The Factoring Dead

Preparing for the Cryptopocalypse

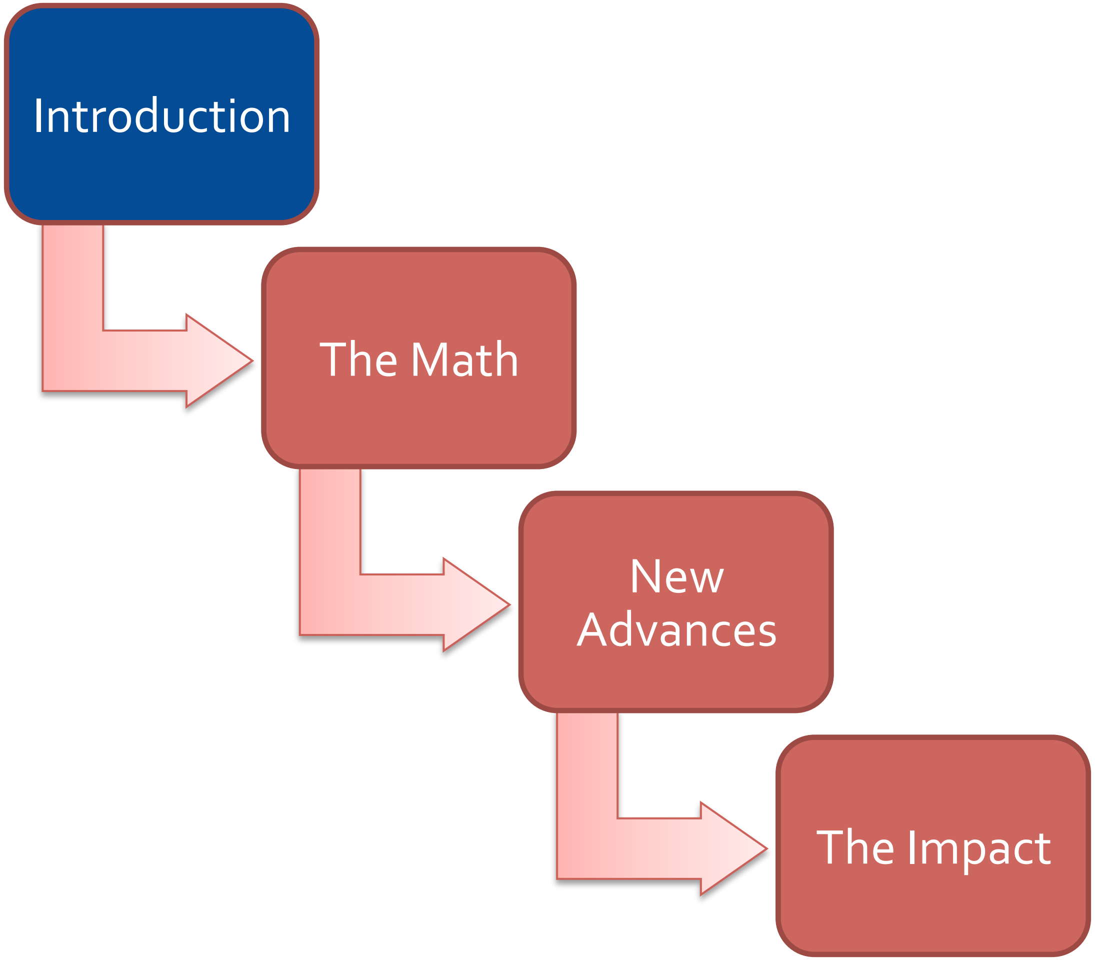
Thomas Ptacek, Matasano

Tom Ritter, iSEC Partners

Javed Samuel, iSEC Partners

Alex Stamos, Artemis Internet

Agenda



Introduction

The Math

New Advances

The Impact

Why are we here?

- There is a significant disconnect between theory and reality in security.
    - Lots of great, continuous academic research in cryptography.
    - Few engineers get beyond Applied Cryptography before shipping code.
    - In the 2010's, it is no longer acceptable to just use standard libraries and claim ignorance.
- We wanted to see if we could bridge this gap a bit.
- We certainly are not the only ones to do so.

Recent TLS Problems

- Numerous attacks on the current TLS infrastructure.
  - BEAST [1]
  - CRIME [2]
  - Lucky 13 [3]
  - RC4 Bias [4]
- Even a new compression oracle attack here at BlackHat USA 2013! [5]
- Were any of these attacks really unpredictable to people paying attention? (Hint: no[6])

[1] http://vnhacker.blogspot.com/2011/09/beast.htm
[2] https://www.isecpartners.com/blog/2012/september/details-on-the-crime-attack.aspx
[3] http://www.isg.rhul.ac.uk/tls/TLStiming.pdf
[4] http://infoscience.epfl.ch/record/152526/files/RC4_1.pdf
[5] http://www.blackhat.com/us-13/briefings.html#Prado
[6] John Kelsey. Compression and information leakage of plaintext. Fast Software Encryption, 9th International Workshop, February 2002!

Comparison to Academic Time Line

- 1998 – EFF Deep Crack defeats DES in 56 hours
- 2005 - Pre-image attacks against MD5 discussed
- 2008- Applebaum, Sotirov et. al. use MD5 attack against CA
- 2011 - CA/Browser Forum forbids MD5
- 2012 - Somebody (*cough*) uses related attack against Microsoft for FLAME
- SIM Card Attack at BlackHat 2013 using DES

Why such a disconnect?

- Most systems are not designed for cryptographic agility
- Cryptography is an ecosystem
- Few companies employ full-time cryptographers
- Hard for InfoSec practitioners to keep up-to-speed
- Lots of momentum in the professional consulting core.

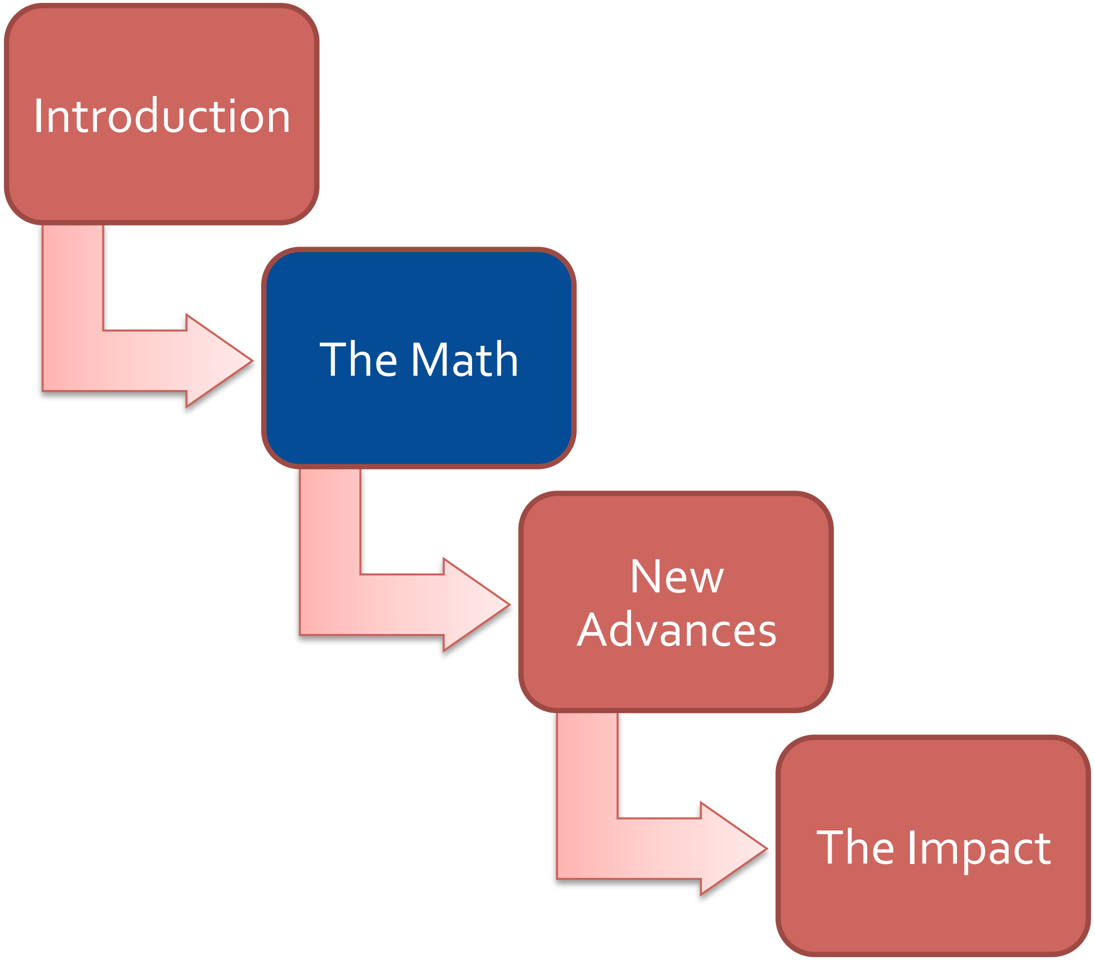
We have failed as an industry
to address these structural problems.

Why are we here?

- Looking for the next crypto black swan.
- Our thesis:
  - Last six months has seen huge leaps in solving the DLP
  - These leaps have parallels to the past.
  - There is a small but real chance that both RSA and non-ECC DH will soon become unusable.
  - Ecosystem currently cannot support a quick pivot to ECC

We want this room to become the seed of change

Agenda


Introduction

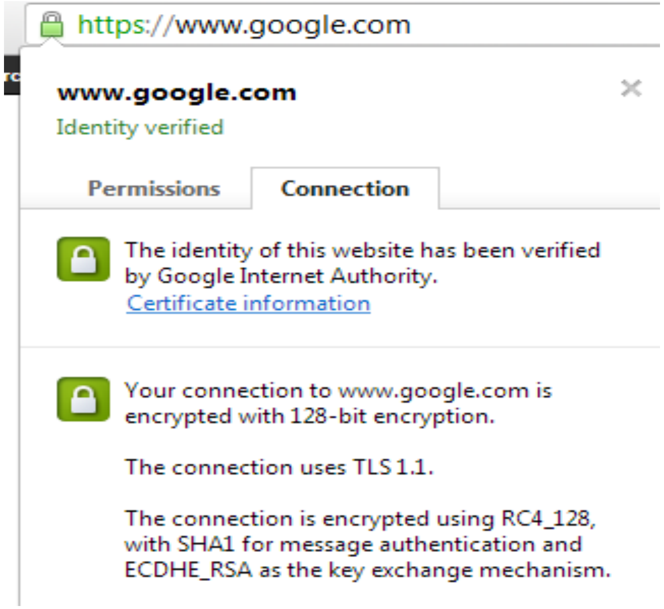
The Math

New Advances

The Impact

Why Asymmetric Cryptography?

- Key part of modern cryptosystems



https://www.google.com

www.google.com

Identity verified

The identity of this website has been verified by Google Internet Authority.

Certificate information

Your connection to www.google.com is encrypted with 128-bit encryption.

The connection uses TLS 1.1.

The connection is encrypted using RC4_128, with SHA1 for message authentication and ECDHE_RSA as the key exchange mechanism.

How does asymmetric crypto work?

- We need a “trap-door” function, something that is easy to do but hard to undo
  - We also need a way to cheat with more information
- Rarely is the difficulty of this function proved, only assumed

What are the common primitives?

iSECpartners part of nccgroup

- **Diffie-Hellman** - 1976 - Secure key exchange
- **RSA** - 1977 - Encryption, signing
- **Elliptic Curve Cryptography**
  - Suite B - 2007 - Key exchange, signing and encryption
  - GOST - 2010 - Key exchange, signing and encryption

Diffe Hellman Overview

- First published by Whitfield Diffie and Martin Hellman in 1976
- Establishes shared secret by exchanging data over a public network.
- Security relies on the hardness of the discrete logarithm problem.

How do I attack DH?

- Solve the discrete logarithm problem:
  - Suppose $h = g^x$ for some g in the finite field and secret integer x.
  - The discrete logarithm problem is to find the element x, when only g and h are known.
- Also how you attack El-Gamal and DSA

RSA Overview

- Key Generation to compute public and private key exponent (*e*, *d*)
- Encryption by raising the message to public key exponent *e*
- Decryption by raising the message to private key *d*
- Security relies on the hardness of factoring.

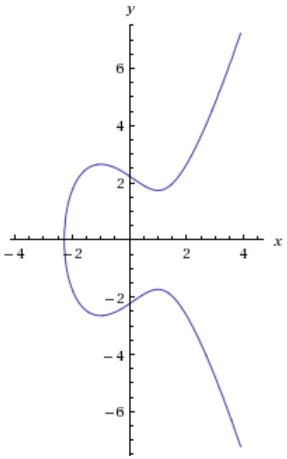
How do I attack RSA?

- Factoring!
  - Find the *p* & *q* such that *p*q* = *N*

- Factoring an RSA modulus allows an attacker to compute the secret *d* and thus figure out the private key.

Elliptic Curve Cryptography Overview

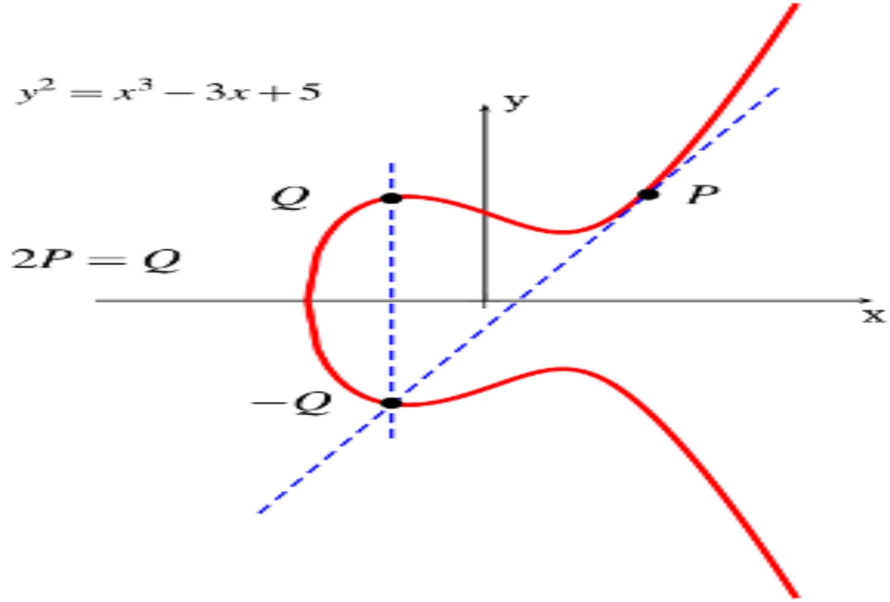
- An elliptic curve *E* over *R* real numbers is defined by a Weierstrass equation eg $y^2 = x^3 - 3x + 5$



- Cryptographic schemes require fast and accurate arithmetic and use one of the following elliptic fields.
  - Prime Field F_p where *p* is a prime for software applications.
  - Binary Field F_2m where *m* is a positive integer for hardware applications.

Elliptic Curve Cryptography

- ECC is secure due to the hardness of the elliptic curve discrete logarithm problem (ECDLP).
  - Given an elliptic curve E defined over a finite field F_q, a point $P \in E(F_q)$ of order n, and a point $Q \in E$
  - Find the integer $d \in [0; n - 1]$ such that $Q = dP$

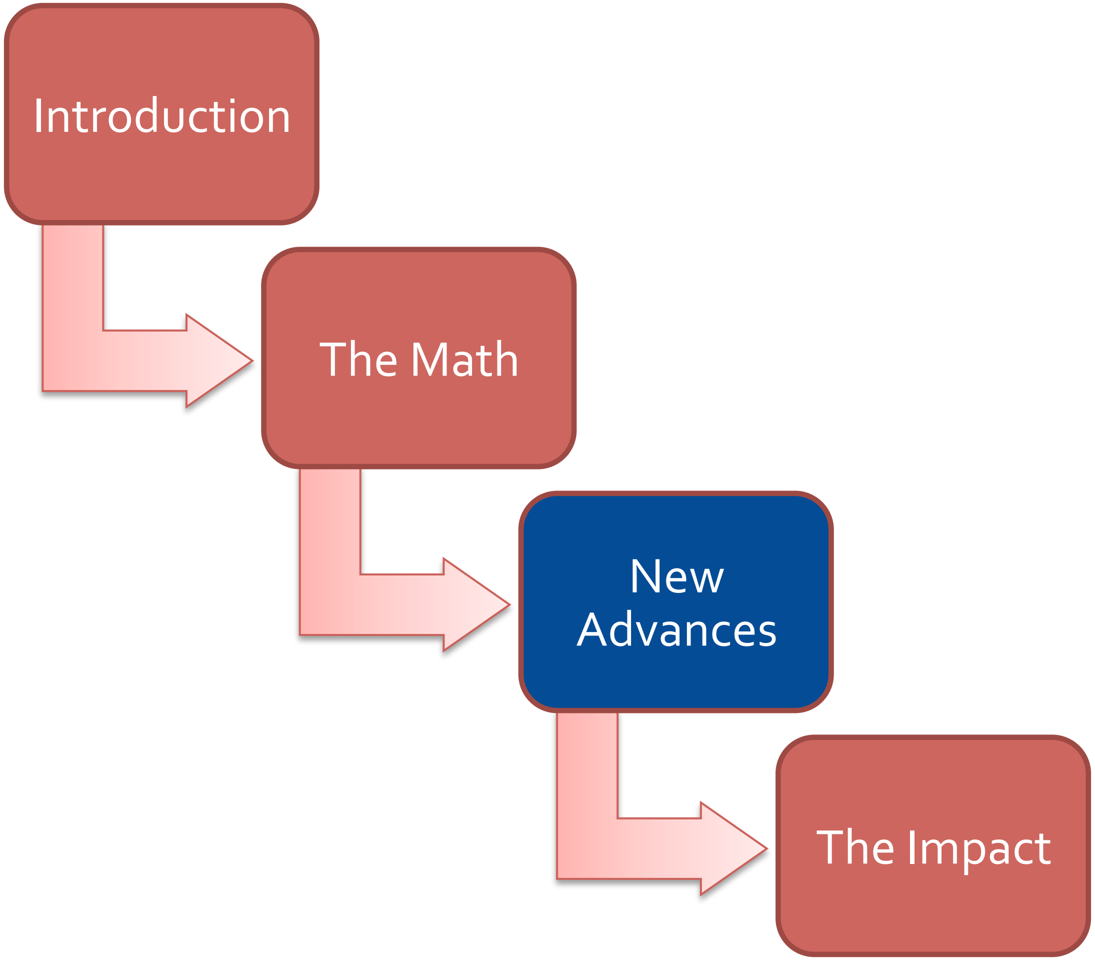


ECC v RSA key sizes

Symmetric	DH or RSA	ECC
56	512	112
80	1024	160
112	2048	224
128	3072	256
192	7680	384
256	15360	521

NIST Recommended Key Sizes

Agenda


Introduction

The Math

New Advances

The Impact

Discrete Logarithm Algorithms

- Generic algorithms (for any G)
  - Example: Pohlig-Hellman
  - Shows that discrete logarithm can be solved by breaking up the groups into subgroups of prime order.
  - Generic algorithms are exponential time algorithms.

- Specific algorithms which make use of group representation
  - Example: Index calculus algorithms
  - They leverage particular properties of the group
  - Result in sub-exponential running time

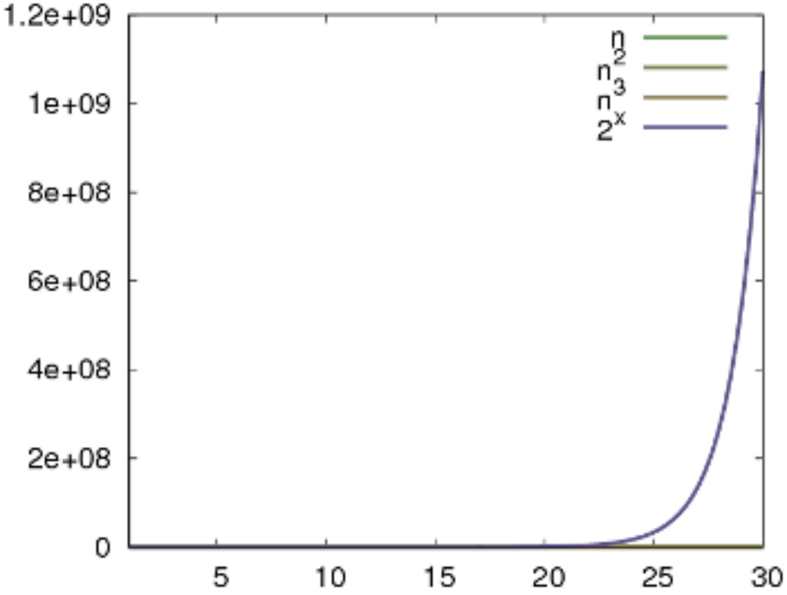
Exponential vs Polynomial

L(0) – Polynomial

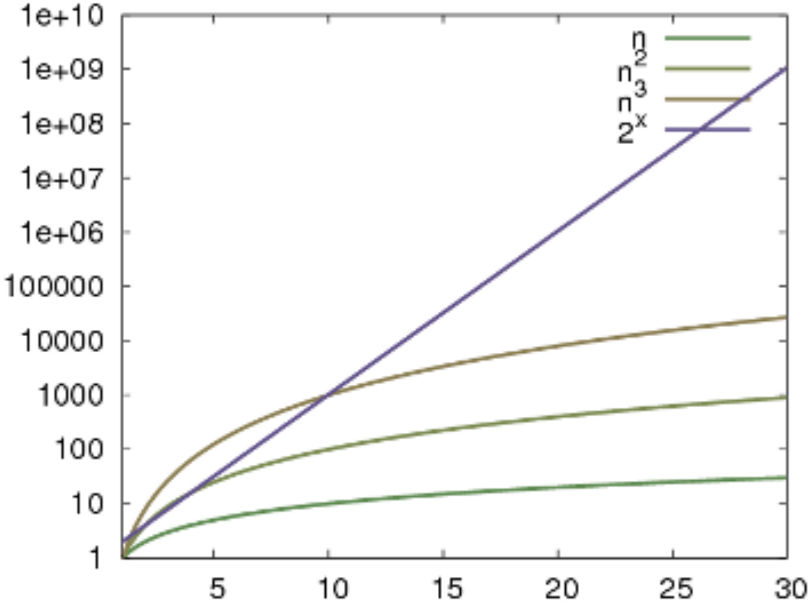
Fast enough to scare you

L(1) – Exponential

Way Too Slow

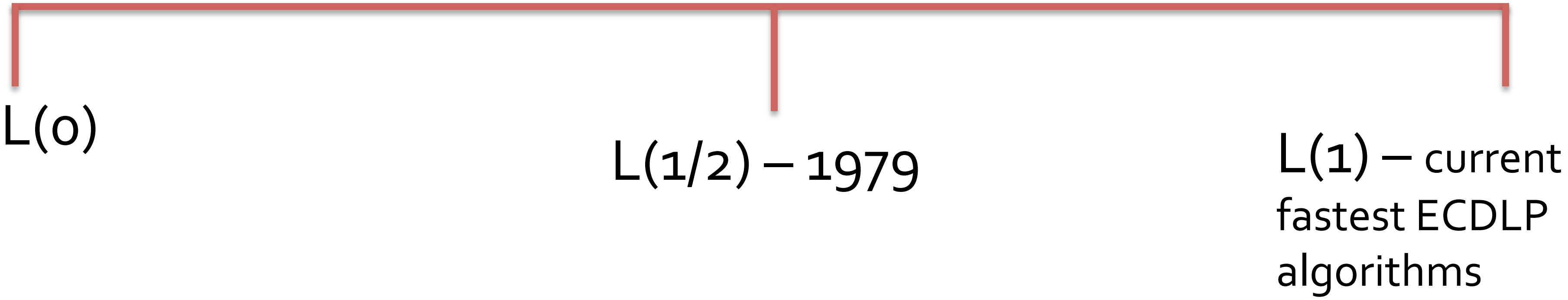


Linear running time plot



Logarithmic running time plot

Exponential vs Polynomial

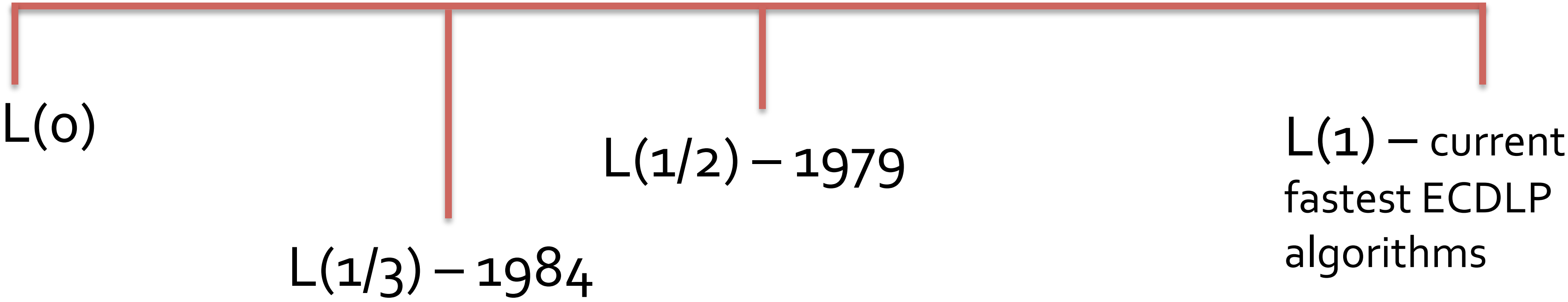


L(0)

L(1/2) – 1979

L(1) – current fastest ECDLP algorithms

Exponential vs Polynomial



L(0)

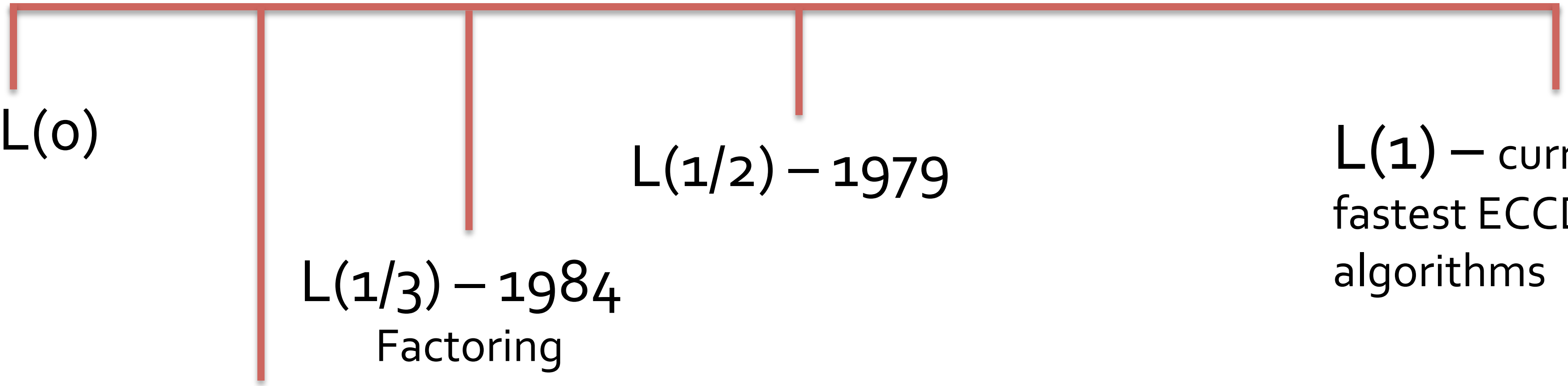
L(1/3) – 1984

L(1/2) – 1979

L(1) – current fastest ECDLP algorithms

Factoring and Discrete Logs stay here for the next 30 years

Exponential vs Polynomial



L(0)

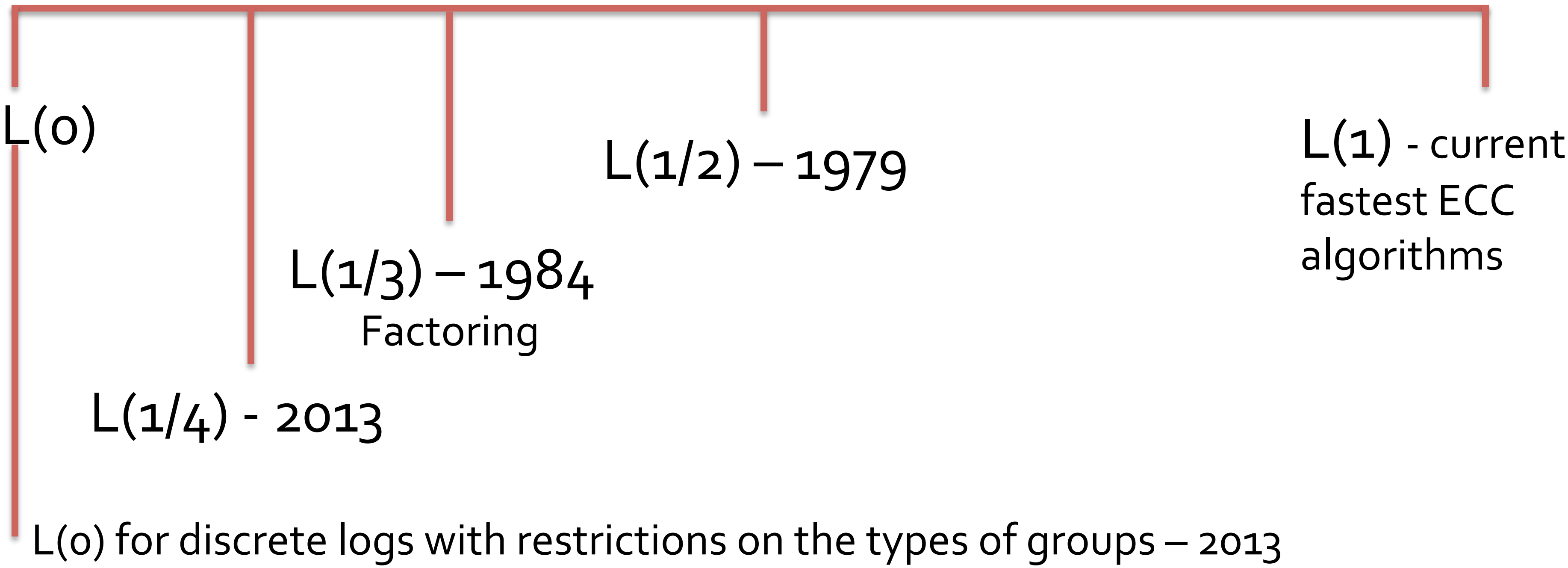
L(1/4) for Discrete Logs with restrictions on the types of group - 2013

L(1/3) – 1984 Factoring

L(1/2) – 1979

L(1) – current fastest ECCDLP algorithms

Exponential vs Polynomial



L(0)

L(1/4) - 2013

L(1/3) – 1984
Factoring

L(1/2) – 1979

L(1) - current fastest ECC algorithms

L(0) for discrete logs with restrictions on the types of groups – 2013

New Developments in 2013

- Rapid progress in DL research in past 6 months
  - February 20, 2013: Joux published a L(1/4) algorithm to solve DLP in small characteristic fields.
  - April 6, 2013: Barbulescu et al solve the DLP in of $F_{2^{809}}$ using the Function Field Sieve algorithm (FFS)
  - June 18, 2013: Barbulescu, Gaudry, Joux, Thomé publish a quasi-polynomial algorithm for DLP in finite fields of small characteristic.

Joux’s New Discrete Log Algorithm (Feb 2013)

- Uses judicious change of variables to find multiplicative relations easier.

- Uses a specific polynomial with linear factors to simplify the computation.

- Uses a new descent algorithm to expresses arbitrary elements in the finite field.

- Complexity is L(1/4 + o(1)) which is considerably faster than any discrete logarithm algorithm published before.

More Improvements

June 2013, Barbulescu, Gaudry, Joux, Thomé

- Quasi-polynomial algorithm for DL in finite fields of small characteristic.

- Improves Joux's February 2013 algorithm using special matrix properties.

- Fastest discrete logarithm has been improved significantly in the past 6 months after marginal progress in 25 years.

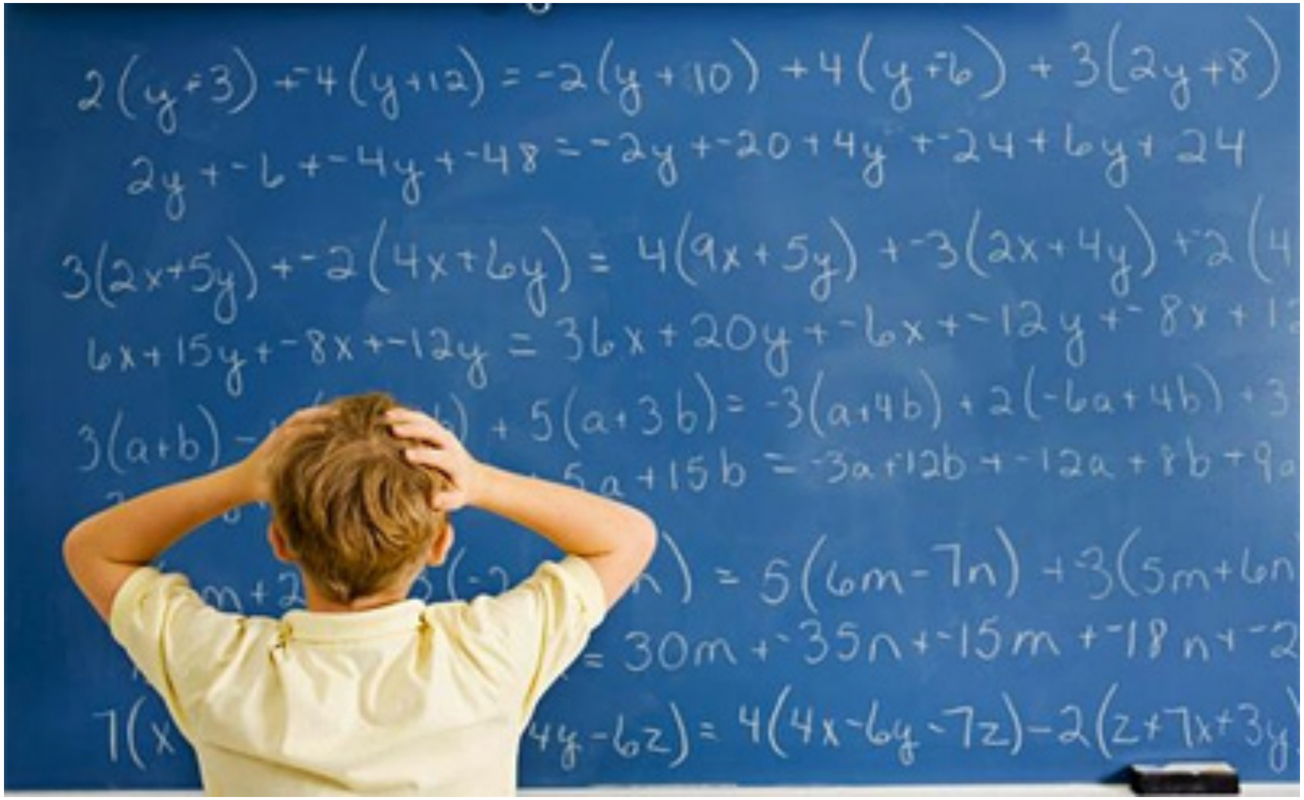
- However; no clear jump to more practical implementations which use finite fields with larger characteristic YET!

Implications of Discrete Log Progress

- Pairing based cryptography (PBC) over small characteristics is no longer secure.
  - PBC can be used for identity-based encryption, keyword searchable encryption where traditional public key cryptography may be unsuitable.
  - Currently used mainly in academic circle.

- Improves the Function Field Sieve (FFS) in most cases.
  - The function field sieve currently can be used to solve for small to medium characteristics fields.

Why Should I Care?


2(y+3) + -4(y+12) = -2(y+10) + 4(y+6) + 3(2y+8)
2y + -6 + -4y + -48 = -2y + -20 + 4y + -24 + 6y + 24
3(2x+5y) + -2(4x+6y) = 4(9x+5y) + -3(2x+4y) + 2(4
6x+15y + -8x + -12y = 36x + 20y + -6x + -12y + -8x + 1
3(a+b) -) + 5(a+3b) = -3(a+4b) + 2(-6a+4b) + 3
5a + 15b = -3a + 12b + -12a + 8b + 9a
m+2) = 5(6m-7n) + 3(5m+6n
= 30m + -35n + -15m + -18n + -2
7(x 4y-6z) = 4(4x-6y-7z)-2(z+7x+3y

Function Field Sieve

- Function Field Sieve has Four Steps
  - Choose a Polynomial
  - Relation Filtering
  - Linear Algebra
  - The Descent

- In the last 6 months, all of them have been improved
  - More likely something can be used on something we care about

- His record setting calculation, in May, took 550 Hours
  - 512 Bit RSA takes 652 Hours

Attacking DH, DSA, ElGamal

- Joux has attacked fields of a small characteristic
- We use fields of a large characteristic

- Joux's…
  - Polynomial choice probably would not help
  - Sieving Improvements may help
  - Descent Algorithm needs tweaking, but definitely helps

- Renewed interest could result in further improvements.

Attacking RSA

- Factoring advances tend to lead to advances in Discrete Log
- Discrete Log advances tend to lead to advances in Factoring
- Degrees of difficulty of both problems are closely linked.

Mutual Advances over the years

- 1975 Pollard's Rho in Factoring -> 1978 Pollard's Rho in Discrete Log.
- 1984 Quadratic Sieve Factoring -> 1987 improvements in Discrete Log Index Calculus Algorithms.
- 1993/4 Discrete Log Number & Function Field Sieves -> 1994 General Number Field Sieve for Factoring.

Factoring vs Discrete Logs

Factoring

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. Square Root

Discrete Logs

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. The Descent

Factoring vs Discrete Logs

Not That Slow

Factoring

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. Square Root

Constant Time

Discrete Logs

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. The Descent

Factoring vs Discrete Logs

Not That Slow

Factoring

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. Square Root

Easy to Parallelize

Constant Time

Discrete Logs

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. The Descent

Factoring vs Discrete Logs

Not That Slow

Constant Time

Factoring

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. Square Root

Discrete Logs

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. The Descent

Easy to Parallelize

Slow & Difficult to Parallelize

Factoring vs Discrete Logs

Not That Slow

Factoring

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. Square Root

Constant Time

Discrete Logs

1. Polynomial Selection
2. Sieving
3. Linear Algebra
4. The Descent

Easy to Parralellize

Slow & Difficult to Parallelize

Very Fast

Very Slow

Attacking RSA

- No obvious technique right now from Joux's improved discrete logarithm algorithm that applies directly to factoring.
- But I'm not a mathematician, I just play one on stage – I wouldn't bet the farm on that
- Public colloquium and publications seem to indicate that NSA/NIST may also already be very concerned.

Public Implementations & Tutorials

- MSIEVE
  - http://sourceforge.net/projects/msieve/
- CADO-NFS
  - http://cado-nfs.gforge.inria.fr/
- GGNFS
  - http://www.math.ttu.edu/~cmonico/software/ggnfs/
- Tutorials
  - http://github.com/tomrittervg/cloud-and-control

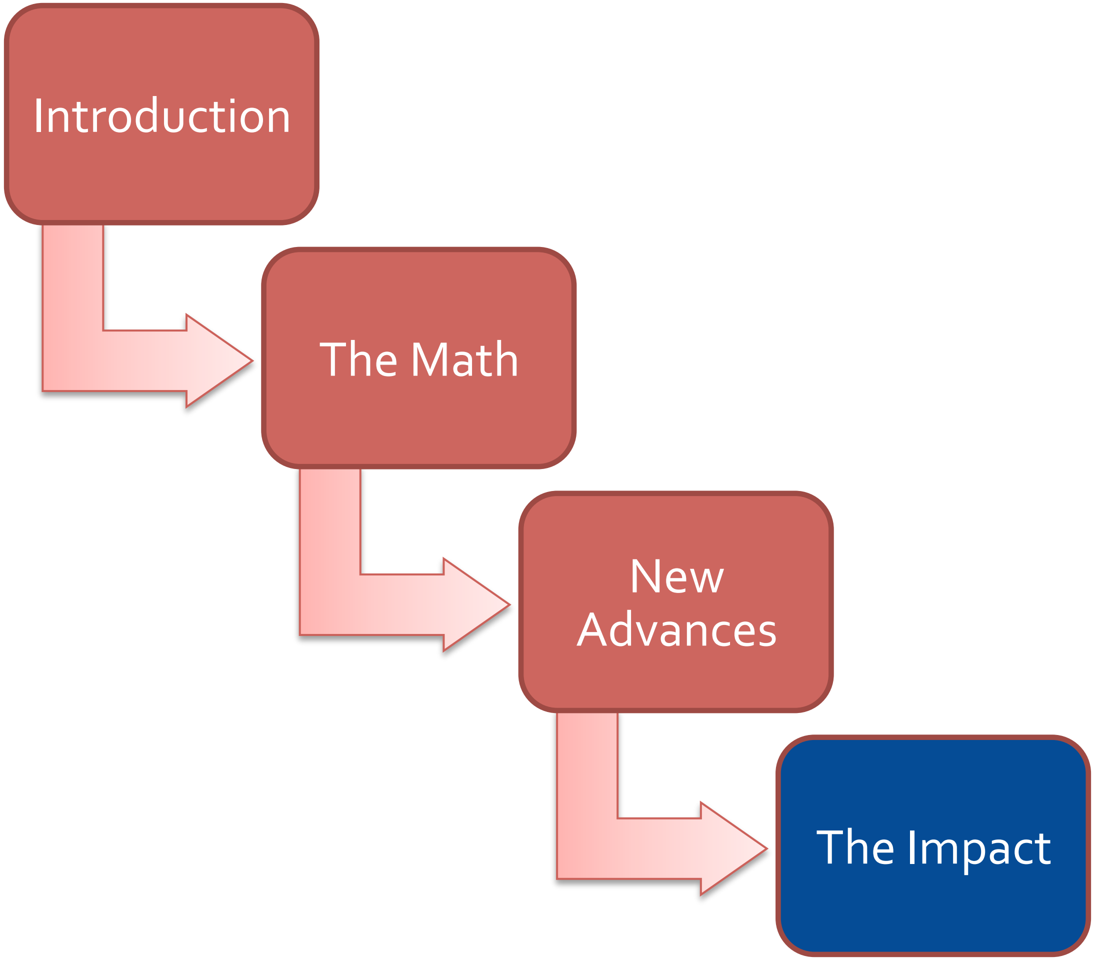
Implications

- ECC is still standing - still requires exponential time algorithms

- If Joux or others hits upon a general purpose discrete logarithm algorithm as fast his special purpose one…
  - Diffie-Hellman, DSA, and El-Gamal are toast
  - If that leaps to factoring - RSA is toast

- Technically not dead, but…
  - RSA key sizes may have to go up to 16,384 bits
  - Wildly impractical for actual use, never mind that nothing supports keysizes that large

Agenda



What Happens If DH or RSA Fails Now?

- Widespread active and passive attacks against live and recorded TLS.
  - PFS not necessarily the panacea
- Failure of code-signing and update mechanisms
  - How do you fix your software
- Failure of PGP, S/MIME and most end-to-end encryption
- Almost total failure of trust in the Internet

So, what now?

- We need to move to ECC, rather quickly
- Alex says that ECC is perfectly secure, YAY!
- Not really
  - <30 years of research versus 400:
  - Uses some of the same ideas

- Right now it's all we have
  - Long-term, we need more research into alternatives
  - RSA was 1977, RC4 was 1984. Give Rivest a break.

Why has ECC uptake been so slow?

- Lots of push from academia and government into ECC
- DH/RSA are here and they are easily understood
- Legal risks have slowed ECC adoption
- ECC had compatibility problems, but NIST has specified 15 standard curves

Overview of Suite B

- In 2005, the NSA released the Suite B set of interoperable standards

- Suite B specifies:
  - The encryption algorithm (AES-256)
  - The key exchange algorithm (Elliptic Curve DH)
  - The digital signature algorithm (Elliptic Curve DSA)
  - The hashing algorithms (SHA-256 and SHA-384)

Hmm, what's missing?

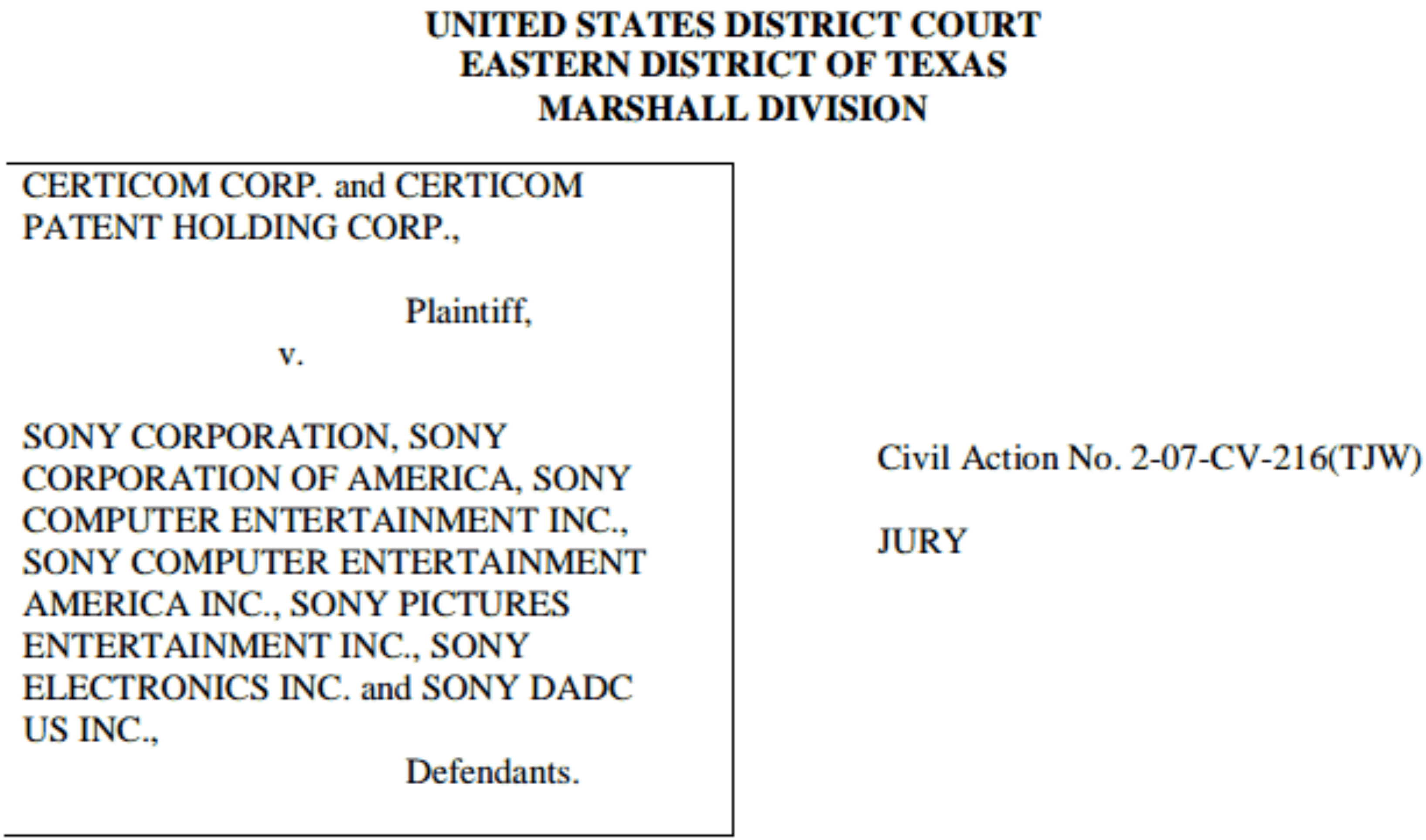
ECC Patents

- The patent issue for elliptic curve cryptosystems is the opposite of that for RSA and Diffie-Hellman.
    - RSA and Diffie-Hellman had patents for the cryptosystems but not the implementation.
- Several important ECC patents owned by Certicom (Blackberry)
    - Efficient $GF(2^n)$ multiplication in normal basis representation.
    - Technique of validating key exchange messages to prevent a man-in-the-middle attack.
    - Technique for compressing elliptic curve point representations.

ECC and Suite B

- NSA purchased from Certicom (now Blackberry) a license that covers all of their intellectual property in a restricted field of use.
- License is limited to implementations that were for national security uses and certified under FIPS 140-2 or were approved by NSA.
- Commercial vendors may receive a license from NSA provided their products fit within the field of use of NSA's license.
- Commercial vendors may contact Blackberry for a license for the same 26 patents.

Maybe Certicom is cool about this?



UNITED STATES DISTRICT COURT
EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CERTICOM CORP. and CERTICOM
PATENT HOLDING CORP.,

Plaintiff,

v.

SONY CORPORATION, SONY
CORPORATION OF AMERICA, SONY
COMPUTER ENTERTAINMENT INC.,
SONY COMPUTER ENTERTAINMENT
AMERICA INC., SONY PICTURES
ENTERTAINMENT INC., SONY
ELECTRONICS INC. and SONY DADC
US INC.,

Defendants.

Civil Action No. 2-07-CV-216(TJW)

JURY

ECC Support on Operating Systems

OS	Library	ECDH	ECDSA	Others	Version
OSX/IOS	ssl-36800	Yes	Yes	None	10.6
OSX/IOS	smime-36873	Yes	Yes	None	10.6
Windows	CNG	Yes	Yes	None	Vista
Windows	TLS	Yes	Yes	None	Vista
Windows	Suite B	Yes	Yes	None	Vista SP1, Windows 7

Table : Windows and OSX ECC Support

ECC Support on Android

OS	Library	ECDH	ECDSA	Others	Version
Android	Bouncy Castle	Yes	Yes	None	4.0
Android	TLS	Yes	Yes	None	3.2.4
Android	CyaSSL	Yes	Yes	None	2.4.6
Android	NSS	Yes	Yes	NTRU	3.11

Android ECC Support

ECC Support on Programing Languages

Programming Language	Library	ECDH	ECDSA	Others	Version
Python	PyECC	Yes	Yes	ECIES	2.4
C	OpenSSL	Yes	Yes	None	3.2.4
Java SE6	Bouncy Castle	Yes	Yes	None	Java 6
Java SE7	Native	Yes	Yes	ECIES, ECDSA, ECHR	Java 7
Ruby	OpenSSL	Yes	Yes	None	1.8

Programming Languages ECC Support

Code Signing

- Windows Code Signing
  - Default is RSA
  - ECC is supported through CSPs but not default
- Android Code Signing
  - Both DSA and RSA are currently supported.
- iOS code Signing
  - Uses CMS
  - Supports ECDH and ECDSA.

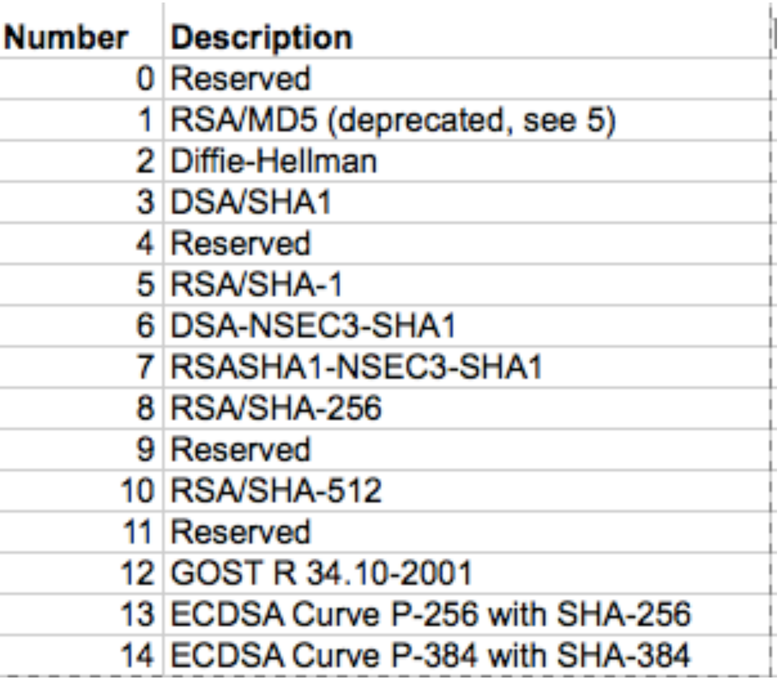
Transport Encryption

- TLSv1.2 is the first to include ECC options
  - Only `TLS_RSA_WITH_AES_128_CBC_SHA` is required
- Before TLS 1.2, CA and Cert had to match.
  - With 1.2 you can cross-sign
  - Can use `DH_DSS`, `DH_RSA`, `ECDH_ECDSA`, and `ECDH_RSA` with either ECC or RSA
- TLS 1.1 supports ECDH(E) for PFS

PKI Infrastructure

- ECC roots exist, buying a cert is not so easy
- There would significant work required in the transition form RSA to ECC certificates.
  - **Thawte Root Certificate6** - Root CA is not used today. Intended for use in the future for SSL certificates.
  - **Verisign/Symantec Root Certificate7** - ECC root certificate for 5 years; just begun offering commercial certificate this year.
  - **Entrust ECC Certificate8** - No global root certificate currently available today. Will use a Public ECC-256 Root.
  - **Comodo9 -** 384 bit ECC Root certificate.

DNSSEC



Number	Description
0	Reserved
1	RSA/MD5 (deprecated, see 5)
2	Diffie-Hellman
3	DSA/SHA1
4	Reserved
5	RSA/SHA-1
6	DSA-NSEC3-SHA1
7	RSASHA1-NSEC3-SHA1
8	RSA/SHA-256
9	Reserved
10	RSA/SHA-512
11	Reserved
12	GOST R 34.10-2001
13	ECDSA Curve P-256 with SHA-256
14	ECDSA Curve P-384 with SHA-384

- Current Root KSK generated in 2010 (algorithm 8)
    - Standard specifies rotated "when necessary" or at five years
- IANA, Verisign, ICANN SSAC looking at options
- ECC being considered
    - Helps with Zone File size
- Interesting enough, check out .ru

Other Popular Applications

- BlackBerry uses ECC extensively
- OpenVPN uses OpenSSL which includes ECC support, doesn’t seem to work
- IPSEC - Cisco, Shiva and Nortel gateways support ECDH IKE.
- OpenSSH has ECC support, not the default.

What do you do now?

If you are a… OS or language vendor

- Make ECC easy to use
  - See NaCl's box() and unbox()
- Update documentation to push developers away from RSA
- Get aggressive about compatibility testing
- Eat your own dogfood

If you are… a browser vendor

- TLS 1.2 needs to be a P1 feature
  - Only IE 11 and Chrome 29 support (both pre-release)
- Push at CA/B Forum for standardized process for cross-signed certificates

If you are a… software maker

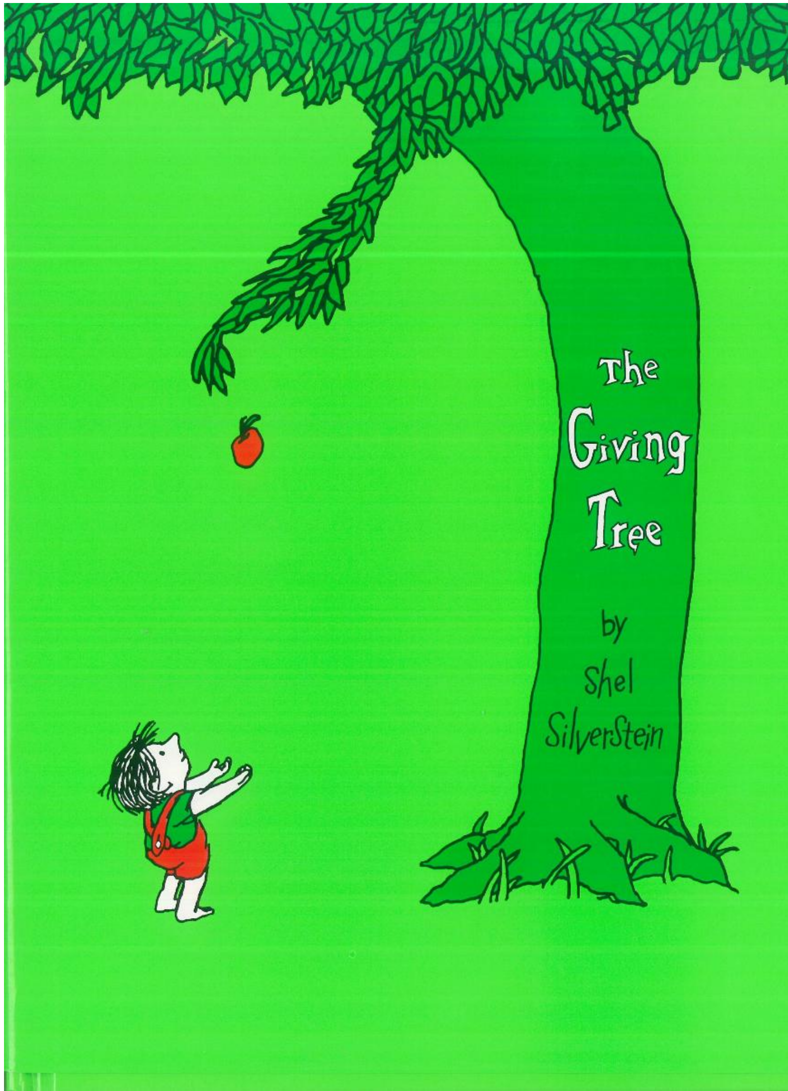
- You need to support TLS 1.2 on endpoints
- Build systems with pluggable primitives
  - Versioning
  - Handshake and negotiation
  - If this sounds too hard use TLS 1.2
- Use ECC for any new cryptosystems
- Retrofit old mechanisms using wrapping
  - ECC signed binary inside of legacy RSA signature

If you are a… Certificate Authority

- Make it easy to buy an ECC cert
- Change documentation to include ECC CSR instructions
- The CA/Browser Forum should promulgate standards pushing this

If you are… BlackBerry

- Make the world a safer place…
- License the ECC patents openly to any implementation of Suite B, regardless of use



If you are… just a normal company

- Use ECC certificates where possible
- Bug vendors for TLS 1.2 and ECC support
- Turn on ECDHE PFS today!
- Survey your exposure, so when the cryptopocalypse comes you are like this guy:

Summary

- Current cryptosystems depend on discrete logarithm and factoring which has seen some major new developments in the past 6 months.

- We need to move to stronger cryptosystems that leverage more difficult mathematical problems such as ECC.

- There is a huge amount of work to be done, so please get started now.

Thank You

- JasonP
- Antonie Joux
- Dan Boneh
- Ryan Winkelmaier (iSEC Partners intern)