Ptolemy's longitudes and Eratosthenes' measurement of the earth's circumference

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A statistical analysis of the longitudes reported in Ptolemy's *Geographia* shows that many of them were obtained by distorting in a linear way data which were known with good accuracy. As a consequence, a new estimate of the value of the stadion used by Eratosthenes is obtained, supporting the thesis that his measurement of the Earth's circumference was remarkably accurate. Some conjectures about possible simplifications introduced by Cleomedes in his account of Eratosthenes' method are also proposed.

1. The distortion of longitudes in Ptolemy's *Geographia*

The longitudes of 6345 localities[1] reported by Ptolemy in his *Geographia* [Stückelberger and Graßhoff 2006] are affected by an error which dilates their differences. While this error has been often remarked, it has not been so far analyzed in a quantitative way. The analysis of the distortion of the longitudes for all 6345 localities considered by Ptolemy is inconvenient for several reasons. First, many of the places are not identifiable with reasonable certainty. Furthermore for some regions the systematic error overlaps errors of different nature, due to the lack of knowledge of the country (this is the case, for example, for Indian localities). I have therefore preferred to consider a sample of eighty towns, chosen with the following criteria.

First, since it is plausible that Ptolemy's error stems from a wrong interpretation of hellenistic data, I have restricted the choice to the following regions, which were well known in the Greek world both in hellenistic and imperial times: Spain, Southern Gaul, Italy, Greece, Mediterranean coast of Africa west of Egypt, Egypt, regions of Asia that had belonged to the other hellenistic kingdoms.

Secondly, in order to minimize the influence of errors due to the lack of geographical knowledge and to enhance the effect of Ptolemy's systematic error, I have selected my (nonrandom) sample by trying to choose for each of the previous regions the most famous towns, as the ones whose coordinates were presumably

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[1] The number of localities has been counted by A. Stückelberger and G. Graßhoff [2006, p. 23].

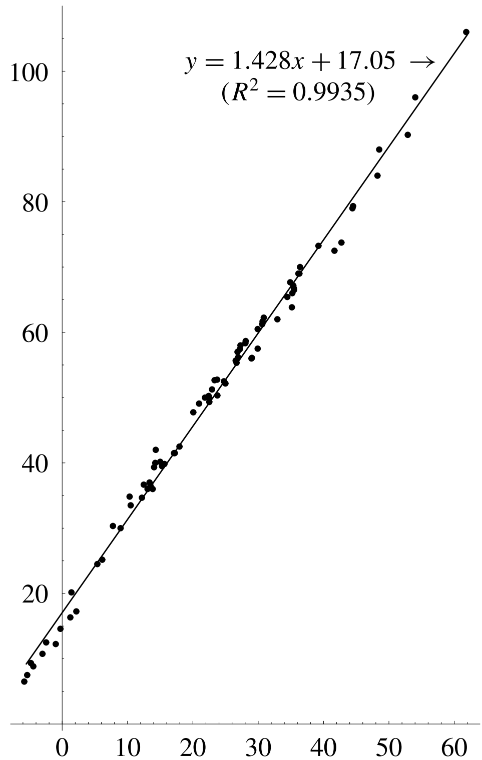

Figure 1. Ptolemy's longitudes (vertical axis) versus actual longitudes from Greenwich (horizonal axis) for the eighty towns in the chosen sample. See also Table 1.

best known, besides being identifiable with certainty. The towns of the sample are listed in Table 1 (page 77) with their longitudes, both actual and as reported by Ptolemy; the data are also plotted in Figure 1.

The regression line has equation

$$y = 1.428x + 17.05.$$

The term 17.05 is of course the longitude that Ptolemy would have assigned to Greenwich and is of no interest to us, while the regression coefficient, 1.428, gives a measure of the dilatation in longitude differences performed by Ptolemy.

We call x_i the actual longitudes from Greenwich of the towns of the sample, y_i their longitudes as reported by Ptolemy and z_i the corresponding values on the regression line ($z_i = 1.428x_i + 17.05$). The variances of the two last series of values are

$$\sigma^2(y_i) = 465.431, \qquad \sigma^2(z_i) = 462.406.$$

The coefficient of determination R^2, which is defined as the ratio $\sigma^2(z_i)/\sigma^2(y_i)$ and is considered a measure of how well empirical data are described by the regression line, is

$$R^2 = 0.9935.$$

A value of R^2 so close to 1 clearly shows that Ptolemy's numbers were obtained by distorting in a linear way data that were known with remarkable accuracy.

2. The value of the stadion used by Eratosthenes

Since antiquity Eratosthenes' measurement of the Earth's circumference is one of the most celebrated achievements of Alexandrian science. The principle of the method used by Eratosthenes in his measurement is well-known and it is not worth to be recalled here.[2] Since we know his result in "stadia", the accuracy of his measure cannot be evaluated without knowing the actual length of the "stadion" used by him. In the Greek world several different "stadia" had been in use and the value of the one used by Eratosthenes is a *vexata questio*. Hultsch, in 1882, had determined it as 157.5 meters [Hultsch 1882] and this measure was accepted by most of the scholars till the first half of the twentieth century. Among the many other values that have been proposed it seems that the most widely accepted nowadays is 185 meters, which is the length of the so-called "Attic stadion".[3] This value is documented in many sources, but not explicitly referring to Eratosthenes, while Hultsch's argument was based essentially only on a single statement by Pliny, which nevertheless refers explicitly to Eratosthenes.[4] If we accept Hultsch's value, the error of Eratosthenes' measure is less than 1%, while if we assume that his stadion was the Attic one the error is about 17%.

Whereas there is no general agreement on the length of the "stadion" used by Eratosthenes, all scholars agree that later geographers, like Hipparchus, Strabo, Marinus and Ptolemy, used his same stadion (as is shown by the fact that many distances in stadia have the same value for all of them). It is well known, on the other hand, that Ptolemy, like Marinus before him, did not accept Eratosthenes' measure of the meridian, corresponding to 700 stadia per degree, adopting instead the measure of 500 stadia per degree.[5] Since our regression coefficient is a fair approximation of the ratio 7/5 between the lengths of the Earth's circumference

[2] The reader is referred to [Russo 2004, pp. 68–69] for some considerations on Eratosthenes' method. While it may seem simple now, it was beyond the understanding of post-hellenistic antiquity.

[3] In [Rawlins 1982] the value of 185 meters for Eratosthenes' stadion is considered a well established fact. The same value is accepted by, among others, Dicks [1960] and by Berggren and Jones [2000].

[4] Pliny, *Naturalis historia* , XII, 53.

[5] The origin of this new measure is unknown. Ptolemy (*Geographia*, I, 11), without mentioning Eratosthenes' measure, simply states that there was a general consensus on the measure of 500 stadia

according Eratosthenes and Ptolemy, our computation shows that (as has often been suggested) the distortion operated by Ptolemy on the longitudes is not independent of the new value he had assumed for the length of the Earth's circumference. We may assume, as it is generally accepted,[6] that he had deduced his differences in longitude from known distances, measured in stadia, along a given circle of latitude, so that his distortion of longitudes compensates for the reduced dimensions of the Earth. We know, in fact, that, given the difficulty of determining longitudes by astronomical methods, hellenistic geographers like Eratosthenes preferred to use, instead of longitudes, distances along a given circle of latitude. Since we know that Ptolemy assumed that one degree of a great circle of the Earth had the length of 500 stadia, we can recover from his longitudes the original distances in stadia between a large number of localities, getting precious information on the actual value of the stadion used in geographical treatises.

We call Δl the difference in longitude between two arbitrary places, Δl_T their difference in longitude according to Ptolemy, d_m and d_s the measures, respectively in meters and in stadia, of the arc of equator comprised between their meridians.

Since $d_m \approx 111{,}100\Delta l$, $d_s \approx 500\Delta l_T$, we get that the value in meters of the stadion is

$$s = \frac{d_m}{d_s} \approx \frac{111{,}100}{500}\,\frac{\Delta l}{\Delta l_T} = 222.2\frac{\Delta l}{\Delta l_T}.$$

By replacing the ratio $\Delta l/\Delta l_T$ with its mean value given by the regression coefficient, 1/1.428, we obtain for the stadion the value of 155.6 meters. Since $155.6 \times 252{,}000 = 39{,}211{,}200$, this value would correspond to an error a little less than 2% on Eratosthenes' measurement of the great circle of the Earth.

A possible objection to this procedure is that we cannot exclude that the distances known to Ptolemy were affected by a significant systematic error (so that their accuracy was small, despite their remarkable precision). I can answer this objection in two ways. First, if all large distances were affected by the same systematic error, the value obtained for the stadion may be very different from the one understood by ancient geographers, but corresponds very well to its value *de facto*; in other words, we can use it to convert effectively to kilometers the large distances in stadia recorded by ancient geographers. Secondly, the circumstance that the value we obtained is remarkably close to the one determined by Hultsch on philological grounds (157.5 meters) makes the previous possibility unlikely, lending strong support to Hultsch's determination and allowing us to exclude, in my opinion, that Eratosthenes had used the Attic stadion of 185 meters or the even

per degree. We know from him that the same measure had been adopted by Marinus (Ptolemy, *Geographia*, I, 7; I, 11).

[6] See for example [Berggren and Jones 2000, p. 30].

larger stadia proposed by some scholars. We have to conclude that the relative measurement error was probably within a few percent.

3. Some conjectures on possible simplifications introduced in Cleomedes' account

According to Cleomedes' account of Eratosthenes' measurement,[7] the difference of latitude between Alexandria and Syene, supposed on the tropic (difference which is equal to the angle between sunbeams and the vertical in Alexandria at noon of the summer solstice), was measured as $\frac{1}{50}$ of a turn and the distance between the two cities (supposed on the same meridian) was estimated as 5000 stadia. The length of the great circle, measured in stadia, was then obtained as the result of the multiplication

$$50 \times 5{,}000 = 250{,}000.$$

The result of the previous section shows an accuracy of Eratosthenes' result which is hardly compatible with such round figures, which have been often considered a clear evidence of the crudeness of Eratosthenes' measure (this argument is used, for example, in [Goldstein 1984]).

On the other hand all sources other than Cleomedes unanimously give for the final result the value of 252,000 stadia.[8] The discrepancy is usually explained (see for example [Roller 2010, p. 143]) assuming that Eratosthenes had obtained the round figures reported by Cleomedes, but afterwards had added 2000 stadia to the final result in order to get a figure divisible by 60. Such a reconstruction is hardly acceptable. What number should have recorded Eratosthenes in his lost treatise "On the measurement of the Earth"?[9] If he had reported only the final figure 252,000, Cleomedes could not have recovered the original result of the measurement. Suppose, instead, that Eratosthenes had written that the measurement result had been 250,000 stadia, but that, in his opinion, it could have been convenient to replace it by 252,000. It would be hardly understandable, in this case, why no other source, except Cleomedes, should have recorded the value 250,000, which had the double advantage of being a round figure and the true result of the measurement.

It appears much more likely that the rounding of the figures was one of the simplifications introduced by Cleomedes in his short account (contained in about

[7] Cleomedes, *Caelestia*, I, 7, ll, 48–120 (pp. 35–37, ed. Todd).

[8] Strabo, *Geographia* (II, v, 7; II, v, 34); Geminus, *Introduction to the Phenomena*, XVI, 6; Macrobius, *Commentarii in Somnium Scipionis*, I, xx, 20; Vitruvius, *De architectura*, I, vi, 9; Plinius, *Naturalis Historia*, II, 247; Censorinus, *De die natali*, xiii, 5; Theon of Smyrna, *De utilitate mathematicae*, 124, 10–12 (ed. Hiller); Heron of Alexandria, *Dioptra*, xxxv, 302, 10–17 (ed. Schöne); Martianus Capella, *De nuptiis Philologiae et Mercurii*, VI, 596.

[9] The title of Eratosthenes' work is quoted by Heron of Alexandria (*Dioptra*, xxxv).

three pages in modern editions[10]) of the lost Eratosthenes' treatise in two books. Whereas all other sources, quoting the figure 252,000, intend to report Eratosthenes' result, Cleomedes clarifies in the beginning of his popularization that his only aim is to explain the "method (ἔφοδος)" used by Eratosthenes to readers unable to follow the geometric technicalities of the original work and the accuracy of the figures is clearly irrelevant for this purpose. By rounding the figures Cleomedes might better have achieved the goal to explain Eratosthenes' method without boring the reader with computations which are not immediately worked out mentally. On the other hand, since Cleomedes also writes that a circumference is three times its diameter[11] and it is not conceivable that Eratosthenes had used such a crude estimate of π we know that the rounding of the figures was actually part of the simplifications introduced by him.

Cleomedes could not round the final result 252,000 without altering at least one of the two factors whose product had given such result. On the other hand we have to exclude the possibility that the original multiplication was $50 \times 5{,}040 = 252{,}000$, because large distances are never recorded by ancient geographers with the accuracy of tens of stadia. Hence, if the product was 252,000, we must exclude the number 50 as first factor. Once excluded 50 itself, the only submultiple of 252,000 which can be reasonably rounded to 50 is 48.

We are thus led to conjecture that the original multiplication performed by Eratosthenes might have been

$$48 \times 5{,}250 = 252{,}000,$$

where 5,250 stadia was the measured distance between Alexandria and the northern tropic and $\frac{1}{48}$ of a turn was the measure of the angle between the vertical and the direction of the sunbeams at noon of the summer solstice in Alexandria.

I think that the conjecture above could be accepted, because it is strengthened by three independent elements:

(a) In Eratosthenes' time the angles $\frac{1}{12}$ of a turn (corresponding to one sign of the zodiac, or 30° in our notations), $\frac{1}{24}$ of a turn (half-sign or "step") and $\frac{1}{48}$ of a turn ("part"), as well as sixtieths of a turn, were privileged as units of measurement,[12] so that $\frac{1}{48}$ of a turn was a very natural result of an angular measurement, while the angle reported by Cleomedes ($\frac{1}{50}$ of a turn) is hard to express in the units then used.

[10] See note 8 above.

[11] Cleomedes, *Caelestia*, I, 7, 119–120.

[12] [Neugebauer 1975, pp. 671–672]; [Roller 2010, p. 151].

(b) 5,250 stadia is a plausible result of the measurement of Eratosthenes, because he used to express large distances as multiples of 250 stadia.[13]

(c) An important piece of evidence is provided by Strabo, who reports that the distance between Syene and the Mediterranean was estimated by Eratosthenes as 5,300 stadia.[14] Since Strabo always expresses large distances as multiples of 100 stadia,[15] his figure has the best possible agreement with the value of 5,250 stadia.

If the present conjecture is accepted, one of the consequences is that the error of the angular measure by Eratosthenes was much smaller than what has been so far supposed. The difference of latitude between Alexandria and the tropic was in fact at the time[16] 7°28′, much nearer to $\frac{1}{48}$ of a turn (7°30′) than to Cleomedes' value, corresponding in our notations to 7°12′.

Another consequence has to do with Eratosthenes' estimate of the error on the measure of the length of the arc of meridian between Alexandria and the tropic. If Eratosthenes assumed the value of 5,250 stadia, we have to think that he was confident to be able to choose the multiple of 250 stadia nearest to the true distance; in other words he may have thought that his error could be less than 125 stadia, or less than about 2.5%, in good agreement with the estimate obtained in Section 2.

In all expositions of Eratosthenes' measurement we read that he supposed that the town of Syene was exactly in the intersection of the tropic with the meridian through Alexandria.[17] Since, as is shown in Figure 2, Syene was actually not far from the tropic,[18] but its difference in longitude with Alexandria is not negligible at all, this assumption, too, seems hardly compatible with the estimate on the error of the result we have found in Section 2.

The universally shared belief that Eratosthenes supposed that Alexandria and Syene were on the same meridian is mainly drawn from Cleomedes' account. Actually, after having exposed Posidonius' method for measuring the Earth, Cleomedes introduces Eratosthenes' measurement with these words:

> ... Eratosthenes' method, being geometrical in nature, is considered more obscure. But what he says will become clear if we premise the

[13] For example the distance between Alexandria and Rhodes was estimated by Eratosthenes as 3,750 stadia (Strabo, *Geographia*, II, v, 24).

[14] Strabo, *Geographia*, XVII, i, 2. In [Rawlins 1982, p. 215], this passage is used, strangely enough, as a proof that Strabo's source was a map pre-dating Eratosthenes and that Eratosthenes had obtained his distance of 5000 stadia just by rounding the ancient value.

[15] [Shcheglov 2003–2007, p. 165].

[16] The latitude of the tropic, i.e., the obliquity of the ecliptic, is about 23°26′ nowadays, but in hellenistic times it was about 23°44′.

[17] See for example [Goldstein 1984; Dutka 1993].

[18] Since the latitude of Syene (today Aswan) is 24°05′N, its distance from the tropic is now almost doubled, but in Eratosthenes' time it was about 21′.

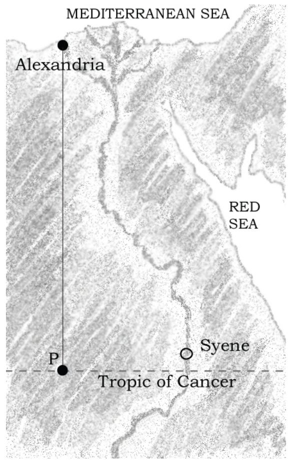

Figure 2. Alexandria, Syene and the tropic (shown in the position it had in Eratosthenes' time).

> following assumptions: suppose first that Alexandria and Syene are on the same meridian . . .[19]

These sentences suggest that the assumptions on the coordinates of Syene might be one of the simplifications introduced by Cleomedes. Eratosthenes, for computing the distance between Alexandria and the tropic, had to identify the point P in our figure, i.e., the intersection of the meridian passing through Alexandria with the tropic. In other words he had to measure the component along the meridian of a segment joining Alexandria with any point on the tropic. The individuation of such a component was an operation usual not only in geometry,[20] but also in Eratosthenes' geography.[21] This is a mathematical operation which Cleomedes understandably might have preferred to avoid in exposing the method of Eratosthenes to readers unable to follow geometrical arguments, by replacing the abstract point P with a very concrete town.

[19] [. . .] ἡ δὲ τοῦ Ἐρατοσθένους γεωμετρικῆς ἐφόδου ἐχομένη καὶ δοκοῦσά τι ἀσαφέστερον ἔχειν. ποιήσει δὲ σαφῆ τὰ λεγόμενα ὑπ᾽ αὐτοῦ τάδε προϋποθεμένων ἡμῶν. ὑποκείσθω ἡμῖν πρῶτον μὲν κἀνταῦθα ὑπὸ τῷ αὐτῷ μεσημβρινῷ κεῖσθαι Συήνην καὶ Ἀλεξάνδρειαν, [. . .] (Cleomedes, *Caelestia*, I, 7, 49–52).

[20] The orthogonal projection of a point on a line is one of the first procedures explained in a text which was certainly very familiar to hellenistic geographers: Euclid's *Elements* (it is the object of prop. 12 of the first book). Modern historians, who are usually less acquainted with this text, are more inclined to recognize Euclid's influence on hellenistic geographers in some geometrical shapes (see, for example, [Roller 2010, p. 26]) than in geometrical procedures.

[21] Strabo in his *Geographia*, often quoting Eratosthenes, reports several discussions concerning right triangles whose legs are aligned with a meridian and a circle of latitude. Although Strabo does not seem able to master the matter, it is clear that his source was considering orthogonal projections along the two directions.

It is true that also Strabo writes in a couple of passages that according to Eratosthenes the Nile flows along the meridian from Syene to Alexandria, but in the same passages the Nile is described as flowing along the same meridian even from Meroë to Syene,[22] while in the book devoted to Egypt, still quoting Eratosthenes, he describes the path of the Nile as far from being a north-south straight line[23] and in more than one instance Strabo appears to confuse distances with their orthogonal projections along a meridian.[24]

The accuracy of Eratosthenes' measurement resulting from our evaluation of the stadion lends strong support to the conjecture that it was based on land surveying. We know, in fact, that Egypt had a cadastre based on detailed surveying[25] and the use of "royal surveyors" even outside of Egypt is documented by Martianus Capella.[26] Furthermore, the fact that the title of Eratosthenes' treatise "On the measurement of the Earth" is transmitted by Heron's *Dioptra* — by a work devoted to the description of a surveying instrument and of its use — suggests the possibility that part of Eratosthenes' treatise had been devoted to surveying techniques.[27] That no measurement of the Earth's circumference was attempted in Europe until the seventeenth century is per se a strong indication that Eratosthenes planned and oversaw an enterprise requiring a degree of collective organization that cannot be taken for granted in other historical contexts.[28]

A possible objection to the reconstruction so far suggested is that it requires the drawing of an accurate map of Egypt and, whereas such a map was attributed to Eratosthenes in the past, in the last decades the appreciation of ancient cartography has been drastically reduced and the opinion has prevailed that Eratosthenes did not in fact prepare a map of Egypt.[29] We only have evidence of locally confined surveying in ancient Egypt and there is no direct evidence of a map of Egypt drawn

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[22] Strabo, *Geographia*, I, iv, 2; II, v, 7.

[23] Strabo, *Geographia*, XVII, i, 2.

[24] For the scant reliability of Strabo in reporting his hellenistic sources, see [Shcheglov 2003–2007]. On our particular subject see also [Rawlins 1982], where some examples of orthogonal components of distances along the path of the Nile, considered by Eratosthenes and misunderstood by Strabo, are recovered.

[25] Valuable information on the Egyptian cadastre is contained in the Oxyrhynchus papyri; see in particular P.Oxy VI 0918 [Grenfell and Hunt 1908, p. 272]. Some useful references on surveying techniques in ancient Egypt are in [Dutka 1993].

[26] Martianus Capella, *De nuptiis Mercurii et Philologiae*, VI, 598.

[27] The use of dioptras by Eratosthenes is well attested (Theon of Alexandria, *Commentaria in Ptolemaei syntaxin mathematicam* i–iv (ed. Rome), 395, 1–2; Simplicius, *In Aristotelis de caelo commentaria*, 246a [CGA 1894, 550]).

[28] On this point see [Russo 2004, pp. 273–277].

[29] Good examples of this new trend are [Harley and Woodward 1987; Brodersen 1995; Rathmann 2007]. (I am indebted to a referee for suggesting these references in this context.) Scholars of ancient science know very well, however, that more recent and better do not always coincide.

by Eratosthenes (apart from the quantitative data reported by Strabo in *Geographia*, XVII, i, 2). The first Greek maps mentioned by our sources date back to the sixth century B.C., but surely did not incorporate quantitative data. On the other hand Ptolemy's *Geographia* is precisely a handbook for drawing maps of the whole *oikoumene* and each of its regions, and for this purpose it stores 12,690 numerical data. We do not know with certainty when the passage from purely symbolic maps to quantitative cartography was accomplished, but it seems reasonable that it was contemporary with the birth of mathematical geography and the introduction of geographical coordinates, i.e., in the time of or shortly before Eratosthenes. On the other hand the thesis that Ptolemy, in his handbook for drawing maps, drew heavily on data from hellenistic sources, in particular incorporating Eratosthenes' material expressed via his value of 700 stadia for a degree of the Earth's circumference, is not only proved by the results in the first two sections of the present paper, but has been shared by other authors on completely different grounds (see [Knobloch et al. 2003; Shcheglov 2004].[30]) Furthermore, the opinion that there were no quantitative maps in Eratosthenes' time is difficult to reconcile with Hipparchus' discussion, in the context of his criticism of Eratosthenes' geographical treatise, of particular directions reported in "ancient maps (ἀρχαῖοι πίνακες)".[31]

Finally, we have a linguistic clue suggesting that Eratosthenes might have extended on a different scale techniques used until then only in local surveying. We know that in hellenistic Egypt officials used a concept analogous to our cadastral sheet, i.e., a portion of land, containing several estates, which was numbered and whose extension and position were described in the cadastral register. Such a portion of land was called σφραγίς.[32] It was Eratosthenes who first introduced the same term σφραγίς in geography, to mean a vastly larger portion of land.[33]

Cleomedes reports an interesting remark made by Eratosthenes in his work. He had observed that at noon of the summer solstice the gnomons gave no shadow not only in the point where the sun was exactly at the zenith, but in a circle around it whose diameter was 300 stadia.[34] It was suggested in [Hultsch 1897] that Eratosthenes had gotten this information from people specifically sent for this purpose, but it is also possible that his estimate had a theoretical basis, being deduced from the knowledge of the angular size of the sun.[35] In either case the remark would

[30] I am indebted to the same anonymous referee for drawing my attention to these references.

[31] Hipparchus' fragment is reported in Strabo, *Geographia*, II, i, 11.

[32] See for example the Oxyrhynchus' papyrus quoted in note 26 above.

[33] See [Roller 2010, pp. 26–27] and Eratosthenes' fragments quoted therein.

[34] Cleomedes, *Caelestia*, I, 7, 101–102 (ed. Todd).

[35] Since the angular size of the sun is about half a degree, the width of the strip where the gnomons gave no true shadow (umbra), but only penumbra, is about half a degree in latitude, or about 350 stadia according to Eratosthenes' measure, but Eratosthenes may have considered that outside a strip 300 stadia wide most of sunlight was intercepted by the gnomons.

Table 1. Longitudes of the towns in the sample

	actual	Ptolemy's		actual	Ptolemy's
Calpe (Gibraltar)	5°21′ W	7°30′	Carthage	10°19′	34°50′
Malaca (Malaga)	4°25′ W	8°50′	Leptis Magna	14°19′	42°
Corduba	4°47′ W	9°20′	Berenice	20°04′	47°45′
Abdara (Adra)	3° 1′ W	10°45′	Ptolemais	20°57′	49°05′
Carthago nova (Cartagena)	0°59′ W	12°15′	Cyrene	21°51′	50°
Tarraco	1°15′	16°20′	Alexandria	29°55′	60°30′
Barcinon (Barcelona)	2°10′	17°15′	Naucratis	30°37′	61°15′
Numantia (Garray)	2°27′ W	12°30′	Oxyrynchus	30°40′	61°40′
Saguntum	0°16′ W	14°35′	Syene (Aswan)	32°56′	62°
Tolosa	1°25′	20°10′	Arsinoe in Eritrea (Assab)	42°44′	73°45′
Massalia (Marseille)	5°23′	24°30′	Chalcedon	29°02′	56°05′
Olbia (Hyères)	6°08′	25°10′	Nicomedia	29°55′	57°30′
Genua	8°56′	30°	Lampsacus	26°41′	55°20′
Populonium	10°29′	33°30′	Pitane	26°56′	56°10′
Roma	12°29′	36°40′	Miletus	27°17′	58°
Cumae (Arco Felice)	14°04′	39°20′	Pergamus	27°11′	57°25′
Paestum	15°00′	40°10′	Sardes	28°02′	58°20′
Croton	17°07′	41°30′	Mytilene	26°33′	55°40′
Rhegium Julium	15°39′	39°50′	Rhodes (Lindos)	28°05′	58°40′
Tarentum	17°14′	41°30′	Samos	26°50′	57°
Brundisium	17°57′	42°30′	Sinope	35°09′	63°50′
Ravenna	12°12′	34°40′	Perga	30°51′	62°15′
Ancona	13°31′	36°30′	Caesarea in Cappadocia	35°29′	66°30′
Camerinum	13°04′	36°	Tarsus	34°54′	67°40′
Capua (Santa Maria C. V.)	14°15′	40°	Phasis in Colchis (Poti)	41°40′	72°30′
Panormus	13°22′	37°	Sidon	35°22′	67°10′
Syracuse	15°17′	39°30′	Antiochia on the Orontes	36°09′	69°
Pola	13°51′	36°	Apamea	36°24′	70°
Abdera	24°59′	52°10′	Carrae	39°13′	73°15′
Byzantium	28°58′	56°	Damascus	36°18′	69°
Philippopolis	24°45′	52°30′	Hierosolyma (Jerusalem)	35°13′	66°
Pella	22°31′	49°20′	Gaza	34°27′	65°25′
Stagira	23°45′	50°20′	Petra	35°27′	66°45′
Athens	23°43′	52°45′	Seleucia on the Tigris	44°31′	79°20′
Thebes (in Boeotia)	23°19′	52°40′	Babylonia (al-Hilla)	44°25′	79°
Delphi	22°30′	50°	Susa	48°15′	84°
Corinth	22°56′	51°15′	Ecbatana (Hamadan)	48°31′	88°
Lacedaemon	22°25′	50°15′	Persepolis	52°53′	90°15′
Tingis Caesarea (Tangier)	5°48′ W	6°30′	Hecatompylon	54°02′	96°
Hippo Regius	7°46′	30°20′	Antiochia Margiana (Merv)	61°50′	106°

Actual longitudes are from Greenwich. Given in parentheses are the current names (or names of nearby modern towns) when they differ from the classical ones.

make no sense if the distance between Alexandria and the tropic was roughly estimated in thousands of stadia. Furthermore, since Syene was 245 stadia away from the tropic, Eratosthenes had determined the distance between Alexandria and the tropic as a multiple of 250 stadia and the central line of a strip 300 stadia wide can certainly be identified with the precision of some tens of stadia, it seems possible that the idea of considering Syene to be on the tropic was another simplification (which, however, Cleomedes shares with many other authors).

I conclude with a remark on a method for measuring large distances which is often recalled in the context of Eratosthenes' measurement. In most of the popular accounts we read that the distance between Alexandria and Syene was reported to Eratosthenes by a "bematist", a man trained to keep a regular pace when marching and to record the number of steps between places. The use of bematists is often presented as the usual method for measuring large distances in the Greek world. But a search of the Thesaurus Linguae Graecae has yielded the result that the word bematist (βηματιστὴς) is attested only once in the entire corpus of Greek literature,[36] in a passage concerning the method used for measuring the distances traveled by the army during the campaign of Alexander the Great, i.e., in circumstances in which usual surveying was hardly practicable.

References

[Berggren and Jones 2000] J. L. Berggren and A. Jones, *Ptolemy's Geography: An annotated translation of the theoretical chapters*, Princeton University Press, 2000.

[Brodersen 1995] K. Brodersen, *Terra cognita: Studien zur römischen Raumerfassung*, Olms, Hildesheim, 1995.

[CGA 1894] J. L. Heiberg (editor), *Commentaria in Aristotelem graeca*, vol. 7, Reimer, Berlin, 1894.

[Dicks 1960] D. R. Dicks (editor), *The geographical fragments of Hipparchus*, Athlone Press, London, 1960.

[Dutka 1993] J. Dutka, "Eratosthenes' measurement of the Earth reconsidered", *Arch. Hist. Exact Sci.* **46**:1 (1993), 55–66.

[Goldstein 1984] B. R. Goldstein, "Eratosthenes on the "measurement" of the earth", *Historia Math.* **11**:4 (1984), 411–416.

[Grenfell and Hunt 1908] B. P. Grenfell and A. S. Hunt (editors), *The Oxyrhynchus papyri*, vol. VI, Egypt Exploration Fund, London, 1908. P.Oxy VI 0918 can be viewed at http://archive.org/stream/oxyrhynchuspappt06grenuoft#page/272/mode/2up.

[Harley and Woodward 1987] J. B. Harley and D. Woodward (editors), *The history of cartography*, vol. 1: *Cartography in prehistoric, ancient, and medieval Europe and the mediterranean*, University of Chicago Press, 1987.

[Hultsch 1882] F. Hultsch, *Griechische und römische Metrologie*, Weidmann, Berlin, 1882. Reprinted 1971.

[36] Athenaeus, *Deipnosophistae*, X, 59, 2.

[Hultsch 1897] F. Hultsch, *Poseidonios über die Grösse und Entfernung der Sonne*, vol. 1, Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen / Philosophisch-historische Klasse. N.F **5**, 1897.

[Knobloch et al. 2003] E. Knobloch, D. Lelgemann, and A. Fuls, "Zur hellenistischen Methode der Bestimmung des Erdumfanges und zur Asienkarte des Klaudios Ptolemaios", *Zeitschrift für Geodäsie, Geoinformation und Landmanagment* **128**:3 (2003), 211–217.

[Neugebauer 1975] O. Neugebauer, *A history of ancient mathematical astronomy*, Springer, Berlin, 1975.

[Rathmann 2007] M. Rathmann (editor), *Wahrnehmung und Erfassung geographischer Räume in der Antike*, Philipp von Zabern, Mainz, 2007.

[Rawlins 1982] D. Rawlins, "The Eratosthenes–Strabo Nile map: Is it the earliest surviving instance of spherical cartography? Did it supply the 5000 stades arc for Eratosthenes' experiment?", *Arch. Hist. Exact Sci.* **26**:3 (1982), 211–219.

[Roller 2010] D. W. Roller (editor), *Eratosthenes' geography: Fragments collected and translated, with commentary and additional material*, Princeton University Press, 2010.

[Russo 2004] L. Russo, *The forgotten revolution: How science was born in 300 BC and why it had to be reborn*, Springer, Berlin, 2004.

[Shcheglov 2003–2007] D. Shcheglov, "Hipparchus' table of climata and Ptolemy's *Geography*", *Orbis Terrarum* **9** (2003–2007), 159–192.

[Shcheglov 2004] D. A. Shcheglov, "Ptolemy's system of seven climata and Eratosthenes' *Geography*", *Geographia Antiqua* **13** (2004), 21–37.

[Stückelberger and Graßhoff 2006] A. Stückelberger and G. Graßhoff (editors), *Klaudios Ptolemaios Handbuch der Geographie: griechisch-deutsch*, Schwabe, Basel, 2006.

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