IHP 2006

Experimental Tests of General Relativity

John Mester
Stanford University

Gravitational Redshift

This was the last of Einstein's 3 or (2 ½) tests to have any confirmation.

From GR Theory the period between ticks of a clock in arbitrary motion and gravitational field (the proper time) is given by

$\Delta t = (- g_{\mu\nu}\, dx^{\mu} dx^{\nu})^{1/2}$

if the clock has velocity $\frac{dx^{\mu}}{dt}$

the time between clicks of clock will be

$$\frac{dt}{\Delta t} = \left(-g_{\mu\nu}\frac{dx^{\mu}}{dt}\frac{dx^{\mu}}{dt}\right)^{-1/2}$$

So for a clock at rest the

$$\frac{dt}{\Delta t} = (-g_{00})^{-1/2}$$

In order to measure a time difference we need to compare clocks at different space-time locations

$$\frac{dt_1}{\Delta t} = (-g_{00}(x_1))^{-1/2} \qquad \frac{dt_2}{\Delta t} = (-g_{00}(x_2))^{-1/2}$$

So for clocks that output measured frequencies

$$\frac{\nu_2}{\nu_1} = \frac{(-g_{00}(x_2))^{-1/2}}{(-g_{00}(x_1))^{-1/2}}$$

In the weak field limit $g_{00} = -1 - 2\phi$ (ϕ is the Newtonian potential, $C^2=1$)

Also $\nu 2/\nu 1 = 1 + \Delta\nu/\nu$ therefore

$$\frac{\Delta\nu}{\nu} = \phi(x_2) - \phi(x_1)$$

Note this is the same equation one would get just from the equivalence principle (and conservation of energy E=hν)

That is, suppose an isolated system of one object at position 1 and another at position 2 is arranged so that object 1 will release a photon that will be absorbed by object 2. The total energy of the system before release should equal the energy after absorption.

gravitational + internal energy before = gravitational + internal energy after

$\phi_1 m_1^* + m_1^* + \phi_1 m_2 + m_2 = \phi_1 m_1 + m_1 + \phi_1 m_2^* + m_2^*$

$m_a^* = m_a + h\nu_a$

➔ $\phi_1 h\nu_1 + h\nu_1 = \phi_2 h\nu_1 + h\nu_2$

And so this gives what we obtained for the full theory $\nu_2/\nu_1 = 1 + \phi_1 - \phi_2$

So tests of gravitational redshift are EP tests.—They do not test the predictions of GR vs alternative metric theories (alternative theories that obey EP)

So how can we measure the gravitational redshift?

Remember form the introduction

At surface of the sun $\Phi = 2\text{x}10^{-6}$
So the frequency of light admitted from an atomic transition on the sun should be shifted by 2 ppm compared to same transition on earth so this should be easy to detect.

Except that:
Doppler shifts can mask the effect.

In order to produce a shift of $2\ \text{x}10^{-6}$ requires a velocity of 600m/sec.

Rotation of sun and earth are known
and can be accounted for

Thermal effects are an issue
At 3000K (surface temps are 6000K) typical velocities of C,N and O
(light elements but heavier than the predominant H and He)
Are ~ 2 km/sec. Now this only causes broadening so with enough signal to noise one could determine the center to higher precision than the line width.

More serious issue is motion due to unknown convection currents
(and these can be seen to differ in different region of the solar disk)
This can be minimized by looking at the limb of the sun since the major convection motion is vertical. Still quantitative results that test EP and not solar models are difficult.

White dwarfs have about the mass of the sun and smaller radii
(1/10 to 1/100 of sun) so the larger ϕ can produce redshift 10 to 100 times higher than that for the sun. Here the problem is knowing ϕ or even knowing the mass of the white dwarfs to make a quantitative test. A partial way out is to find white dwarf binaries to get an independent mass determination. Still white dwarf models are needed to determine ϕ.

What about the earth? The effect is much smaller but one can take advantage of a controlled experiement.

Pound Rebka Experiement

In 1959 Professor Robert Pound of Harvard University with graduate student Glen Rebka began investigating the possibility of detecting the gravitational redshift in an earth bound experiment.

The Jefferson Physical Laboratory at Harvard has an enclosed, isolated tower that is 22.5 meters high.

Over this height the gravitational potential of the earth varies by $2.5\text{x}10^{-15}$ (gh/C^2)

In order to measure a relative frequency shift this small Pound needed to find an EM transition of narrow line width

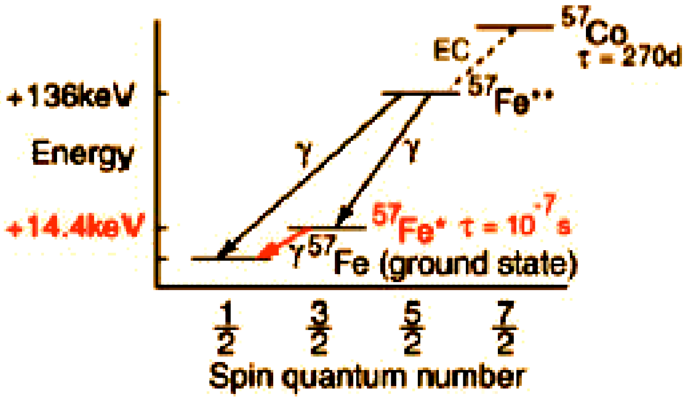
The 14kev γ ray transition in Fe 57 proved sufficient.

This transition has an intrinsically narrow line width –

Natural Lifetime $\tau = \sim 10^{-7}$ sec

Natural Linewidth $\Gamma = h/\tau = 10^{-8}$ eV

➔ a fractional FWHM $\Gamma/E = 1\text{x}10^{-12}$



So the idea was to have an emitter source at one end of the tower and an absorber at the other

And then to divide line width further by studying the resonant absorption.

Such a scheme was made possible by the discovery just the year prior of recoil free resonant absorption by Mössbauer.

The Mössbauer effect

The typical resonant absorption or fluorescence observed in optical transitions is hampered by the large

recoil energy of nuclear transitions.

For our case of Iron a 14.4 keV γ ray has a momentum P = 14.4 keV/C

So for and isolated Fe atom conservation of momentum leads to a recoil energy of

$E_R = P^2C^2/2m_{Fe}C^2 = 0.002eV$

This is much larger (5 orders of magnitude) than the natural linewidth so the γ ray produced has an energy too small to be absorbed by another Fe nucleus (which would also recoil on absorption, doubling the problem).

Mössbauer discovered that if the emitting atom were contained in a crystal lattice, then at sufficiently low temperatures the atom could not recoil on its own. That is, the quantization of the lattice vibrational states requires that the recoil energy be taken up by the whole lattice.

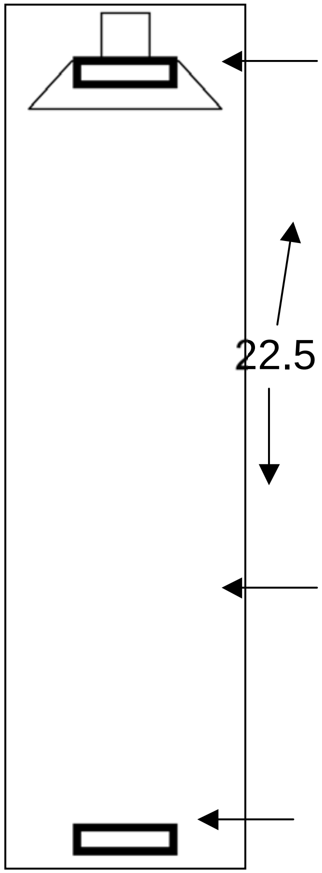
So if we have a lattice of greater than 10^5 atoms then the recoil energy lost

$E_R = P^2C^2/2m_{\text{lattice}}C^2$ will be less than the natural linewidth

Pound Rebka Experiement

SO now that we have a source and absorber, how can we detect the fraction change in frequency over the tower height.

To do this Pound and Rebka mounted the source on a voice coil motor (a speaker) and modulated it up and down at between10–50 Hz



Source mounted on speaker peak velocity greater than the doppler velocity
Corresponding to a $\Delta\nu = 2.5x10^{-15}$, that is > $7x10^{-7}$ m/s

22.5 m tower

He filled Mylar bag

Absorber mount on
NaI Scintillation detector

Scintillation counter was correlated with velocity of source.
Difference in counts recorded is a measure of frequency shift between emission and absorption lines

Source and absorber were switched and compared to double the effect

Full scheme from Pound and Rebka, PRL 4 7 April 1 1960 *The Apparent Weight of Photons*

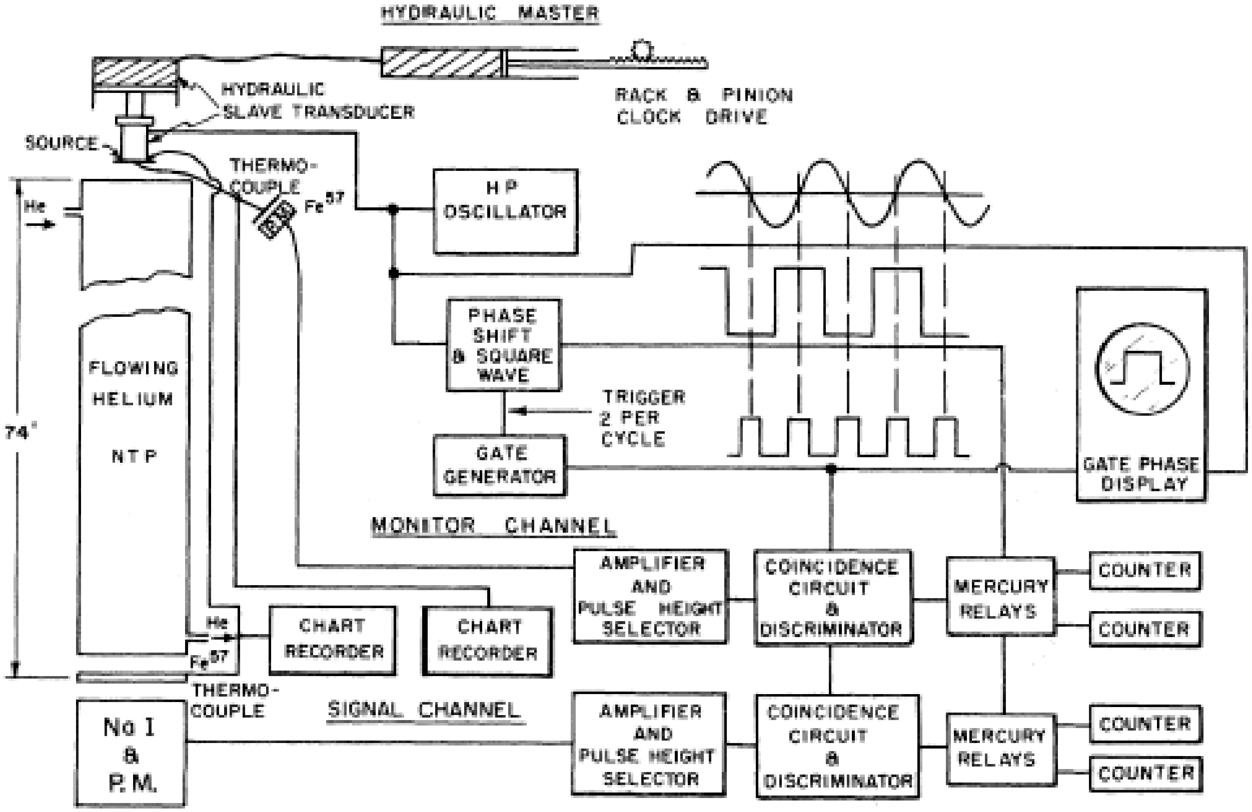


FIG. 1. A block diagram of the over-all experimental arrangement. The source and absorber-detector units were frequently interchanged. Sometimes a ferroelectric and sometimes a moving-coil magnetic transducer was used with frequencies ranging from 10 to 50 cps.

Now as I said the expected redshift, $\Delta\nu = 2{,}46 \times 10^{-15}$

While the linewidth Γ is 1.1×10^{-12} so for a stationary source and absorber the change in counting rate one would expect is

$$\Delta C = \frac{\Gamma^2}{\Delta\nu^2 + \Gamma^2}$$ or about 1 part in 2×10^{-5}

This would make the measurement very difficult.

Fortunately the effect of moving the source introduces a further shift

$$\Delta C = \frac{\Gamma^2}{\Delta\nu_{Grav}{}^2 + \Delta\nu_{Dop}{}^2 + \Gamma^2}$$ And for $V_{source}(t) = V_0 \cos\omega t$

$$C(t) = \frac{\Gamma/\nu^2}{V_{source}{}^2 + \Gamma/\nu^2}\left[1 + \frac{2\frac{\Delta\nu}{\nu}V_0\cos\omega t}{V_{source}{}^2 + \Gamma/\nu^2}\right]$$

then one can find $\Delta\nu$ by comparing the count rate when the source is moving up to that when the source is moving down.

Pound and Rebka did this and found $2\Delta\nu = (5.13 \pm 0.51) \times 10^{-15}$ a result good to 10%
Later Pound and Snider reduced the uncertainty to 1%, PRL 13 539 (1964)

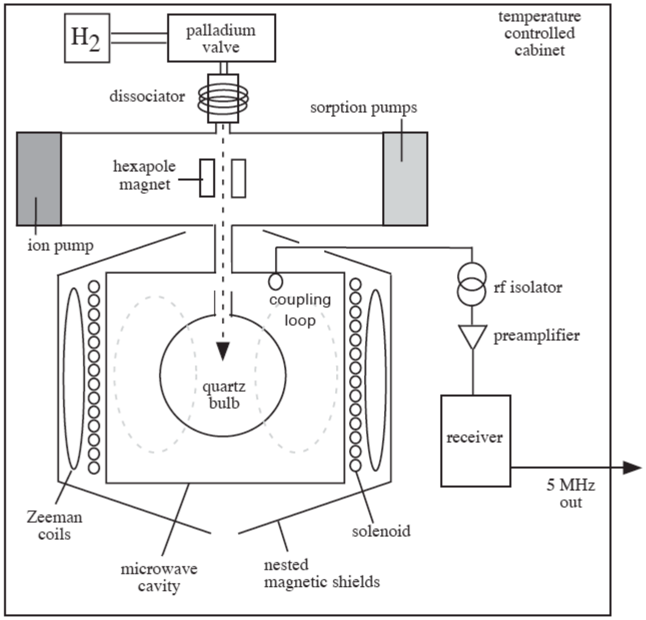
Gravity Probe A

In 1976 R. F.C. Vessot and others at the Harvard Smithsonian Astrophysical Lab succeeded in measuring the gravitational redshiift to 70 ppm by means of hydrogen maser . The maser was launched nearly vertically on a Scout rocket to a height of 10,000 km.

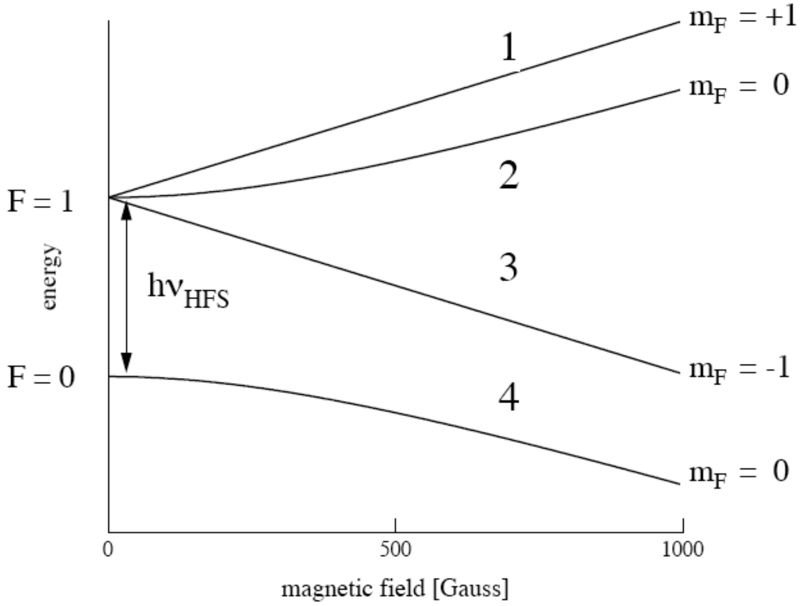
The maser GP-A employed had a frequency stability of better than 1 part in 10^{14} over 100 seconds.

Hydrogen Maser

The atomic hydrogen maser was developed by Norman Ramsey in the 1960s.



In the maser a beam of H atoms flow into a storage bulb inside a microwave avity tuned to the 1420MHz hyperfine transition. The atoms are magnetically state selected on the way in into the low field seeking high energy state.



Hydrogen hyperfine diagram

With suitable storage bulb wall coatings (like Teflon) the atoms reside in the cavity (interacting with the microwave field) for periods of 1 second or more resulting in narrow line widths. This can bee seen as an extension or the Ramsey separated oscillatory field technique.

The microwave field in the cavity stimulates a macroscopic magnetization of the atomic ensemble-and this magnetization stimulates the cavity field. This continuous interaction in called masing.

Quality H masers have frequency stabilities of 10^{-15} over periods of 1000 sec to 1 day.

Gravity Probe A Experiment Procedure

The Gravity Probe A experiment employed a multi-transponder scheme to simultaneously downlink the maser oscillation frequency, determine the trajectory, and subtract out first order Doppler shifts (df/f ~10^{-5}).
A clever choice of frequency allowed ionospheric dispersion to be eliminated.

Since the rocket is in motion relative to the earth for most of the flight the frequency of the maser on the rocket compared to that on the earth is given by:

$$\frac{\nu_{rocket}}{\nu_{earth}} = \frac{\left(-g_{\mu\nu}\frac{dx^{\mu}}{dt}\frac{dx^{\mu}}{dt}\right)_{rocket}^{-1/2}}{g_{00\,earth}}$$

In the weak field

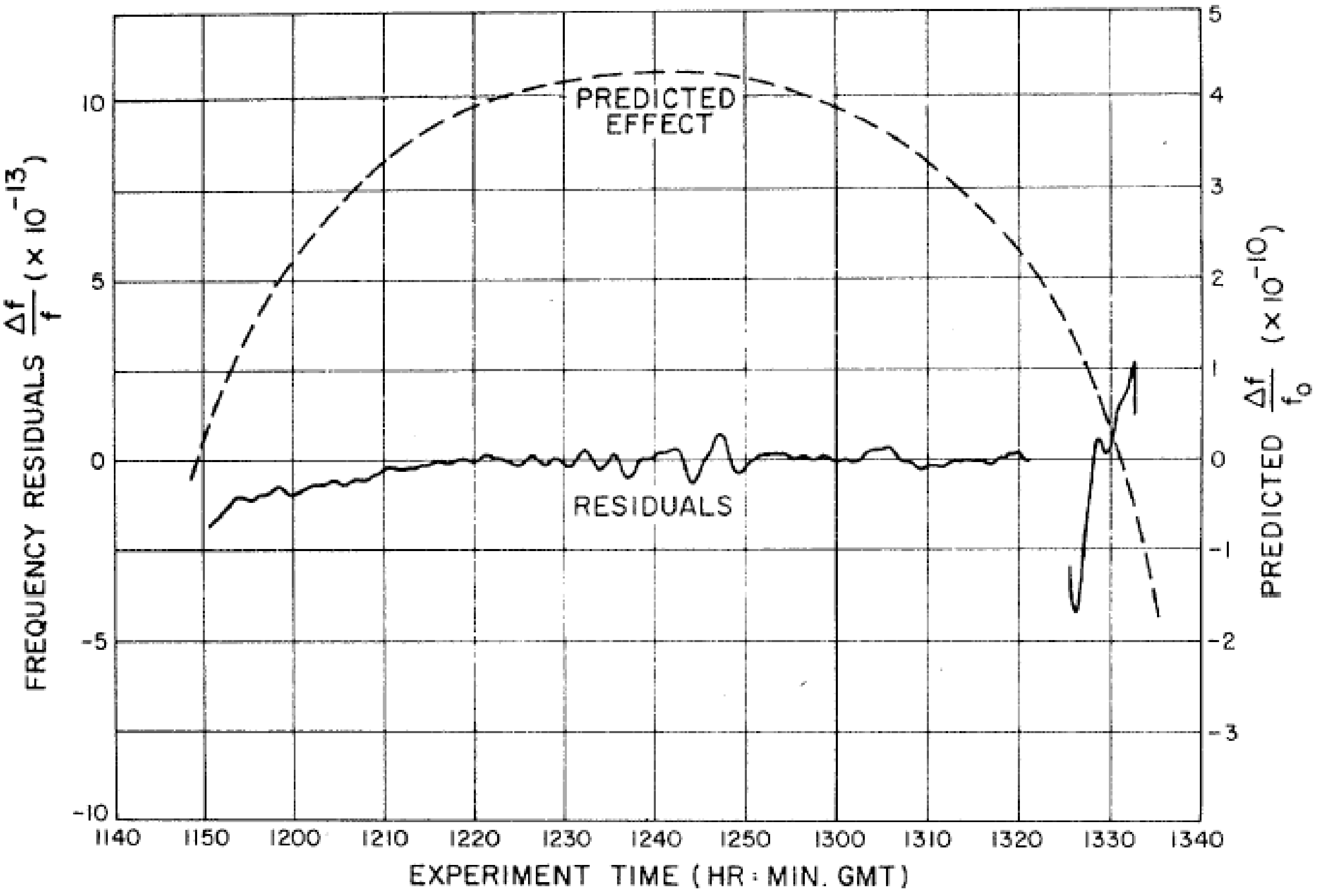
$$(-g_{\mu\nu}\frac{dx^{\mu}}{dt}\frac{dx^{\mu}}{dt})_{rocket} = -g_{00rocket} - v^{2}_{rocket} = 1 + 2\phi_{rocket} - v^{2}_{rocket}$$

where the ϕ term is the gravitation potential and the V^2 term is the special relativistic 2nd order Doppler term.

In fact the analysis included a third term due to residual first order Doppler shift caused by acceleration of the ground station (Vessot et al. PRL 45 26 (1980))

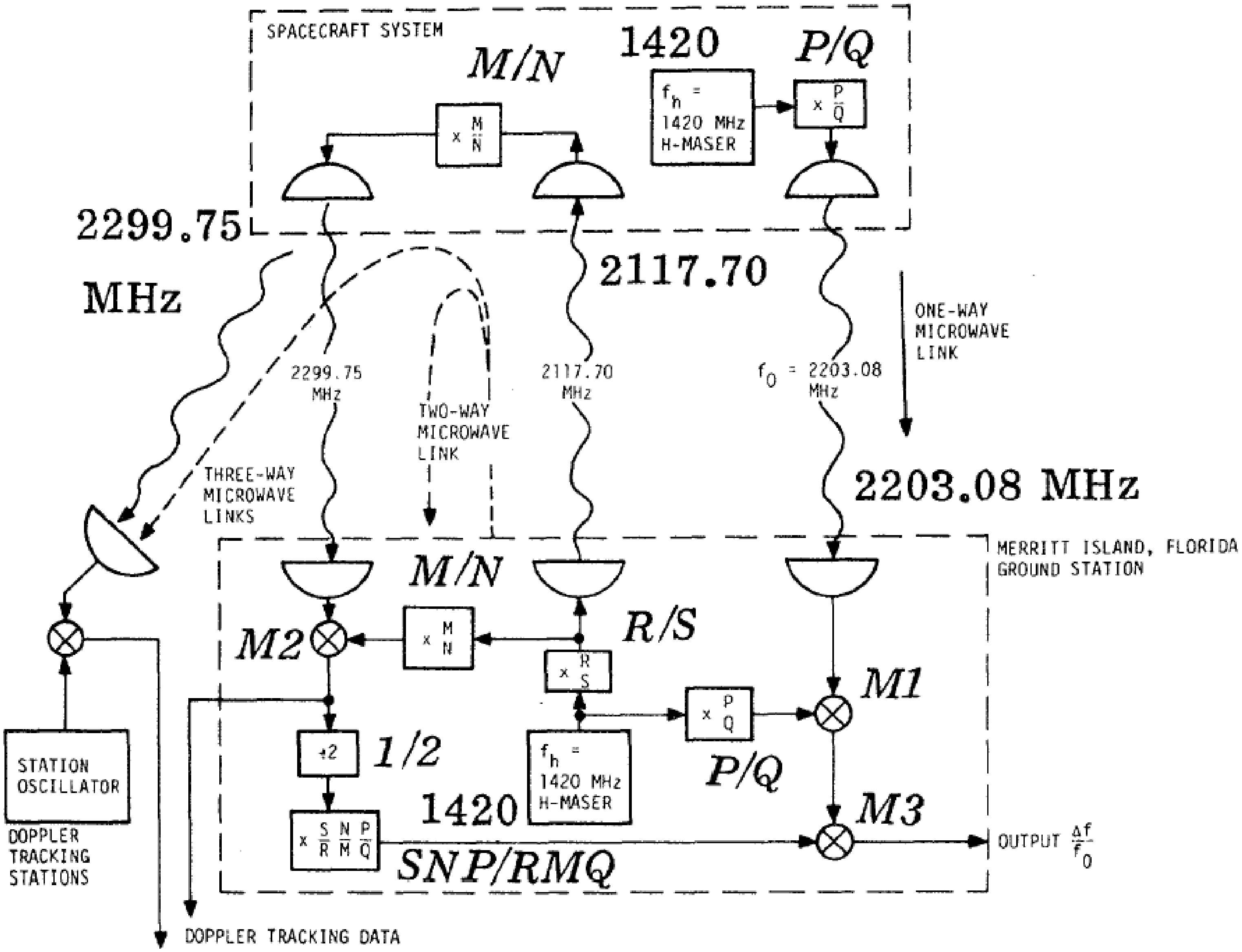
$$\frac{\Delta f}{f_0} = \frac{\varphi_s - \varphi_e}{c^2} - \frac{|\vec{v}_e - \vec{v}_s|^2}{2c^2} - \frac{\vec{r}_{se} \cdot \vec{a}_e}{c^2}$$

From this one can notice that one should see a blue shift in the lower altitude parts of the flight and them this crosses over into a redshift.



Vessot et al. PRL 45 26 (1980)

The flight data form GP-A. Overall residuals of 70 parts per million.


SPACECRAFT SYSTEM
M/N
1420
P/Q
x M/N
f_h = 1420 MHz H-MASER
x P/Q
2299.75 MHz
2117.70
ONE-WAY MICROWAVE LINK
2299.75 MHz
TWO-WAY MICROWAVE LINK
2117.70 MHz
f_0 = 2203.08 MHz
THREE-WAY MICROWAVE LINKS
2203.08 MHz
MERRITT ISLAND, FLORIDA GROUND STATION
M/N
R/S
M2
x M/N
x R/S
M1
x P/Q
÷2
1/2
f_h = 1420 MHz H-MASER
P/Q
STATION OSCILLATOR
DOPPLER TRACKING STATIONS
1420
M3
x S/R N/M P/Q
SNP/RMQ
OUTPUT $\frac{\Delta f}{f_0}$
DOPPLER TRACKING DATA

Future Gravitational Redshift Experiments

ESA and CNES are supporting the ACES mission to fly on the ISS.

ACES, Atomic Clock Ensemble in Space, will employ 2 precision clocks and a microwave links for clock comparisons on board and between the ISS and ground stations.

The Clocks:

SHM - Space Hydrogen Maser

PHARAO – Laser cooled atomic Cs Clock

Scientific Objectives rely on space –ground comparisons and include:

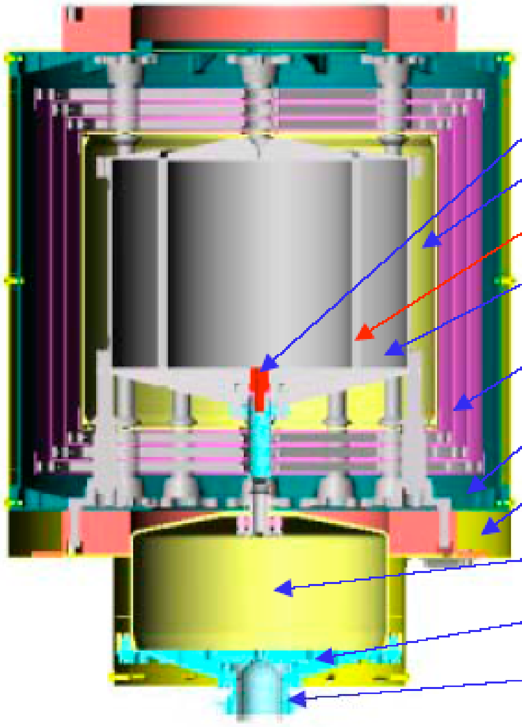
Special relativity tests, LLI
Variation of α
GR tests-red shift,

SHM view from Moriond Workshop

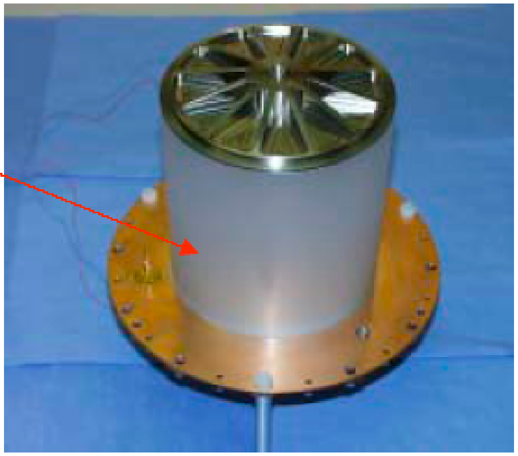
SHM PHYSICS PACKAGE

(Neuchâtel Observatory, Switzerland)

physics package cut view

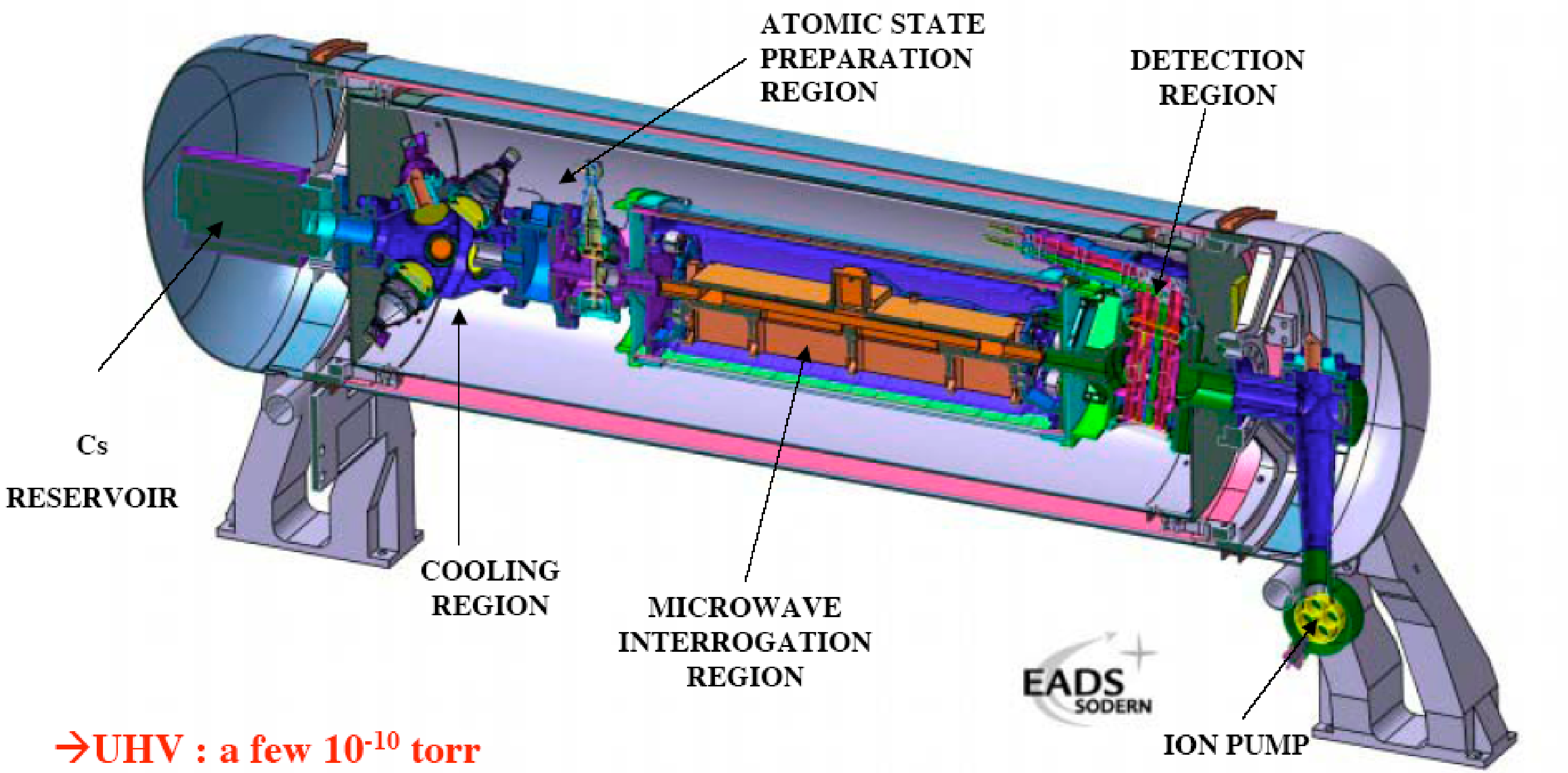


- PTFE storage bulb collimator
- cavity thermal shield & c-field
- sapphire storage bulb
- microwave cavity
- cavity magnetic shields (4x)
- thermal shield
- external magnetic shield
- vacuum main structure assembly
- hydrogen beam assembly
- dissociator assembly



PHARAO CESIUM TUBE DESIGN

(SODERN, France)



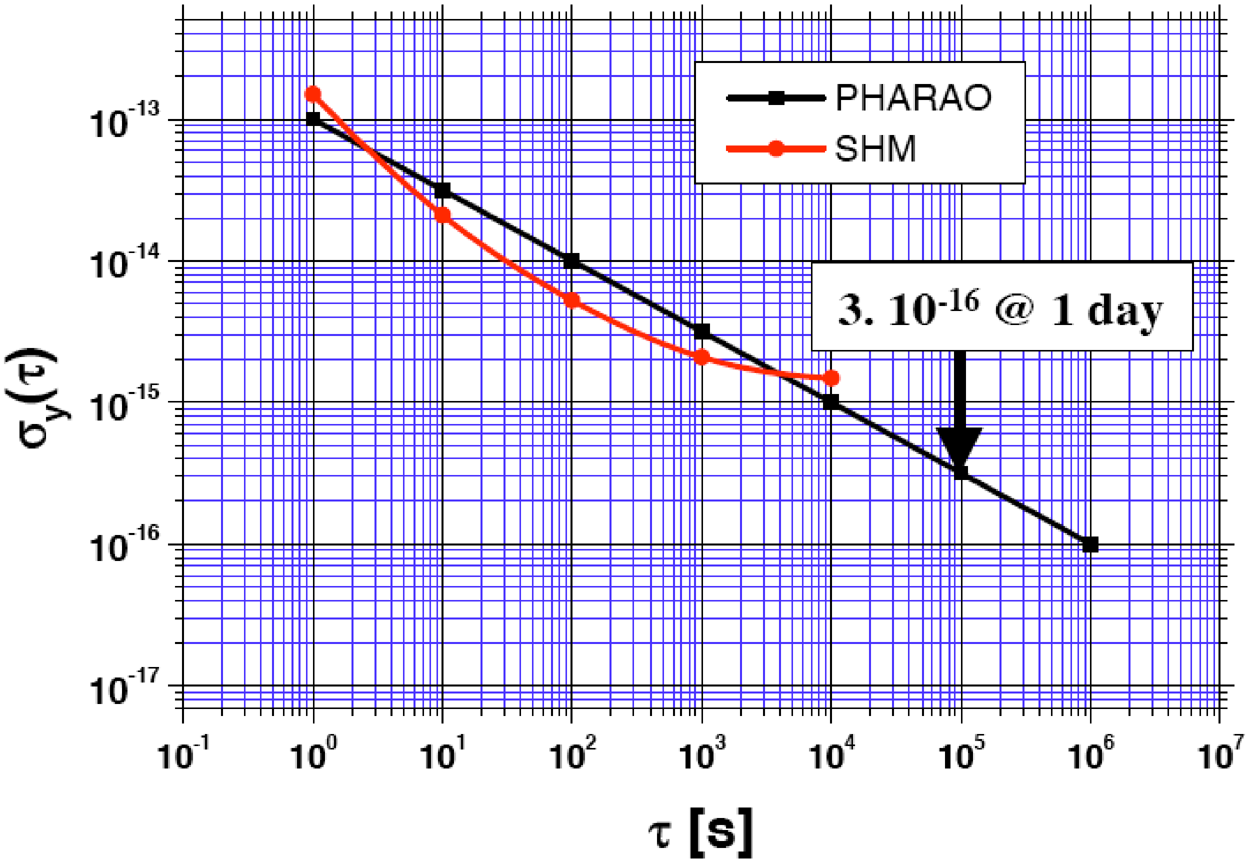
→UHV : a few 10^{-10} torr

→Cavity temperature control (0.1 °C), magnetic control ($< 1\ \mu G \approx 0.1\ nT$)

Comparison With Fountain Clock

Fountain:	v = 4 m/s	T=0.5 s	Δν =1Hz
PHARAO:	v = 0.05 m/s	T=5 s	Δν = 0.1Hz

Expected Frequency Stability of the ACES Clocks



Stability at one day: 3 10^{-16}

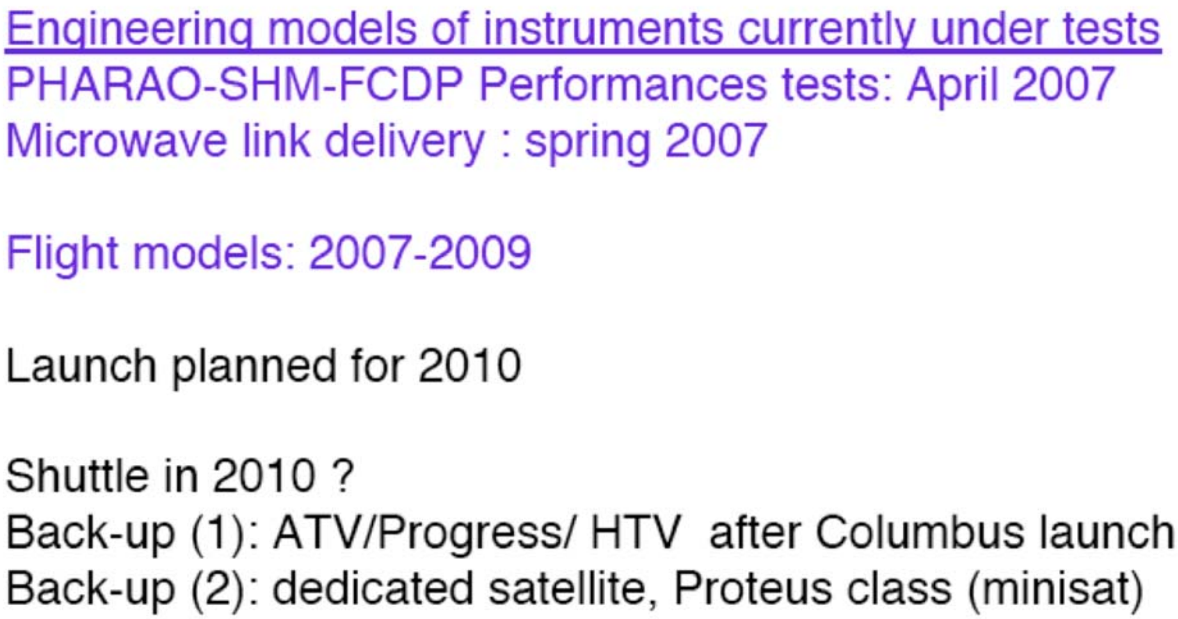
at 10 days: 1 10^{-16}

PREDICTED EFFECTS

1st order Doppler	2×10^{-5}
2nd order Doppler	3×10^{-10}
Gravitational red shift	4×10^{-11}

Goal of red shift measurement to a few ppm!	x35 Improvement over GP-A
Drift of fine structure constant α to 10^{-16} per year	x10 Improvement over present
Tests of Lorentz invariance SME	x10-100 Improvement over present

ACES Status



Engineering models of instruments currently under tests
PHARAO-SHM-FCDP Performances tests: April 2007
Microwave link delivery : spring 2007

Flight models: 2007-2009

Launch planned for 2010

Shuttle in 2010 ?
Back-up (1): ATV/Progress/ HTV after Columbus launch
Back-up (2): dedicated satellite, Proteus class (minisat)