal.” 14 May 2009 <http://www.essentialvermeer.com/catalogue/lady_standing_virginal.html>.

[10] “The Dutch are the World’s Tallest People.” Spry Health. 16 September 2006. 21 May 2009 <http://www.spryhealth.com/12>.

[11] Max, Arthur. “Dutch Reach New Heights.” USA TODAY. 16 September 2006. 21 May 2009 <http://www.usatoday.com/news/offbeat/2006-09-16-dutch-tall_x.htm>.

APPENDICES

Appendix A

List of terms and their corresponding notations as used in this paper and in the papers by Tomas Garcia-Salgado

Terms (as used in this paper)	**Notations used in this paper**	**Notations used by Garcia-Salgado**
Observer point	*O*	-
Vanishing point	*vp*	-
Central vanishing point	*cvp*	*vp*
Left vanishing point	*lvp*	*vdp, dvp*
Right vanishing point	*rvp*	*vdp, dvp*
Middle vanishing point	*mvp*	-
Right vanishing point of the Perspective Plane on the floor of *The Music Lesson*	$rvpML_0$	*dvpML,* *ML dvp*
Right vanishing point of the Perspective Plane of the back wall of *The Music Lesson*	*rvpML'*	*ML dvp*
Right vanishing point of the Perspective Plane of *A Lady Standing at a Virginal*	*rvpLSV*	*dvpLSV,* *LSV dvp*

Right vanishing point of the Perspective Plane of the back wall of *A Lady Standing at a Virginal*	*rvpLSV'*	*LSV' dvp*
Distance between any two points *A* and *B*	*A-B*	-
Perspective Plane	*PP*	-
Perspective Plane on the floor	*PPf*	PP
Perspective Plane of the back wall	*PPbw*	PPbw
The Music Lesson	ML	ML
Actual length of tile diagonal at the same depth as the Perspective Plane of *The Music Lesson*	*m*	m
Length of tile diagonal at the same depth as the Perspective Plane on the floor of *The Music Lesson*, as viewed from the observer point	m_0	m
Length of tile diagonal at the same depth as the Perspective Plane of the back wall of *The Music Lesson*, as viewed from the observer point	m_0'	m'
Actual length of tile diagonal for *The Music Lesson*	m_a	m
Actual length of tile diagonal on the painting for *The Music Lesson*	m_a'	m
A Lady Standing at a Virginal	LSV	LSV

Actual length of tile diagonal on the Perspective Plane of *A Lady Standing at a Virginal*	l	m
Length of tile diagonal on the Perspective Plane on the floor of *A Lady Standing at a Virginal*, as viewed from the observer point	l_0	m
Length of tile diagonal on the Perspective Plane of the back wall of *A Lady Standing at a Virginal*, as viewed from the observer point	l_0'	m'
Actual length of tile diagonal for *A Lady Standing at a Virginal*	l_a	m
Actual length of tile diagonal on the painting for *A Lady Standing at a Virginal*	l_a'	m
Point on the Perspective Plane such that O-O_{PP} is perpendicular to horizon	O_{PP}	-
Horizon	-	vh

Appendix B

The Music Lesson and *A Lady Standing at a Virginal*

The Music Lesson Version 1

The Music Lesson

Dimensions:

73.3 cm by 64.5 cm (1485 pixels by 1296 pixels)

Source: http://www.essentialvermeer.com/catalogue_xxl/music_xxl.html

***The Music Lesson* Version 2**

The Music Lesson

Dimensions: 74.6 cm by 64.1 cm (990 pixels by 870 pixels)

Source: http://en.wikipedia.org/wiki/The_Music_Lesson

***The Music Lesson* used by Tomas Garcia-Salgado**

The Music Lesson

Dimensions stated by Garcia-Salgado: 74 cm x 63 cm

Source: “Some Perspective Considerations on Vermeer’s The Music Lesson” by Tomas Garcia-Salgado

There are three different sets of dimensions of *The Music Lesson* (ML) given by three sources as shown above. In this paper, the dimensions of ML have been taken to be that of Version 1 rather than Version 2, because Garcia-Salgado seems to have used a painting of ML that is cropped similarly to Version 1. The three versions of ML will be compared in the following paragraphs.

The height of Version 2 of ML is greater than the height of Version 1 of ML. The fraction of the leftmost tile at the bottom edge of Version 2 of ML that is visible in ML is smaller than that in Version 1 of ML. The table cloth touches the bottom edge of Version 2 of ML but not the bottom edge of Version 1 of ML. In addition, a smaller portion of the room ceiling is depicted in Version 2 than in Version 1 of ML. By these comparisons, the height of Version 2 of ML would be smaller than the height of Version 1 of ML. However, it should be noted that the actual dimensions of both versions imply that the height of Version 1 of ML is smaller than that of Version 2 of ML. This means that either the height of Version 1 of ML or the height of Version 2 of ML may be incorrect.

The width of Version 2 of ML is shorter than that of Version 1 of ML. In Version 2 of ML, the porcelain white vase is closer to the left edge of the picture of ML than in Version 1 of ML. Less of the window closer to the observer is also depicted in Version 2 of ML.

From these observations, we may deduce that Version 1 and Version 2 of ML are cropped differently. Hence, they have different dimensions.

A Lady Standing at a Virginal

A Lady Standing at a Virginal

Dimensions: 51.7 cm by 45.2 cm (1319 pixels by 1149 pixels)

Source: http://www.essentialvermeer.com/catalogue_xxl/standing_xxl.html

Appendix C

MATLAB commands

(1) To upload image to MATLAB Workspace (e.g. image_essentialvermeer2.jpg).
Import Data
=> Browse for the required picture
=> Finish

(2) To insert picture as figure in MATLAB.
>> imagesc(image_essentialvermeer2)
>> axis image
(“>> axis image” command makes the pixels square / of equal length)

(3) To label the picture.
Insert => X Label, Y Label, Title

(4) To collect one set of coordinates as data points on a figure.
Zoom in to desired scale
=> Data cursor => Click first point
=> Alt + Click to make second point (without “alt”, the first data point disappears)
=> When one set is complete, right click on one point
=> Export cursor data to workspace (ends up in workspace in main page)
=> Right click on one point to clear all data points in figure and restart before collecting next set of data points

(5) To get best fit line of a set of data.

(a) In Excel:
Insert chart
=> XY Scatter
=> Right click, add trendline
=> Options: display equation

(b) In Matlab:
>> X = [1231 1248 1249 ... 2487];
>> Y = [9875 0985 8734 ... 0986];

```
>> polyfit(X,Y,1)
```

(Use 1 in "polyfit" command because we want a line, which is a polynomial of degree 1)

Example of Result:

p_r1 = 1.0e+03 * -0.0005 3.0841

Means Y= (1.0e+03) (-0.0005)X + (1.0e+03)(3.0841) = -0.5X+3084.1

(6) To solve for intersection points (vanishing points).

Y = mX + c => -mX + Y = c

Use '\' or 'mldivide' to solve x in Ax=B, where A consists coefficients, -m, of X and Y, and B consists values of c (each row of values in A and B is from each line)

(a) To solve for left vanishing point,

L1 = 0.7878X+1458.7
L2 = 0.6261X +1484.5
L3 = 0.5178X +1505.5
L4 = 0.4364X +1529
L5 = 0.3754X +1549.9
L6 = 0.3321X +1560.7

```
>> A=[-0.7878 1; -0.6261 1; -0.5178 1; -0.4364 1; -0.3754 1; -0.3321 1]
>> B=[1458.7; 1484.5; 1505.5; 1529; 1549.9; 1560.7]
>> x=A\B
```

(b) To solve for central vanishing point,

C1 = -0.8878X +3230.2
C2 = -1.0735X +3553.3
C3 = -1.3692X +4066
C4 = -1.7757X +4801.2
C5 = -2.6458X +6340.5
C6 = -5.1042X +10696

```
>> A=[0.8878 1; 1.0735 1; 1.3692 1; 1.7757 1; 2.6458 1; 5.1042 1]
>> B=[3230.2; 3553.3; 4066; 4801.2; 6340.5; 10696]
>> x=A\B
```

(c) To solve for right vanishing point,

R1 = -0.2324X+2426.3
R2 = -0.2547X+2500.1
R3 = -0.2841X+2593.6
R4 = -0.3081X+2689.9
R5 = -0.3577X+2843
R6 = -0.4203X+3036.5

```
>> A=[0.2324 1; 0.2547 1; 0.28411; 0.3081 1; 0.3577 1; 0.42031]
>> B=[2426.3; 2500.1; 2593.6; 2689.9; 2843; 3036.5]
```

```
>> x=A\B
```

(7) To find the best fit line of vanishing line of the floor.
Use Step 5, using the coordinates of the vanishing points.

(8) To solve for k and θ in Fig 16. k is denoted as x and θ is denoted as y in MATLAB (“S.x” gives the answer for x, “S.y” gives the answer for θ in radians).

```
>> S=solve('x*(tan(y)+tan((pi/4)-y))= 1491.4','x*(tan((pi/4)+y)-tan(y))= 1545.2')
   S.x, S.y
```

(9) To draw the lines obtained on the tile to check whether the lines are on the tile lines. This following set of equations defines the tile diagonal length of the second half tile at the bottom of the picture.
Import Data => Browse for the required picture => Finish

```
>> imagesc(image_essentialvermeer2)
>> axis image
>> figure(gcf)
>> hold on
>> X = [0:500:4000]
L1 = 0.7878*X+1458.7
L2 = 0.6261*X +1484.5
R6 = -0.4203*X+3036.5
R7 = -0.488*X+3277.6
plot (X,L1,X,L2,X,R6,X,R7), xlabel('x'), ylabel('y'), title ('The Music Lesson')
hold off
```

(10) To solve for left and right coordinates of tile diagonal of the second half tile at the bottom of the picture.
Use Step 6 on the following equations:
For left coordinate of tile diagonal:

```
L1 = 0.7878*X+1458.7
R6 = -0.4203*X+3036.5
```

For right coordinate of tile diagonal:

```
L2 = 0.6261*X +1484.5
R7 = -0.488*X+3277.6
```

(11) To check the visible depth of the room against Salgado's value of 9.4 tiles, find the coordinates of tile vertices for the end tiles in the 2^{nd} row and compute the distance. First plot the lines to verify the position of the vertice, then use Step 6 to find the coordinates of the intersection of the relevant lines.

Import Data => Browse for the required picture => Finish

```
>> imagesc(image_essentialvermeer2)
>> axis image
```

For top vertice of lower tile:

```
>> figure(gcf)
>> hold on
>> X = [0:500:4000]
L3 = 0.5178*X +1505.5
R6 = -0.4203*X+3036.5
plot (X,L3,X,R6), xlabel('x'), ylabel('y'), title ('The Music Lesson')
```

For bottom vertice of lower tile:

```
>> figure(gcf)
>> hold on
>> X = [0:500:4000]
L2 = 0.6261*X +1484.5
R7 = -0.4937*X + 3284.9
plot (X,L2,X,R7), xlabel('x'), ylabel('y'), title ('The Music Lesson')
```

For the intersection between the tile diagonal of the lower tile and the lower boundary of the painting of *The Music Lesson*:

```
>> figure(gcf)
>> hold on
>> X = [0:500:4000]
C6 = -5.1042*X +10696
Y = 0*X + 2485
plot (X,C6,X,Y), xlabel('x'), ylabel('y'), title ('The Music Lesson')
```