

Diving Deep into the Preimage Security of AES-like Hashing

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Abstract. Since the seminal works by Sasaki and Aoki, Meet-in-the-Middle (MITM) attacks are recognized as an effective technique for preimage and collision attacks on hash functions. At Eurocrypt 2021, Bao *et al.* automated MITM attacks on AES-like hashing and improved upon the best manual result. The attack framework has been furnished by subsequent works, yet far from complete. This paper introduces three key contributions dedicated to further generalizing the idea of MITM and refining the automatic model on AES-like hashing. (1) We introduce *S-box linearization* to MITM pseudo-preimage attacks on AES-like hashing. The technique works well with superposition states to preserve information after S-boxes at affordable cost. (2) We propose *distributed initial structures*, an extension on the original concept of initial states, that selects initial degrees of freedom in a more versatile manner to enlarge the search space. (3) We exploit the *structural similarities* between encryption and key schedule in constructions (*e.g.*, Whirlpool and Streebog) to model propagations more accurately and avoid repeated costs. Weaponed with these innovative techniques, we further empower the MITM framework and improve the attack results on AES-like designs for preimage and collision. We obtain the first preimage attacks on 10-round AES-192, 10-round Rijndael-192/256, and 7.75-round Whirlpool, reduced time and/or memory complexities for preimage attacks on 5-, 6-round Whirlpool and 7.5-, 8.5-round Streebog, as well as improved collision attacks on 6- and 6.5-round Whirlpool.

Keywords: Meet-in-the-Middle Attack · S-box Linearization · Distributed Initial Structures · Structural Similarities · AES · Rijndael · Whirlpool · Streebog

1 Introduction

1.1 Hash Functions

Hash functions map arbitrary long inputs to fixed-length hash values and have been used in a myriad of applications. There are three fundamental security requirements for a cryptographic hash function to fulfill, namely preimage, second-preimage, and collision resistance. This work focuses on the notion of preimage resistance: given a hash function H and a random hash value t , it should be computationally infeasible to find a preimage x such that $H(x) = t$.

To make use of the coexistence of encryption and hashing in embedded systems, a conventional strategy is to construct a hash function from a secure block cipher to minimize hardware or software costs: the encryption function of a block cipher is first transformed into a one-way compression function and then iterated following the Merkle-Damgård design. In 1993, Preneel, Govaerts, and van de Walle [36] identified 12 secure modes for the encryption-compression-function conversion, later known as the PGV modes.

The strategy is highly practical if the underlying block cipher is widely used and has seen a long record of withstanding cryptanalysis, which makes AES the perfect candidate. The MMO mode (one of the PGV modes) instantiated with AES-128 have been standardized by the Zigbee [2] protocol suite and ISO/IEC [24]. Given the high security of AES, several dedicated hash functions are designed with AES-like structures, *e.g.*, the ISO standards Whirlpool [9,23] or the ISO and GOST standard Streebog [1,25,15], which are collectively referred to as AES-like hashing.

1.2 Meet-in-the-Middle Attacks on Block-cipher-based Hashing

The Meet-in-the-Middle (MITM) attack is well-known for its effectiveness in cryptanalysis of Double-DES [14] and key recovery. In a series of pioneer works [4,5,39,40], Sasaki and Aoki enlightened the community with MITM attacks applied to the security analyzation of cryptographic hash functions. The core attack framework had been extended ever since by numerous techniques, such as splice-and-cut [4], initial structures [40], indirect and partial matching [4,40], biclique as a formalization of initial structures [28], sieve-in-the-middle [12] and match-boxes [17].

MITM attack on block-cipher-based hash functions is, in essence, a pseudo-preimage attack: the attack splits the computation of the compression function into two chunks, the forward and the backward chunk, so that two portions of input bits, called neutral bits, affect only one of the sub-functions. In such a setting, the chunks are computed independently and end at a common state where their (partial) values are matched. Usually, a third set of bits is shared by both chunks, which is captured in the notion of a 3-subset MITM attack [11].

Sasaki was the first to apply this to a preimage attack on AES hashing modes [38]. However, to avoid the complex relations from the round keys, the key was still fixed to a constant. Sasaki *et al.* then introduced the guess-and-determine

strategy [41]. Bao *et al.* [6] revisited the attacks by introducing the degree of freedom from the key space.

At Eurocrypt 2021, Bao *et al.* [7] automated the search for efficient MITM preimage attacks with Mixed-integer Linear Programming (MILP) and applied it to AES hashing modes and Haraka v2. Dong *et al.* [16] later extended this automation model to search for key-recovery and collision attacks and introduced nonlinear constraints for the neutral bits. Later in 2022, Bao *et al.* [8] brought up the concept of superposition bytes, which allowed forward and backward neutral words to propagate simultaneously and independently at a common byte through linear operations in the encryption and key schedule. They also proposed bi-directional attribute propagation and cancellation, *i.e.*, the known values in each chunk are propagated not only in the direction of the chunk but in both directions. Moreover, they integrated the guess-and-determine method into their models. Hua *et al.* [22] then combined guess-and-determine with nonlinearly constrained neutral words in their search for preimage attacks. More recently, Qin *et al.* [37] applied the new framework to Sponges. As a contrast to those very detailed frameworks, Schrottenloher and Stevens proposed a simpler MILP-modeling approach for preimage attacks against keyless permutations [42] and was later extended to ciphers with very light key schedule [43]. While their model was considerably more lightweight and applicable to AES-like permutations, its exclusion of the key schedule made it less effective against the AES than the detailed frameworks.

1.3 Gaps

While previous works on automating MITM on AES-like hashing already provided a groundlaying seminal framework [7,8,16,37], the complexity of the task has left several gaps. Among those, we identified three core challenges:

1. The preimage security evaluation on AES-like hashing has always been at byte-level as the S-box details are abstracted away. Recently, Zhang *et al.* [45] studied the field inversion S-box with algebraic properties and thus can consider the S-box details. However, quoting their words, *this linearizes the non-linear layer of AES, but unfortunately, no attacks better than the current state-of-the-art has been found based on this fact.*
2. The selection of initial states in previous works was limited to two full states, one of them in the encryption function and the other in the key schedule. Initial states could be more scattered, even across several intermediate states. Thus, the artificial limits on initial states had discarded a fraction of the solution pool.
3. It has been a long endeavor to address the dependencies in the model that lead to incorrect measures on the degree of freedom consumption. Particularly, the dependencies due to the structural similarity between the encryption and key schedule in some designs have been overlooked and may lead to duplicate costs.

1.4 Our Contributions

In this work, we have proposed and incorporated three techniques to fill the gaps and improve the state-of-the-art attacks. We would like to point out that the core ideas behind the techniques are fairly generic and expected to have more applications beyond this paper.

Linearizing S-boxes. We introduce *S-box linearization* (**LIN**) to MITM attacks on AES-like hashing and efficiently incorporate it with the superposition structure. Both propagations at a superposition byte are preserved through the S-box at the cost of guessing over a small pool of *hints*. Making use of the linear relation between propagations, **LIN** checks if a guessed hint is correct efficiently using for- and backward values at S-box input.

In comparison, the study in [45] exploited the algebraic properties of field inversion S-boxes thus resulting in guess-and-determine and announced no improved result on AES. A similar conclusion on their work was also drawn by Liu *et al.* in [31]. The checking phase in plain guess-and-determine requires full information on forward and backward neutral bytes and leads to a cost on degrees of matching, while **LIN** in our proposal uses only local information and spares such cost. To conclude, **LIN** serves as a lightweight alternative to plain guess-and-determine and introduces a new trade-off rule. The technique enables us to mount the first bit-level preimage attacks on AES-like hashing and improve the state-of-the-art.

Distributed Initial Structures. In this work, we further generalize the concept of initial structures, originally proposed by Sasaki and Aoki [40]. We lift the artificial limitation on selecting two full states as initial states and introduce *distributed initial structures* (**DIS**). We now allow the initial states to be distributed in a combination of encryption states and round keys, as long as the total initial degrees of freedom remain the same. An important reflection of this idea is to assign more superposition bytes in the AES key schedule. As only a portion of bytes is propagated through the AES S-box in each key schedule round, more superposition information can be allowed in round keys by the introduction of **DIS**. This has expanded the solution pool by adding more alternatives to the invocation of constraints and allowing more valid propagation patterns.

Structural Similarities. The *structural similarities* (**SIM**) between encryption and key schedule may lead to dependencies across multiple rounds. We observe that values injected into an encryption state by round key addition may propagate through similar sets of operators in encryption and key schedule. Therefore, certain costs of degrees of freedom can be traced back to the same constraints and previous attacks may be suboptimal due to double counting such costs. By modeling the degree consumption more accurately, we enlarged the search space by sparing unnecessary double costs of earlier approaches. Moreover, the approach potentially finds attacks with high concentrations of constraints around the starting points, which could help reduce the memory complexity of the attack.

Table 1. Results of our improved attacks on AES-like Hashing.

| Preimage Attacks | | | | | | |
|-------------------------------|----------------|-----------------------------|--------------------------------|-----------------------------|--------------------------|------------|
| Cipher (target) | #Rounds | T_1^\dagger | T_2^\ddagger | Memory | Essential technique(s) | References |
| AES-192 (Hash) | 8/12 | 2^{112} [§] | 2^{116} | 2^{16} | MITM | [6] |
| | 8/12 | 2^{100} | 2^{115} | 2^{96} | LIN, DIS , BiDir* | App. B |
| | 9/12 | 2^{120} | 2^{125} | — | MILP | [7] |
| | 9/12 | 2^{112} | 2^{121} | — | BiDir | [8] |
| | 10/12 | 2^{124} | 2^{127} | 2^{124} | LIN, DIS , BiDir | Sect. 5.1 |
| Rijndael-192/192 (Hash) | 9/12 | 2^{184} | 2^{189} | — | BiDir | [46] |
| | 9/12 | 2^{180} | 2^{187} | 2^{180} | LIN , BiDir | App. C.1 |
| Rijndael-192/256 (Hash) | 9/12 | 2^{168} | 2^{181} | — | BiDir | [46] |
| | 10/12 | 2^{180} | 2^{187} | 2^{180} | LIN , BiDir | App. C.2 |
| Whirlpool (Hash) | 5/10 | 2^{416} | 2^{448} | 2^{96} | Dedicated | [41] |
| | 5/10 | 2^{352} | 2^{433} | 2^{160} | BiDir, MulAK* | [8] |
| | 5/10 | 2^{320} | 2^{417} | $O(1)$ | SIM , BiDir | Sect. 5.2 |
| | 6/10 | 2^{448} | 2^{481} | 2^{256} | Dedicated, GnD* | [41] |
| | 6/10 | 2^{440} | 2^{477} | 2^{192} | GnD | [8] |
| | 6/10 | 2^{416} | 2^{465} | 2^{288} | SIM , BiDir, GnD | App. D.1 |
| | 7/10 | 2^{480} | 2^{497} | 2^{128} | GnD, MulAK | [8] |
| | 7.75/10 | 2^{480} | 2^{497} | 2^{256} | SIM , BiDir, GnD | App. D.2 |
| Streebog-512 (Compression) | 7.5/12 | 2^{496} | — | 2^{64} | Dedicated method | [32] |
| | 7.5/12 | 2^{441} | — | 2^{192} | GnD, MulAK | [22] |
| | 7.5/12 | 2^{433} | — | 2^{177} | SIM , GnD | App. E.1 |
| | 8.5/12 | 2^{481} | — | 2^{288} | GnD, MulAK | [22] |
| | 8.5/12 | 2^{481} | — | 2^{129} | SIM , GnD | App. E.2 |
| Streebog-512 (Hash) | 7.5/12 | — | 2^{496} | 2^{64} | Dedicated method | [32] |
| | 7.5/12 | — | $2^{478.25}$ | 2^{256} | MITM + Multi-collision* | [22] |
| | 7.5/12 | — | $2^{474.25}$ | 2^{256} | MITM + Multi-collision | App. E.1 |
| | 8.5/12 | — | $2^{498.25}$ | 2^{288} | MITM + Multi-collision | [22] |
| | 8.5/12 | — | $2^{498.25}$ | 2^{256} | MITM + Multi-collision | App. E.2 |
| Collision Attacks | | | | | | |
| Cipher (target) | #Rounds | Time | Memory | Essential technique(s) | References | |
| | 4.5/10 | 2^{120} | 2^{16} | Rebound | [34] | |
| | 4.5/10 | 2^{64} | 2^{16} | Rebound | [30] | |
| Whirlpool (Hash) | 5/10 | 2^{120} | 2^{64} | Super-SBox | [18,29] | |
| | 5.5/10 | 2^{184-s} | 2^s | Rebound | [30] | |
| | 6/10 | 2^{228} | 2^{228} | Quantum | [20] | |
| | 6/10 | 2^{248} | 2^{248} | MILP, MITM | [16] | |
| | 6/10 | 2^{240} | 2^{240} | New MILP model, MITM | App. A | |
| | 6.5/10 | 2^{240} | 2^{240} | New MILP model, MITM | App. A | |

[†] T_1 represents the time complexity of the pseudo-preimage attack on compression function.

[‡] T_2 represents the time complexity of the preimage attack on hash function.

[§] We list only single-target result in [6] for comparison in this table.

* BiDir, MulAK and GnD are techniques introduced to the MITM framework in [8], respectively, short for bi-directional attribute propagation and cancellation, multiple ways of AddRoundKey and guess-and-determine.

* The attack on the compression function of Streebog is converted into a preimage attack on its hash function using the technique from [3].

1.5 Application Results

Our results are as summarized in Table 1. The effectiveness of our proposed techniques is well demonstrated through improved attacks on standards including AES-192 hashing, Whirlpool, and Streebog, as well as the Rijndael hashing family. We argue that the techniques are significant and essential to the breakthroughs.

Both **LIN** and **DIS** are critical for attacking one additional round of AES-192 hashing, i.e. excluding either technique would not yield an attack. Simply using guess-and-determine strategy combined with the AES S-box property also could not improve the attack on Rijndael-192/256. Moreover, the attack advantage on (pseudo-)preimage is non-trivial, *i.e.*, proportional to the size of a subset rather than a fixed constant.

Incorporating **SIM**, we improve the (pseudo-) preimage attacks on 5- and 6-round Whirlpool in terms of time and/or memory complexity. In particular, we achieve a memoryless attack on 5-round Whirlpool. Besides, we present the first preimage attack on 7.75-round Whirlpool, which extends the state-of-the-art by almost a full round while maintaining the same time complexity. What is more, our efficient MILP-based search model improves the 6-round collision attack on Whirlpool and extends it to 6.5 rounds. For Streebog, our approach reduces the time and/or memory complexity on 7.5- and 8.5-round Streebog compared to previous best (pseudo-)preimage attacks.

1.6 Organization

The remainder of this work is structured as follows. In Section 2, we provide preliminaries on the MITM attacks and the target designs. Then we elaborate the proposed techniques and their significance in Section 3. Thereupon, we present the enhanced MITM framework and MILP modeling in Section 4. We detailed pseudo-preimage results of the first attack on 10-round AES-192 hashing and the memoryless attack on 5-round Whirlpool in Section 5. Furthermore, we provide the attacks and details of Whirlpool, Rijndael, 8-round AES-192 and Streebog in the appendix. Finally, we conclude and discuss in Section 6.

2 Preliminaries

2.1 MITM Attacks: Notations and Principle

We provide a high-level overview of MITM pseudo-preimage attacks in Figure 1 and a list of notations common in all our attack descriptions in Table 2.

In block-cipher-based hashing, the MITM technique is often used to mount only a pseudo-preimage attack, *i.e.*, a preimage that uses a chaining value different from the fixed initial value of the hash function specification. The pseudo-preimage is later transformed into a preimage on the hash function.

The MITM attack divides the computations into two independent chunks, forward and backward. A byte is called neutral if its value is used only (*i.e.*, is

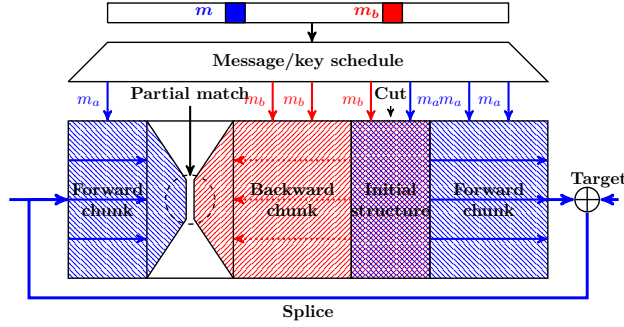


Fig. 1. A high-level overview of MITM attacks [38].

known only) in one of the chunks and has a constant influence on the respective other. Both chunks end at E^+ and E^- , respectively, which are the input and output of a matching operation M . Note that the attack exploits the feed-forward of start and end values in block-cipher-based compression functions. Based on the properties of M , an MITM attack can invoke certain constraints to filter ineligible candidates, which are called partial-match constraints. Those pairs that satisfy these constraints are checked thereupon on larger parts of their states if their combination constitutes a valid pseudo-preimage.

Table 2. Common notations.

| | |
|--|---|
| DoF | Degree(s) of freedom. |
| S^{ENC} | Starting state in the encryption. |
| S^{KSA} | Starting state in the key schedule. |
| E^+/E^- | Ending states of forward and backward computations, respectively. |
| M | Matching operation between E^+ and E^- . |
| $d_{\mathcal{B}}/d_{\mathcal{R}}$ | DoF of the forward and backward chunk, respectively. |
| $g_{\mathcal{B}}/g_{\mathcal{R}}/g_{\mathcal{B}\mathcal{R}}$ | DoF of guessed values in the forward, backward, and in both chunks, respectively. |
| $d_{\mathcal{M}}$ | Degrees of matching. |
| V^+/V^- | Sets of values for forward and backward neutral bytes satisfying the predefined constraints, with $ V^+ = 2^{d_{\mathcal{B}}}$ and $ V^- = 2^{d_{\mathcal{R}}}$, respectively. |
| $G^+/G^-/G$ | Sets of guessed values in forward/backward/both chunk(s), with $ G^+ = 2^{g_{\mathcal{B}}}$, $ G^- = 2^{g_{\mathcal{R}}}$, and $ G = 2^{g_{\mathcal{B}\mathcal{R}}}$. |
| T^+/T^- | Lookup tables constructed at E^+/E^- . |
| $\mathcal{B}^{\text{KSA}}/\mathcal{R}^{\text{KSA}}/\mathcal{G}^{\text{KSA}}$ | Sets of indices of forward neutral, backward neutral, and constant bytes in S^{KSA} , respectively. |
| $\mathcal{B}^{\text{ENC}}/\mathcal{R}^{\text{ENC}}/\mathcal{G}^{\text{ENC}}$ | Sets of indices of forward neutral, backward neutral, and constant bytes in S^{ENC} , respectively. |
| $\vec{v}/\overleftarrow{v}$ | Initial DoF of the forward and backward chunk, respectively, with $\vec{v} = \mathcal{B}^{\text{ENC}} + \mathcal{B}^{\text{KSA}} $ and $\overleftarrow{v} = \mathcal{R}^{\text{ENC}} + \mathcal{R}^{\text{KSA}} $. |
| $\vec{\sigma}/\overleftarrow{\sigma}$ | The consumed DoF of the forward and backward chunk, respectively. |

The MITM attack framework [8] with guess-and-determine is described in the following. Without loss of generality, we assume $d_{\mathcal{B}} + g_{\mathcal{B}} \leq d_{\mathcal{R}} + g_{\mathcal{R}}$:

1. Assign arbitrary values to the constants in pre-defined constraints.
2. Compute V^+ and V^- based on the constants.
3. For all tuples $(v^+, g^+, g) \in V^+ \times G^+ \times G$, compute to E^+ , obtain m^+ for matching, store (v^+, g^+) in $T^+[m^+, g]$. We have $|T^+| = 2^{d_{\mathcal{B}}+g_{\mathcal{B}}+g_{\mathcal{B}\mathcal{R}}}$.
4. For all tuples $(v^-, g^-, g) \in V^- \times G^- \times G$, compute to E^- , obtain m^- for matching.
5. For all (v^+, g^+) in $T^+[m^-, g]$, compute to check if (g^+, g^-, g) is compatible with v^+ and v^- .
6. For compatible (v^+, v^-) , check for a full match.
7. If a full match is discovered, compute and return the preimage. Otherwise, revert to Step 1, change the arbitrary values, and repeat the rest.

The computational complexity of the MITM pseudo-preimage attacks is

$$\begin{aligned} & 2^{n-(d_{\mathcal{B}}+d_{\mathcal{R}})} \cdot (2^{d_{\mathcal{B}}+g_{\mathcal{B}}+g_{\mathcal{B}\mathcal{R}}} + 2^{d_{\mathcal{R}}+g_{\mathcal{R}}+g_{\mathcal{B}\mathcal{R}}} + 2^{d_{\mathcal{B}}+g_{\mathcal{B}}+d_{\mathcal{R}}+g_{\mathcal{R}}+g_{\mathcal{B}\mathcal{R}}-d_{\mathcal{M}}}) \\ \simeq & 2^{n-\min(d_{\mathcal{B}}-g_{\mathcal{R}}-g_{\mathcal{B}\mathcal{R}}, d_{\mathcal{R}}-g_{\mathcal{B}}-g_{\mathcal{B}\mathcal{R}}, d_{\mathcal{M}}-g_{\mathcal{B}}-g_{\mathcal{R}}-g_{\mathcal{B}\mathcal{R}})} := 2^l. \end{aligned} \quad (1)$$

A pseudo-preimage attack with a computational complexity of 2^l ($l < n - 2$) can be converted to a preimage attack with a computation complexity of $2^{(n+l)/2+1}$ [35]. First, a total of $2^{(n-l)/2}$ pseudo-preimages is obtained. Then, a total of $2^{(n+l)/2+1}$ random values are inserted after the initialization vector IV to obtain $2^{(n+l)/2+1}$ chaining values. Then, one can expect a match between a chaining value and a pseudo-preimage with non-negligible probability, which yields a preimage for the hash function.

2.2 AES-like Hashing

To start with, we list some common notations in AES-like hashing:

- **Nb/Nk**: number of columns of a state in the encryption procedure or the secret key. When **Nb** and **Nk** are identical, we will denote both by **NCOL**.
- **NROW**: number of rows in an encryption or key state.

AES-like hashing refers to hash functions whose compression function follows an AES-like round structure. In this section, we will focus on recalling the necessary details of **AES** and **Whirlpool** that are used in this work.

AES. In 2001, the NIST selected a subset of the Rijndael family of block ciphers [13] with a block size of 128 bits and key sizes of 128, 192, or 256 bits (**Nb** = 4, **Nk** \in {4, 6, 8}, and **NROW** = 4) as the Advanced Encryption Standard. Conventionally, the transposed of a column is referred to as a word, and the word size is thus fixed to $4 \times 8 = 32$ bits. As shown in Figure 2, an AES round consists of the following operations:

- **SubBytes (SB)**: A non-linear byte-wise substitution.

- **ShiftRows** (SR): A cyclic left shift on the i -th row by i bytes, for $i \in \{0, 1, 2, 3\}$.
- **MixColumns** (MC): A column-wise left multiplication of a 4×4 maximum-distance-separable matrix.
- **AddRoundKey** (AK): A bitwise XOR of the round key to the state.

The final round differs in the sense that it omits the **MixColumns** operation. Before the first round, a whitening key is added to the plaintext.

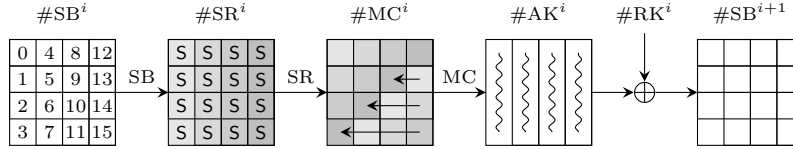


Fig. 2. AES-like round function.

The round keys are expanded from the master key key : Let w be an array of bytes, when $i < Nk$, the key words are derived directly from the secret key $w[i] = key[i]$. Otherwise, $w[i]$ is calculated as follows:

$$\begin{cases} w[i - Nk] \oplus \text{Rot}(S(w[i - 1])) \oplus C[i/Nk] & i \bmod Nk \equiv 0 \text{ and } Nk < 8 \\ w[i - Nk] \oplus S(w[i - 1]) & i \bmod Nk \equiv 4 \text{ and } Nk = 8 \\ w[i - Nk] \oplus w[i - 1] & \text{otherwise,} \end{cases} \quad (2)$$

where S denotes the AES S-box, Rot is a left rotation of the input by one byte, and C represents the list of round constants.

The **AES-128** in the Matyas-Meyer-Oseas (MMO) mode is used in the standards of the Zigbee protocol suite [2] and ISO/IEC [24]. The MMO mode is defined as the mapping $f : f(H_i, M_i) = E_{H_i}(M_i) \oplus M_i$, where H_i stands for the i -th chaining value, M_i as the i -th message block, and E_k stands for the block cipher encryption under key k .

Whirlpool. In 2000, Rijmen and Barreto [9] designed **Whirlpool** as a submission to the NESSIE competition that was later tweaked and adopted as an ISO/IEC standard [23]. **Whirlpool** is a block-cipher-based hash function with a 512-bit hash value, which adopts a 10-round AES-like block cipher with 8×8 -byte ($NROW = NCOL = 8$) keys and plaintexts in Miyaguchi-Preneel mode [36] (MP mode) as its compression function (CF). The MP mode is defined as $f(H_i, M_i) = E_{H_i}(M_i) \oplus M_i \oplus H_i$. It takes the 512-bit chaining value H_i as the key and the 512-bit message block M_i as its plaintext input. Encryption and key schedule essentially use the same round function, except for the fact that the key state has additions with round constants and the encryption state sees additions with the round keys. The round function is depicted in Figure 3 and consists of:

- **SubBytes (SB)**: applies the Substitution-Box to each byte.
- **ShiftColumns (SC)**: cyclically shifts the j -column downwards by j bytes.
- **MixRows (MR)**: multiplies each row of the state by an MDS matrix.
- **AddRoundKey (AK)**: XORs the round key to the state.

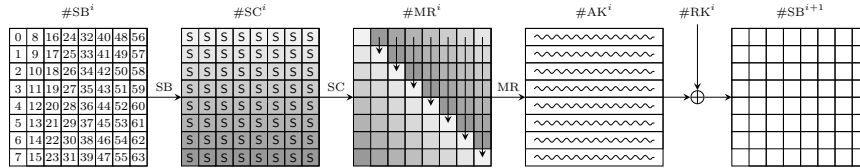


Fig. 3. The round function of **Whirlpool**.

Note that the final round is a complete round unlike that in the **AES**; a whitening key is added before the first round of encryption as in the **AES**. However, in the **MP** mode, the whitening key cancels in splice-and-cut MITM attacks due to the feed-forward operation. The key schedule shares the same operations, but replaces **AddRoundKey** by **AddRoundConstants (AC)**, which XORs the round constants to the first row of the key state before the result of **AC** is used as the round key that is added to the state. For more details, we refer the readers to the design paper [9].

Remark 1. Given that the *transposition between row and column* has no impact on attack results, for convenience, we use **ShiftRows** and **MixColumns** instead of **ShiftColumns** and **MixRows** in the rest of paper for **Whirlpool** hereafter. Thus, the states will be *transposed* to correspond with the states of **Whirlpool**.

Remark 2. In the remainder, we will denote a state by the operation that it is used as the direct input for, and will superscript the round index. For example, $\#SB^i$ denotes the state before the **SB** operation in Round i , as is shown in Figures 2 and 3.

Remark 3. The Russian national standard **Streebog** follows a similar structure as **Whirlpool**, due to the space limit, we provide the specification of **Streebog** in Appendix E.

3 Advanced Techniques in MITM Attacks

We advance the existing automated MITM frameworks with three generic techniques. Here, we detail the ideas and integration into the augmented framework.

3.1 S-box Linearization (LIN)

In MITM attacks, we have two sets of states $V^+ = (v_0^+, \dots, v_{2^{d_b}-1}^+)$ and $V^- = (v_0^-, \dots, v_{2^{d_r}-1}^-)$ propagating through the cipher. The superposition structure allows any cell of any state $v_{i,j}$ to be represented as the sum of its forward and backward neutral components: $v_{i,j} = v_i^+ \oplus v_j^-$, such that the components v_i^+ and v_j^- can be propagated independently through linear operations. Though, the nonlinear operations, *i.e.*, an S-box S in AES-like ciphers, prevent such trivial linear combinations.

Earlier works in the series of automated MITM attacks on AES-like ciphers had to define propagation rules which either lost knowledge about the cell after the S-box or which consumed one byte degree of freedom for forcing at least one neutral value to be constant before and after the nonlinear operation. We can linearize certain S-boxes partially or fully by restricting the input space or by guessing a hint from a set that is smaller than the input space.

In this work, we consider full linearization with a hint. Thus, we aim at finding a decomposition of S , more precisely, functions $F, G, H : \mathbb{F}_2^b \times \mathbb{F}_2^b \rightarrow \mathbb{F}_2^b$ with F and G being linear over \mathbb{F}_2 such that

$$S(v^+ \oplus v^-) = F(v^+, H(v^+, v^-)) \oplus G(v^-, H(v^+, v^-)).$$

The range of H is the set of space of hints. Assuming balancedness of H , *i.e.*, $d_{\mathcal{L}} = \dim(\text{range}(H))$, then $2^{d_{\mathcal{L}}}$ elements have to be guessed at most to linearize S , which is beneficial if we find such a function H with $d_{\mathcal{L}} < b$. Then, we add a complexity term of $2^{d_{\mathcal{L}}}$ to the attack for guessing the hint for each combination of (v_i^+, v_j^-) but can propagate a superposition through the S-box.

The S-box of the AES is given by $S(v) = \mathbf{A} \cdot v^{254} \oplus \text{0x63}$ for a fixed $\mathbf{A} \in \mathbb{F}_2^{8 \times 8}$. The power map and the XOR is in the field \mathbb{F}_{2^8} with a fixed irreducible polynomial; only the affine layer \mathbf{A} is not defined over this field. At Asiacrypt 2023, Zhang *et al.* [45] observed that one can decompose 254 into $17 \cdot 14 + 16$ and obtain

$$\begin{aligned} (v^+ + v^-)^{254} &= ((v^+ + v^-)^{17})^{14} \cdot (v^+ + v^-)^{16} \\ &= (H(v^+, v^-))^{14} \cdot ((v^+)^{16} + (v^-)^{16}), \end{aligned}$$

where the last equality holds since exponentiation with any power of 2 is linear over \mathbb{F}_2 . Then according to the following Theorem 1, the hint $H(v^+, v^-)$ can take $|\text{range}(H)| = |\{v^{17} : v \in \mathbb{F}_{2^8}\}| = 16$ values (including the zero element). Thus, one can linearize the AES S-box by guessing at most 16 candidates.

Theorem 1. *Let d be a divisor of $|\mathbb{F}_q^*| = p^n - 1$ where $q = p^n$, and let $X = \{x^d : x \in \mathbb{F}_q^*\}$. The size of X is $|X| = |\mathbb{F}_q^*|/d = (p^n - 1)/d$.*

3.2 Distributed Initial Structures (DIS)

It has been a common approach in MILP-based MITM models to select two independent initial states: S^{ENC} in the encryption function and S^{KSA} in the key

schedule, as the generation of round keys is independent of the encryption in most designs. In this work, we generalize the selection of initial states.

In essence, the initial states in MITM attacks are composed of some intermediate bytes in the compression function where we distribute initial DoFs for forward and backward computations. There should be no limitations on where the initial DoF is located, as long as the values of those states can be chosen independently of each other in the actual attack. In other words, the initial DoFs can be distributed to several scattered intermediate states, rather than rigidly selecting two full states in encryption and key schedule, respectively. Previous models implicitly limited the key bytes to depend only on S^{KSA} and not on S^{ENC} , and consequently, shrunk the solution pool.

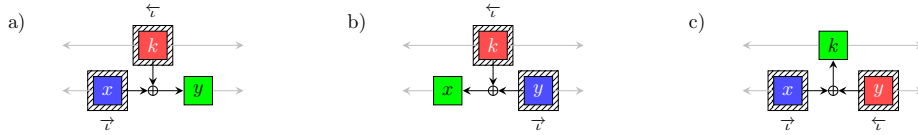


Fig. 4. Conceptual strategies of distributing initial states in two states and a key byte.

Consider an intuitive toy example in Figure 4. Assume x and y denotes bytes in the encryption and k denote a byte in the key, and we distribute initial DoF in this system for forward and backward computation. The whole system has a total initial DoF of 2. We use \vec{v} and \overleftarrow{v} to denote the initial DoF for forward and backward respectively, and the color green \blacksquare to denote a byte in superposition. Without loss of generality, there are three possible scenarios as depicted in Figure 4. The previous models covered the first two cases while excluding the third one, wherein the key byte is dependent on the initial DoF from the encryption.

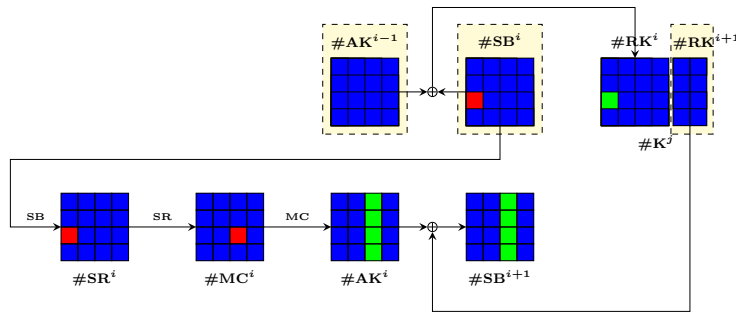


Fig. 5. Example of a distributed initial structure on AES-192.

We extend the above insight to MITM attacks by the introduction of **DIS**. We now distribute the initial DoF to several intermediate states in the compres-

sion function, provided that the bytes are independent and there is sufficient information to define all the round keys and all the intermediate encryption states. For example, Figure 5 describes an example to distribute initial DoF in AES-192. We will distribute initial DoFs in $\#AK^i$, $\#SB^{i+1}$ and the rightmost two columns of $\#K^j$. In this way, we have straightforwardly defined a full intermediate encryption state $\#SB^{i+1}$, and we can squeeze out a full $\#K^j$ for key schedule propagations.

The technique is useful in AES, since the key schedule has relatively low confusion and more linear relations can be preserved. In this work, we realize **DIS** in a heuristic manner. We still select S^{ENC} and S^{KSA} respectively to search for attack configurations, but make the exception that superposition bytes are now allowed in S^{KSA} and remain refrained from S^{ENC} . When a configuration is obtained, we check if the initial states can be equivalently chosen to properly define the superposition bytes in S^{KSA} within the maximum available DoF of the target cipher. Our realization of **DIS**, though heuristic, is more tailored to the key schedule and helps extend the analysis of AES-192 by one round.

3.3 Structural Similarities (SIM)

The models by Bao *et al.* and Dong *et al.* could find longer attacks than the manual attacks *e.g.*, by Sasaki [38] since the former effectively used the degree of freedom in key space to obtain reductions in the encryption state. From the XOR of the state with a round key, state bytes in superposition could become single-colored, and forward or backward neutral bytes could become constants. Such concessions are useful and often necessary before and after non-linear operations so that the knowledge of a byte can be propagated further. However, they come at the price of consuming a degree of freedom from the possible solution space.

In previous works, the effects of multiple round-key additions on the state have been usually modeled to be *independent* from each other and the state values. However, constraints from some consecutive rounds may stem from the same source of the neutral words in the key and state, *e.g.*, as depicted in Figure 6, constraints in states Y^{r_0} and Y^{r_1} are set on the same neutral words in states X^r and $K^{r'}$. Thus, tracing constraints back to such shared sources may enlarge the search space by avoiding duplicate DoF consumption. However, modeling all such dependent constraint relations can become challenging since the relations between all state and key bytes would have to be considered, which can render models infeasible to compute. Nevertheless, we can efficiently model certain special cases, and consider here the *structural similarities* of encryption and key schedule, *e.g.*, Whirlpool and Streebog use almost the same functions for updating key and message, differing only in the usage of round constants. Considering previous best MITM attacks on Whirlpool [8], Bao *et al.* already observed such dependency between encryption and key schedule, however, in their 7-round attacks on Whirlpool, they only used it in a post-processing step to generate the solution space of neutral words. In this work, we include this structural similarity explicitly in our MILP models.

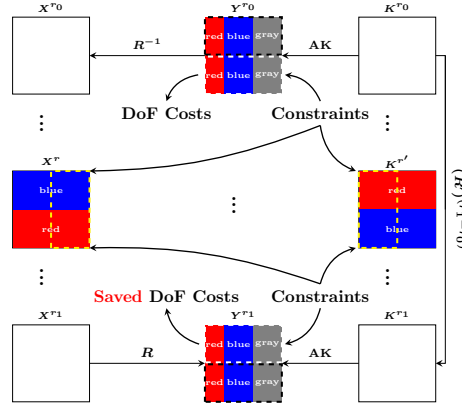


Fig. 6. High-level view of the different constraints connected by key schedule.

General Concept. For SPNs, we can write the round function as a composition of a nonlinear S-box layer SB, an affine layer \mathbf{A} , and a key addition. Assume, the key schedule employs the same S-box layer SB, an affine layer \mathbf{A}' , and a constant addition. Consider an interval of rounds from i to $i+1$ and assume that we can split the key $\#K^i$ and the state in the encryption $\#AK^i$ into an active part, subscripted by a , and a constant part subscripted by c , each: $\#K^i = \#K_a^i \parallel \#K_c^i$ and $\#AK^i = \#AK_a^i \parallel \#AK_c^i$. Figure 7 illustrates this setting. If the active parts of the key and encryption state are equal before the S-box layer, then the same values will also be the results in both the encryption and key schedule:

$$\#SB_a^{i+1} = \#K_a^{i+1} \Leftrightarrow \#A_a^{i+1} = \#A_a^{i+1}.$$

If $\#A_a^{i+1}$ and $\#A_a^{i+1}$ are mapped to the same positions of the state after \mathbf{A} and \mathbf{A}' , respectively, then the nonlinear contributions will cancel. Thus, we can define a nonlinear function G and a linear function H such that the active part of the message after the round is given by

$$\#SB_a^{i+1} = G(\#K_a^i) \oplus H(\#K_c^i, \#K_a^i, \#AK_c^i).$$

Note that it does not depend on $\#AK_a^i$.

We can generalize this observation to multiple rounds if the similarities between key schedule and message schedule allow. Given that the diffusion of AES-like ciphers is usually strong, *i.e.*, an MDS matrix, the number of subsequent rounds where this approach can be employed seems limited to two or three rounds, depending on the round constants and the linear layers. However, it will enlarge the search space and lead to high concentrations of constraints around the starting points, which could help reduce the memory complexity of the attack, as we will demonstrate later in our application results.

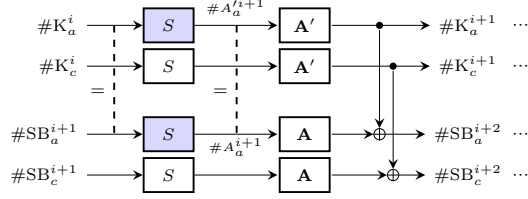


Fig. 7. Exploiting similar operations in the encryption and key schedule.

4 Enhanced Attack Framework and MILP Model

This section demonstrates our enhanced attack framework, that we call the exceptional MITM framework, and the equipped MILP-based search model.

4.1 Exceptional MITM Framework

We append the following two new notations to Table 2 to reflect the use of **LIN**:

Table 3. Additional notations.

| | |
|-------------------|---|
| $d_{\mathcal{L}}$ | DoF consumed by S-Box linearizations. |
| H | Space or set of hints from linearized S-boxes, with $ H = 2^{d_{\mathcal{L}}}$. |

Again, without loss of generality, we assume $d_{\mathcal{B}} + g_{\mathcal{B}} \leq d_{\mathcal{R}} + g_{\mathcal{R}}$. The exceptional MITM attack framework is formulated as follows:

1. Assign arbitrary values to the constants in pre-defined constraints.
2. Compute V^+ and V^- based on the constants.
3. For all tuples $(v^+, g^+, g, h^+) \in V^+ \times G^+ \times G \times H$, compute to E^+ , obtain m^+ for matching, store (v^+, g^+) in $T^+[m^+, g, h^+]$.
4. For all tuples $(v^-, g^-, g, h^-) \in V^- \times G^- \times G \times H$, compute to E^- , obtain m^- for matching.
5. For all (v^+, g^+) in $T^+[m^-, g, h^-]$ and (v^+, v^-) consistent with h^- , compute to check if (g^+, g^-, g) is compatible with (v^+, v^-) .
6. For compatible (v^+, v^-) , check for a full match.
7. If a full match is discovered, compute and return the preimage. Otherwise, revert to step 1, change the arbitrary values, and repeat the rest.

The computational complexity of the above attack is evaluated as follows:

$$\begin{aligned}
 & 2^{n-(d_{\mathcal{B}}+d_{\mathcal{R}})} \cdot (2^{d_{\mathcal{B}}+g_{\mathcal{B}}+g_{\mathcal{B}\mathcal{R}}+d_{\mathcal{L}}} + 2^{d_{\mathcal{R}}+g_{\mathcal{R}}+g_{\mathcal{B}\mathcal{R}}+d_{\mathcal{L}}} + 2^{d_{\mathcal{B}}+g_{\mathcal{B}}+d_{\mathcal{R}}+g_{\mathcal{R}}+g_{\mathcal{B}\mathcal{R}}-d_{\mathcal{M}}}) \\
 & \simeq 2^{n-\min(d_{\mathcal{B}}-g_{\mathcal{R}}-g_{\mathcal{B}\mathcal{R}}-d_{\mathcal{L}}, d_{\mathcal{R}}-g_{\mathcal{B}}-g_{\mathcal{B}\mathcal{R}}-d_{\mathcal{L}}, d_{\mathcal{M}}-g_{\mathcal{B}}-g_{\mathcal{R}}-g_{\mathcal{B}\mathcal{R}})}.
 \end{aligned} \tag{3}$$

4.2 MILP-based Search Model

Now we introduce an enhanced MILP model to integrate proposed techniques, including new coloring schemes and corresponding propagation rules in detail.

Color-encoding Scheme of Neutral Words. We encode different byte types in the superposition structure with three binary variables b , r , and w :

- A blue cell ■ denotes a forward neutral byte, encoded as $(b, r, w) = (1, 0, 0)$.
- A red cell ■ denotes a backward neutral byte, encoded as $(b, r, w) = (0, 1, 0)$.
- A white cell □ denotes an arbitrary byte, encoded as $(b, r, w) = (0, 0, 1)$.
- A gray cell ■ denotes a constant byte, encoded as $(b, r, w) = (0, 0, 0)$.
- A green cell ■ denotes a superposition byte, encoded as $(b, r, w) = (1, 1, 0)$.

In the encoding system, $b = 1$ and $r = 1$ denote that a byte contains forward and backward neutral information, respectively. And $w = 1$ uniquely identifies an arbitrary byte, whose value is unknown, against other byte types. Such construction has high clarity and interpretability, rather than a simple enumeration of the five possible byte types in the superposition structure, and allows a more straightforward realization of propagation rules and efficient counting of DoF. For example, we can easily obtain the initial DoFs $|\mathcal{B}^{\text{ENC}}|$, $|\mathcal{R}^{\text{ENC}}|$, $|\mathcal{B}^{\text{KSA}}|$, and $|\mathcal{R}^{\text{KSA}}|$ by simply summing up b, r encoders in corresponding states.

Our model achieves high efficiencies and makes it possible for better attacks on designs with large state sizes. The new model can formulate attacks on **Whirlpool** and **Streebog** in full-sized versions (8×8), while Bao *et al.*'s attack on **Whirlpool** is limited to 4×4 versions with symmetry patterns [8]. Specifically, our improved MITM attack configurations for **Whirlpool** can be found within 200 seconds, and the optimization of MITM collision attack models of 6-round **Whirlpool** can be finished within just 300 seconds⁷.

In the rest of this chapter, notions b_α , r_α , and w_α are used to represent the encoders of a byte α .

Propagations through SubBytes. We formulate the SB-rule, a byte-wise propagation rule for **SubBytes**, with **LIN** integrated.

- When the input byte is not green, the color of the output byte is identical to that of the input byte.
- When the input byte is green, the output byte is either white by default or green with one cost of linearization ($d_{\mathcal{L}}$ incremented by one).

Modeling the SB-rule requires both encoders of the input and output byte as well as one additional encoder to indicate linearization cost. The rule can be converted to MILP constraints with the convex-hull method [44].

⁷ We ran our MILP models with Gurobi 9.5.2 on a desktop computer with 3.6GHz Intel Core i9 and 16GB 2667MHz DDR4.

Propagations through MixColumns. The `MixColumns` operation takes a column as input and outputs a column. Assume that the input is a mix of n_b blue bytes, n_r red bytes, n_c gray bytes, n_g green bytes, and n_w white bytes, the basic rule for the `MixColumns` operation, MC-rule in short, is formulated as follows:

- When $n_w > 0$, the output contains only white bytes.
- When $n_w = 0$ and $n_b + n_r + n_g = 0$, the output contains only gray bytes.
- When $n_w = n_r = 0$ and $n_b + n_g > 0$, the output contains n'_b blue bytes and n'_c gray bytes, with n'_c consumed DoF from the forward chunk.
- When $n_w = n_b = 0$ and $n_r + n_g > 0$, the output contains n'_r red bytes and n'_c gray bytes, with n'_c consumed DoF from the backward chunk.
- Otherwise, the output is a mix of n'_b blue bytes, n'_r red bytes, n'_c gray bytes, and n'_g green bytes, with $n'_b + n'_c$ consumed DoF from the backward chunk and $n'_r + n'_c$ consumed DoF from the forward chunk.

We use α to denote a byte in the input column and β in the output. To realize the above functionality, we introduce three column-wise encoders Eb , Er , and EW , which is constructed based on the encoding of input bytes:

$$Eb = \max_{\alpha} b_{\alpha}, \quad Er = \max_{\alpha} r_{\alpha}, \quad \text{and} \quad EW = \max_{\alpha} w_{\alpha}. \quad (4)$$

Let further $\bar{\sigma}_{\beta}^{\rightarrow}$ and $\bar{\sigma}_{\beta}^{\leftarrow}$ be binary variables that respectively track the DoF consumption at byte β (in the output column) for the forward and backward chunk. Then, the MC-rule can be formulated as:

$$\begin{cases} \sum_{\beta} w_{\beta} = \text{NROW} \cdot EW \\ \sum_{\alpha} b_{\alpha} + \sum_{\beta} b_{\beta} = \text{NROW} \cdot (Eb - EW) \\ \sum_{\alpha} r_{\alpha} + \sum_{\beta} r_{\beta} = \text{NROW} \cdot (Er - EW) \\ \text{NROW} \cdot (Eb - EW) \leq \sum_{\beta} b_{\beta} + \sum_{\beta} \bar{\sigma}_{\beta}^{\rightarrow} \leq \text{NROW} \cdot \min(Eb, 1 - EW) \\ \text{NROW} \cdot (Er - EW) \leq \sum_{\beta} r_{\beta} + \sum_{\beta} \bar{\sigma}_{\beta}^{\leftarrow} \leq \text{NROW} \cdot \min(Er, 1 - EW) \end{cases}. \quad (5)$$

Integrating Guess-and-Determine into MixColumns. We introduce a light-weight realization of GnD by integrating its functionality into MC-rule, which is named GnD-MC-rule. We introduce four GnD encoders for an input byte α , g_{α}^w , g_{α}^b , g_{α}^r , and g_{α}^{br} , which satisfy:

$$w_{\alpha} = g_{\alpha}^w + g_{\alpha}^b + g_{\alpha}^r + g_{\alpha}^{br}. \quad (6)$$

The simple constraint ensures that, when an input byte α is non-white, all GnD encoders are 0, meaning no GnD is incurred. Otherwise, when α is white, exactly one GnD encoder equals to 1 with the following meaning:

- $g_{\alpha}^w = 1$: GnD is not activated and byte α remains unknown,
- $g_{\alpha}^b = 1$: α is guessed as blue for forward propagation,
- $g_{\alpha}^r = 1$: α is guessed as red for backward propagation,
- $g_{\alpha}^{br} = 1$: α is guessed as green for both forward and backward propagations.

The GnD-MC-rule is formulated based on MC-rule by a simple tweak on the construction of the column-wise encoders:

$$Eb' = \max_{\alpha} \{b_{\alpha}, g_{\alpha}^b, g_{\alpha}^{br}\}, \quad Er' = \max_{\alpha} \{r_{\alpha}, g_{\alpha}^r, g_{\alpha}^{br}\}, \quad \text{and} \quad Ew' = \max_{\alpha} g_{\alpha}^w. \quad (7)$$

We count the guessed DoF by summing up the GnD encoders. Moreover, GnD can be turned off easily for efficiency by adding a simple constraint $w_{\alpha} = g_{\alpha}^w$.

XOR with Two Inputs. The XOR-rule models the propagation of variables through a simple XOR operation with two inputs:

- When the input involves a white byte, the output is white.
- When the input contains only gray bytes, the output is gray.
- When the input contains only blue bytes, the output is either blue with no consumption of DoF or gray consuming one DoF from the forward chunk.
- When the input contains only red bytes, the output is either red with no DoF consumption or gray consuming one DoF from the backward chunk.
- When the input is a mixture of red and blue bytes or involves green bytes, the output is green.

Generating the constraints to account for the XOR-rule in MILP is well understood and therefore omitted here.

XOR with Multiple Inputs. In addition to sequentially deriving the round keys following Equation (2), we propose a new approach to model the AES key schedule. We find the expression of intermediate bytes in terms of bytes in KSA by invoking a sourcing function, which is designed to recursively obtain the parents of an intermediate byte and cancels whenever a byte is XORed an even number of times. Then we propose the n-XOR-rule to determine the coloring of an intermediate byte and the consumed DoF, detailed as follows:

- When the input involves a white byte, the output is white.
- When the input contains only gray bytes, the output is gray.
- When the input contains only blue bytes, the output is either blue with no DoF consumed or gray with 1 DoF consumed from the forward chunk.
- When the input contains only red bytes, the output is either red with no DoF consumed or gray with 1 DoF consumed from the backward chunk.
- When the input contains both red and blue bytes, the output is one of the following:
 - green, with no consumption of DoF,
 - blue, with 1 DoF consumed from the backward chunk,
 - red, with 1 DoF consumed from the forward chunk, or
 - gray, with 1 DoF consumed from each forward and backward chunk.

We denote a byte in the expression of an intermediate byte as γ and introduce three encoders Pb , Pr , and Pw for the intermediate byte that satisfy:

$$Pb = \max_{\gamma} b_{\gamma}, \quad Pr = \max_{\gamma} r_{\gamma}, \quad \text{and} \quad Pw = \max_{\gamma} w_{\gamma}. \quad (8)$$

The constraints for the n -input XOR rule can be obtained by using the convex-hull method on Pb , Pr , Pw , the encoders of the intermediate byte, and two encoders for DoF costs.

Matching. We deploy two types of matching in our attacks: the XOR-match and the MC-match. The XOR-match is used at the feed-forward that checks E^+ , E^- , and $\#RK^{-1}$ byte by byte. If $w_{\alpha} = 0$ holds for position α in E^+ , E^- , and $\#RK^{-1}$, then $m_{\alpha} = 1$. Otherwise, $m_{\alpha} = 0$. Then, $d_{\mathcal{M}}^{\text{XOR}}$ results from:

$$d_{\mathcal{M}}^{\text{XOR}} = \sum_{\alpha} m_{\alpha}. \quad (9)$$

Besides, the MC-match takes the input and output of a `MixColumns` operation as E^+ and E^- and counts the cumulative non-white bytes at a common column index. Let Δ be a column index and denote the cumulative non-white bytes in E_{Δ}^+ and E_{Δ}^- as t_{Δ} . If there exist $t_{\Delta} > \text{NROW}$, then we have a $t_{\Delta} - \text{NROW}$ degrees for matching at column Δ . Otherwise, there are no degrees of matching at column Δ . Then $d_{\mathcal{M}}^{\text{MC}}$ is given by the sum over all columns:

$$d_{\mathcal{M}}^{\text{MC}} = \sum_{\Delta} \max(0, t_{\Delta} - \text{NROW}). \quad (10)$$

Objective Function. Our search model aims to maximize:

$$\min\{d_{\mathcal{B}} - g_{\mathcal{R}} - g_{\mathcal{BR}} - d_{\mathcal{L}}, d_{\mathcal{R}} - g_{\mathcal{B}} - g_{\mathcal{BR}} - d_{\mathcal{L}}, d_{\mathcal{M}} - g_{\mathcal{B}} - g_{\mathcal{R}} - g_{\mathcal{BR}}\}. \quad (11)$$

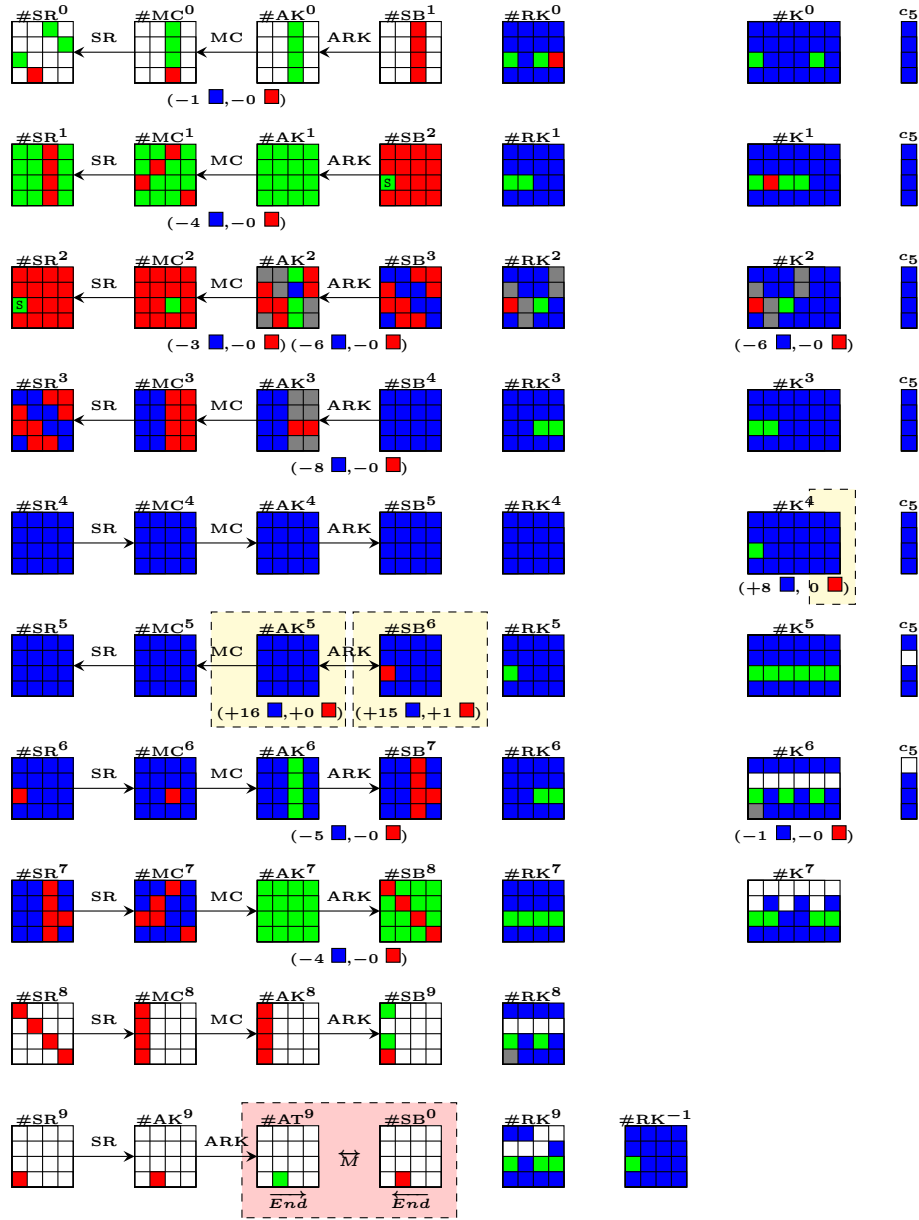
According to Equation (3), $\min\{\vec{d}_b, \overleftarrow{d}_r, \overleftrightarrow{m}\}$ determines the complexity of an MITM attack. Thus, the search for the optimal MITM attack pattern of given *config* is converted to a maximization problem on objective τ_{obj} :

5 Applications to AES and Whirlpool

In this section, we briefly describe the *first* 10-round MITM pseudo-preimage attack on the compression function of `AES-192` and the *memoryless* 5-round MITM pseudo-preimage attack on the compression function of `Whirlpool`.

5.1 First MITM Pseudo-preimage Attack on 10-round AES-192

The previous best MITM pseudo-preimage attack on `AES-192` reaches 9 rounds [8]. Adopting **LIN** and **DIS**, we obtain the first MITM pseudo-preimage attack on 10-round `AES-192`, which is provided in Figure 8 and summarized below:



- $(\vec{v}, \vec{v}) = (39 \blacksquare, 1 \blacksquare)$ $\vec{S}^{\text{ENC}} = (31 \blacksquare, 1 \blacksquare)$ $\vec{S}^{\text{RSA}} = (8 \blacksquare, 0 \blacksquare)$
- $(d_{\mathcal{B}}, d_{\mathcal{R}}, d_{\mathcal{M}}, d_{\mathcal{L}}) = (1, 1, 1, 0.5)$
- $(d_{\mathcal{B}} - d_{\mathcal{L}}, d_{\mathcal{R}} - d_{\mathcal{L}}, d_{\mathcal{M}}) = (0.5, 0.5, 1)$

Fig. 8. An MITM pseudo-preimage attack of 10-round AES-192.

Algorithm 1: Computing forward neutral words (blue) for 10-round AES-192.

```

1 Initialize a table  $T_{\text{blue}}^{\text{neutral}}$ ;
2 Fix 6 bytes  $\#K^2[1, 2, 6, 7, 12, 13]$ , 6  $\blacksquare$  cells in  $\#AK^2$ , 3 bytes  $\#MC^2[8, 9, 11]$ , 2
   bytes  $\#MC^1[8, 15]$  and  $\#AK^3[8..15]$  be all zero.
3 for 4  $\blacksquare$  blue cells  $\#K^4[4, 5, 6, 7] \in (\mathbb{F}_2^8)^4$  do
4   Use 4 constants  $\#K^2[6, 7, 12, 13]$  to derive  $\#K^4[12, 13]$ ,  $\#K^3[20, 23]$ ;
5   for 3  $\blacksquare$  blue cells  $\#K^3[18, 19, 22] \in (\mathbb{F}_2^8)^3$  do
6     Use 2 constant  $\#K^2[1, 2]$  to derive  $\#K^4[1, 2]$ ;
7     for 6  $\blacksquare$  blue cells  $\#K^4[8, 9, 10, 11, 14, 15] \in (\mathbb{F}_2^8)^6$  do
8       Derive  $\#RK^2[14, 15]$  from  $\#K^4[6, 7, 14, 15]$ ;
9       Use 1 constant  $\#MC^1[15]$  to derive  $\#K^3[21]$  from
          $\#K^1[20..23] = \#K^4[12..15] \oplus \#K^4[8..11] \oplus \#K^3[20..23]$  and
          $MC^{-1} \cdot \#K^1[20..23] = (*, *, *, 0)$ ;
10      Derive  $\#RK^2[4, 5]$  from  $\#K^4[4, 5]$ ,  $\#K^3[21, 22]$ ;
11      Use 4 constant  $\#AK^2[4, 5, 14, 15]$  and  $\#RK^2[4, 5, 14, 15]$  to derive
          $\#SB^3[4, 5, 14, 15]$ ;
12      for 2  $\blacksquare$  blue cells  $\#K^4[3], \#K^3[16] \in (\mathbb{F}_2^8)^2$  do
13        Derive  $\#RK^2[3]$  from  $\#K^3[16, 20]$ ,  $\#K^4[3]$ ;
14        Use 1 constant  $\#AK^2[3]$  to derive  $\#SB^3[3]$ ;
15        Use 1 constant  $\#MC^1[8]$  to derive  $\#K^3[17]$  from
           $\#K^1[16..19] = \#K^3[16..19] \oplus \#K^4[4..7] \oplus \#K^4[8..11]$  and
           $MC^{-1} \cdot \#K^1[16..19] = (0, *, *, *)$ ;
16        Derive  $\#K^4[16..23]$ ;
17        Use 4 constants  $\#AK^2[0], \#MC^2[8, 9, 11]$  to derive
           $\#K^4[0], \#SB^3[0, 9, 10]$  by solving 4 linear Equation (12);
          // Now we know all  $\blacksquare$  blue cells in  $\#SB^3$  and  $\#K^4$ 
18        Derive the blue part of  $\#SR^2[2]$ ;
          // Linearization of S-boxes
19        for  $2^{8 \times 0.5}$  values of  $c_0 = (\#SR^2[2])^{17}$  do
20          Compute 14 constants  $(\#MC^0[11], \#MC^1[2, 5], \#SB^6[2],$ 
             $\#SB^7[8..11, 14], \#SB^8[0, 5, 10, 15], \#K^6[3]) = (c_1, \dots, c_{14})$ ;
21          Update the table  $T_{\text{blue}}^{\text{neutral}}[c_0, \dots, c_{14}]^{\pm}$  ( $\blacksquare$  in  $\#K^4$  and
             $\#SB^3$ );
            // For each value of  $(c_0, \dots, c_{14})$ ,  $2^{8 \times (15.5 - 14.5)} = 2^{8 \times 1}$ 
            candidates expected
22 return  $T_{\text{blue}}^{\text{neutral}}$ ;

```

- Initial DoF for forward neutral words \vec{v} (■): 39 bytes (16 in $\#AK^5$, 15 bytes in $\#SB^6$, and 8 bytes $\#K^4[16\dots 23]$);
- Initial DoF for backward neutral words \overleftarrow{v} (■): 1 byte ($\#SB^6[2]$);
- Consumed DoF in forward computation $\vec{\sigma}$: 38 bytes;
- Consumed DoF in backward computation $\overleftarrow{\sigma}$: zero bytes;
- Guessed bytes for blue, red, and both colors g_B, g_R, g_{BR} : $g_R = g_{BR} = 0$ bytes and $g_B = 0$ bytes;
- Guessed byte equivalents for linearization: $d_L = 0.5$ bytes.
- Matching DoF d_M : 1 byte between $\#AT^9$ and $\#SB^0$.
- Remaining DoF: $d_B - d_L = 1 - 0.5 = 0.5$ and $d_R - d_L = 1 - 0.5 = 0.5$.

Compute initial values forward neutral bytes (Blue). Note that the value of a byte in this phase represents the value computed from the blue parts. For example, fixing $\#K^2[2]$ to zero means that the blue initial bytes have a zero impact on this byte. To get the initial values of the blue neutral bytes, the following constraints among states $\#SB^3, \#K^4, \#K^3$ will be enforced.

$$\left\{ \begin{array}{l} \#SB^3[0] \oplus \#K^4[0] \oplus S(\#K^3[21]) \oplus S(\#K^3[17] \oplus \#K^3[21]) = \#AK^2[0] \\ MC^{-1} \cdot \begin{pmatrix} \#K^4[0] \oplus \#K^4[8] \\ \#SB^3[9] \oplus \#K^4[1] \oplus \#K^4[9] \\ \#SB^3[10] \oplus \#K^4[2] \oplus \#K^4[10] \\ \#K^4[3] \oplus \#K^4[11] \end{pmatrix} = \begin{pmatrix} \#MC^2[8] \\ \#MC^2[9] \\ * \\ \#MC^2[11] \end{pmatrix} \end{array} \right. , \quad (12)$$

where $\#AK^2[0]$ and $\#MC^2[8, 9, 11]$ are fixed as zeroes. Algorithm 1 generates the solution space of blue neutral words. The time complexity is upper bounded by $2^{8 \times 15.5} = 2^{124}$ operations.

The MITM attack procedure for 10-round AES-192. For backward neutral words (■ cells), we will iterate over $\#SB^6[2]$. We provide the main attack procedure derived from Figure 8 in Algorithm 2.

The attack complexity. The time complexity of the above MITM pseudo-preimage attack of 10-round AES-192 is about $2^{128 - 8 \times \min(0.5, 0.5, 1)} = 2^{124}$. The table T^+ dominates the memory complexity with $2^{8 \times 1.5} \approx 2^{12}$. The pre-computation table $T_{\text{blue}}^{\text{neutral}}$ dominates the memory complexity with $2^{8 \times 15.5} = 2^{124}$.

5.2 Improved MITM Preimage Attacks of Whirlpool

We use the **SIM** technique to search for attack configurations of Whirlpool, improved MITM (pseudo-)preimage attacks are obtained for both 5- and 6-round Whirlpool. In particular, we could find an attack on 5-round Whirlpool with $O(1)$ memory and present the first MITM attack on 7.75-round Whirlpool, including the SB, SR, and MC operations in the last round. The previous best (pseudo-)preimage attacks are the 7-round MITM attacks on Whirlpool presented by Bao *et al.* [8] at Crypto 2022.

Algorithm 2: MITM attack on 10 rounds of the AES-192 compression function.

```

1 for  $(c_1, \dots, c_{14}) \in (\mathbb{F}_2^8)^{14}$  do
2   Initialize  $T^+$ ;
   // For blue neutral words
3   for the  $2^{8 \times 0.5}$  values  $c_0$  of the hint pool do
4     Lookup the table  $T_{\text{blue}}^{\text{neutral}}[c_0, \dots, c_{14}]$  to get candidates of  $\blacksquare$  blue cells in
        $\#K^4$  and  $\#SB^3$ ;
5     for the values of  $2^{8 \times 1}$  in  $\#K^4$  and  $\#SB^3$  do
6       Derive  $m^+$  (the blue part of  $\#AT^9[7] = \#RK^9[7] \oplus \#RK^{-1}[7]$ ) and
         update the table  $T^+[m^+, c_0] \stackrel{\pm}{=} [\text{blue neutral bytes}]$ ;
         //  $2^{8 \times 0}$  entries for each index in  $T^+$ 

       // For red neutral words
7       for the  $2^{8 \times 0.5}$  values of  $c_0$  do
8         for  $\#SB^6[2] \in \mathbb{F}_2^8$  do
9           Compute  $m^-$  (the red part of  $\#AT^9[7] \oplus \#SB^0[7]$ ), which equals the
             blue part of  $\#AT^9[7]$ ;
10          Check for entries in  $T^+[m^-, c_0]$  to derive 1 blue neutral byte;
11          Check if the red and blue parts of  $\#SR^2[2]$  produce  $c_0$ ;
             //  $2^{8 \times (14 + 0.5 + 1 + 1 - 1 - 0.5)} = 2^{8 \times 15}$  candidates expected
12          Compute the full  $\#AT^9$  and  $\#SB^0$  in both colors to check the
             remaining 15 cells;
13          if the full match is found then
14            //  $2^{8 \times (15 - 15)} = 1$  candidate expected
             Output the preimage and stop;

```

Improved and Memoryless Preimage Attack of 5-round Whirlpool. We directly perform search on the full-size 8×8 version for Whirlpool, and our improved search result for 5-round Whirlpool is given in Figure 9. Compared to the previous best result of 5-round Whirlpool found by Bao *et al.* [8, Figure 13], GnP is still not required but BiDir is utilized (48 \blacksquare cost at Round 0). However, the DoF cost for neutral words is more concentrated at the starting point, *e.g.*, 48 red cells canceled at Round 0 (48 bytes DoF cost will be compensated at Round 1), and another 24 red cells will be canceled at the MC operation for $\#KMC^2$, which makes it more efficient to generate the red neutral words and can improve our attack on 5-round Whirlpool further to *memoryless* when combined with the *same color match*. We elaborate on the attack configuration below.

- Initial DoF for forward neutral words \vec{v} (\blacksquare): 24 bytes (8 blue cells in $\#SB^1$ and 16 blue cells in $\#KK^0$);
- Initial DoF for backward neutral words \overleftarrow{v} (\blacksquare): 48 bytes (48 red cells in $\#KK^0$ are set to be equal to the corresponding cells in $\#SB^1$, then all red

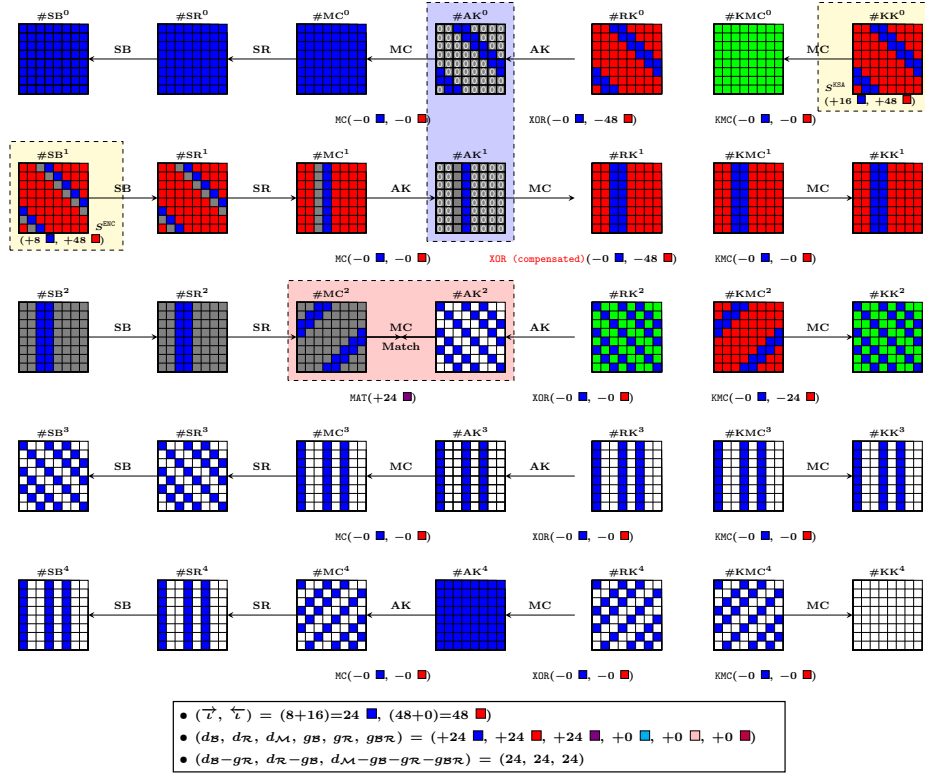


Fig. 9. An MITM pseudo-preimage attack of 5-round Whirlpool.

- bytes can be canceled to constants marked as zero constant⁸ \square in $\#AK^0$ and $\#AK^1$, with just 48 bytes DoF cost for XOR operation);
- Consumed DoF for forward $\vec{\sigma}$: zero;
 - Consumed DoF for backward $\overleftarrow{\sigma}$: 24 bytes for $\#KMC^2 \xrightarrow{MC} \#KK^2$;
 - Guessed bytes for blue, red and both colors g_B, g_R, g_{BR} : all zero byte;
 - Matching DoF d_M : 24 bytes between $\#MC^2$ and $\#AK^2$.

Then, the remaining DoF for the MITM attack is $d_B = 24, d_R = 24, d_M = 24$.

Compute initial values for forward neutral words (Blue). As there is no DoF cost for blue neutral words, to obtain the corresponding initial values, one only needs to enumerate values of $24(16 + 8)$ blue cells in $\#KK^0$ and $\#SB^1$.

⁸ The AddRoundConstant in the key schedule of Whirlpool is the last operation for each round (SB, SR, MC, AC), that is the subkey added into the encryption already involved with the round constant. When using the **SIM** technique, the corresponding cells related to the XOR compensation here will be canceled to all zero constants.

Compute initial values for backward neutral bytes (Red). To get the initial values of red neutral words, the constraints among $\#KMC^2$ and $\#KK^2$ are as below

$$\begin{pmatrix} a_0 & - & - & a_9 & - & a_{15} & - & - \\ - & a_3 & - & - & a_{12} & - & a_{18} & - \\ - & - & a_6 & - & - & a_{16} & - & a_{21} \\ a_1 & - & - & a_{10} & - & - & a_{19} & - \\ - & a_4 & - & - & a_{13} & - & - & a_{22} \\ a_2 & - & a_7 & - & - & a_{18} & - & - \\ - & a_5 & - & a_{11} & - & - & a_{21} & - \\ - & - & a_8 & - & a_{14} & - & - & a_{23} \end{pmatrix} = MC \cdot \begin{pmatrix} \#KMC_0^2 & \#KMC_8^2 & \dots & \#KMC_{48}^2 & \#KMC_{56}^2 \\ \#KMC_1^2 & - & \dots & \#KMC_{49}^2 & \#KMC_{57}^2 \\ - & - & \dots & \#KMC_{50}^2 & \#KMC_{58}^2 \\ - & \#KMC_{11}^2 & \dots & \#KMC_{51}^2 & - \\ \#KMC_2^2 & \#KMC_{12}^2 & \dots & - & - \\ \#KMC_3^2 & \#KMC_{13}^2 & \dots & - & \#KMC_{61}^2 \\ \#KMC_4^2 & \#KMC_{14}^2 & \dots & \#KMC_{54}^2 & \#KMC_{62}^2 \\ \#KMC_5^2 & \#KMC_{15}^2 & \dots & \#KMC_{55}^2 & \#KMC_{63}^2 \end{pmatrix}, \quad (13)$$

where a_i ($0 \leq i \leq 23$) are the chosen constants for $\#KK^2$.

The MITM attack procedure for 5-round *Whirlpool*. We provide the *memoryless* attack procedure derived from Figure 9 in Algorithm 3. The *same-color match* is employed, which is firstly observed by Guo *et al.* [19] in MITM preimage attacks and recently also utilized by Hou *et al.* [21] for MITM attacks on Feistel constructions. It means a match with only blue ■ and gray ■ (or only red ■ and gray ■) at the matching point, rather than a mixture of red ■ and blue ■ cells. While gray ■ cells are fixed as constants, and thus are known in both forward (blue) or backward (red) computations, a same-color match, *e.g.*, only blue or gray, can be performed independently of the red neutral words.

Algorithm 3: MITM attack on 5-round *Whirlpool* compression function

```

1 Fix 8 ■ constant cells in  $\#SB^1$  to all zero;
2 Fix 8 constants  $(a_{16}, a_{17}, \dots, a_{23})$  in Equation (13) to all zero;
3 for  $C = (a_0, a_1, \dots, a_{15}, a_{16}, \dots, a_{23}) \in (\mathbb{F}_2^8)^{16}$  do
    // For red neutral words
    // As there is no ■ red cell in matching states  $\#MC^2$  and
    //  $\#AK^2$ , one does not need a store table  $T^+$  and thus does not
    // need to solve the Equation (13) here for back neutral words,
    // which can be done after the partial match
    // For blue neutral words
4 for 8 ■ blue cells in  $\#SB^1 \in (\mathbb{F}_2^8)^8$  and 16 ■ blue cells in  $\#KK^0 \in (\mathbb{F}_2^8)^{16}$  do
5     Compute forward to the matching state  $\#MC^2$ ;
6     Compute backward to the matching state  $\#AK^2$ ;
    // Only ■ gray and ■ blue cells in matching states here
7     if  $\#MC^2$  and  $\#AK^2$  pass the partial match then
8         //  $2^{8 \times (16+24-24)} = 2^{8 \times 16}$  candidates expected
            Solve Equation (13) according to current constant  $C$  and obtain
             $2^{8 \times 24}$   $\#KMC^2$  for red neutral words;
9         Compute forward and backward to match the rest 40 cells;
10        if the full match is found then
11            //  $2^{8 \times (16+24-40)} = 1$  candidate expected
                Output the preimage and stop;

```

The attack complexity. The time complexity of the above MITM pseudo-preimage attack on 5-round `Whirlpool` is about $2^{512-8\times\min(24,24,24)} = 2^{320}$.⁹ Thanks to the *same color match* between $\#MC^2$ and $\#AK^2$, the partial-matching is independent of backward red neutral words, and the constraints on backward neutral words are *linear*, then the store tables for the partial-matching can be *saved* and thus the memory complexity is $O(1)$.

6 Conclusion

In this work, we advanced the state-of-the-art MITM attacks on AES-like hashing. Among the new techniques, **LIN** and **DIS** contributed to the first 10-round preimage attack on AES-192 and assorted improved results on `Rijndael`-based hashing, while **SIM** better addressed the dependencies and reduced the attack complexities on `Whirlpool` and `Streebog`. We argued that the ideas behind the new techniques are generic and expected them to have more applications in other attack scenarios.

Open Problems. The fact that the non-linear part of the AES S-box is a single monomial x^{254} allows us to obtain the full output from guessing a space with dimension four. We also investigated the S-boxes of `Whirlpool` and `Streebog` and found that their S-boxes possess fewer-dimensional subspaces (yielding parts of the output thus less powerful) for which we could not find better attacks. If more properties of S-boxes are revealed for AES-like hashing, *i.e.*, algebraic properties, better MITM attacks exploiting the **LIN** technique on these target ciphers can be expected. Furthermore, we applied the techniques to all AES/`Rijndael` variants but could not find other improvements compared to [7,8,16,46]. The relation between the security margin and block-key ratio remains to be further investigated.

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⁹ For comparisons, we directly use the calculation method in [8] to provide the time complexity for `Whirlpool`.

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Appendix A Improved Collision Attacks of Whirlpool

Besides our results on preimage attacks, we also improve collision attacks on round-reduced `Whirlpool`. Our attack on 6-round is shown in Figure 10. Compared to the previous best collision attack on 6 rounds of `Whirlpool` presented by Dong *et al.* at Crypto 2021 [16], we reduced the time complexity from 2^{248} to 2^{240} . We also extended it to a collision attack on 6.5-round `Whirlpool`, which is presented in Figure 11 and has the same complexity of 2^{240} operations. When converting an MITM preimage attack into a collision attack, the input for key schedule will be fixed for `Whirlpool`, thus there will be no DoF introduced in key schedule in our search model. To demonstrate, we only provide the attack details for 6-round `Whirlpool` as below.

- Initial DoF for forward neutral words \vec{v} (■): 36 bytes (in $\#MC^3$);
- Initial DoF for backward neutral words \overleftarrow{v} (in ■): 4 bytes (in $\#MC^3$);
- Consumed DoF for forward $\vec{\sigma}$: 32 bytes (16 for $\#MC^4 \xrightarrow{MC} \#AK^4$ and 16 for $\#AK^1 \xrightarrow{MC^{-1}} \#MC^1$);
- Consumed DoF for backward $\overleftarrow{\sigma}$: zero bytes;
- Guessed bytes for blue, red and both colors g_B, g_R, g_{BR} : all zero bytes;
- Matching DoF d_M : 4 bytes between $\#MC^5$ and $\#AK^5$.

Then, the remaining DoF is $d_B = d_R = d_M = 4$.

Compute initial values for forward neutral words (Blue). To get the initial values of forward neutral words, one should focus on the constraints among states $\#MC^4$ and $\#AK^4$, $\#MC^1$ and $\#AK^1$ as below

- For $\#MC^4$ and $\#AK^4$, there are 16 bytes MC DoF cost for forward neutral words, which applies the constraints in Equation (14):

$$\begin{pmatrix} - & - & - & a_6 & a_8 & - & - & - \\ - & - & - & a_9 & a_{10} & - & - & - \\ - & - & - & - & a_{11} & a_{12} & - & - \\ a_0 & - & - & - & - & - & a_{13} & a_{14} \\ a_1 & a_2 & - & - & - & - & - & - \\ - & a_3 & a_4 & - & - & - & - & - \\ - & - & a_5 & a_7 & - & - & - & - \end{pmatrix} = MC \cdot \begin{pmatrix} \#MC_0^4 & \#MC_8^4 & \dots & \#MC_{56}^4 \\ \#MC_1^4 & \#MC_9^4 & \dots & \#MC_{57}^4 \\ \#MC_2^4 & \#MC_{10}^4 & \dots & \#MC_{58}^4 \\ \#MC_3^4 & \#MC_{11}^4 & \dots & \#MC_{59}^4 \\ \#MC_4^4 & \#MC_{12}^4 & \dots & \#MC_{60}^4 \\ \#MC_5^4 & \#MC_{13}^4 & \dots & \#MC_{61}^4 \\ \#MC_6^4 & \#MC_{14}^4 & \dots & \#MC_{62}^4 \\ \#MC_7^4 & \#MC_{15}^4 & \dots & \#MC_{63}^4 \end{pmatrix}, \quad (14)$$

where a_i ($0 \leq i \leq 15$) are the chosen constants for $\#AK^4$.

- For $\#MC^1$ and $\#AK^1$, there are also 16 bytes MC DoF cost for forward neutral words, which applies the constraints in Equation (15) as below

$$\begin{pmatrix} - & b_2 & - & - & - & b_{10} & - & - \\ b_0 & - & - & b_8 & - & - & - & - \\ - & - & b_6 & - & - & - & b_{14} & - \\ - & b_3 & - & - & b_{11} & - & - & - \\ b_1 & - & - & b_9 & - & - & - & - \\ - & - & b_7 & - & - & - & b_{15} & - \\ - & - & b_5 & - & - & b_{12} & - & - \end{pmatrix} = MC^{-1} \cdot \begin{pmatrix} \#AK_0^1 & \#AK_8^1 & \dots & \#AK_{56}^1 \\ \#AK_1^1 & \#AK_9^1 & \dots & \#AK_{57}^1 \\ \#AK_2^1 & \#AK_{10}^1 & \dots & \#AK_{58}^1 \\ \#AK_3^1 & \#AK_{11}^1 & \dots & \#AK_{59}^1 \\ \#AK_4^1 & \#AK_{12}^1 & \dots & \#AK_{60}^1 \\ \#AK_5^1 & \#AK_{13}^1 & \dots & \#AK_{61}^1 \\ \#AK_6^1 & \#AK_{14}^1 & \dots & \#AK_{62}^1 \\ \#AK_7^1 & \#AK_{15}^1 & \dots & \#AK_{63}^1 \end{pmatrix}, \quad (15)$$

where b_i ($0 \leq i \leq 15$) are the chosen constants for $\#MC^1$.

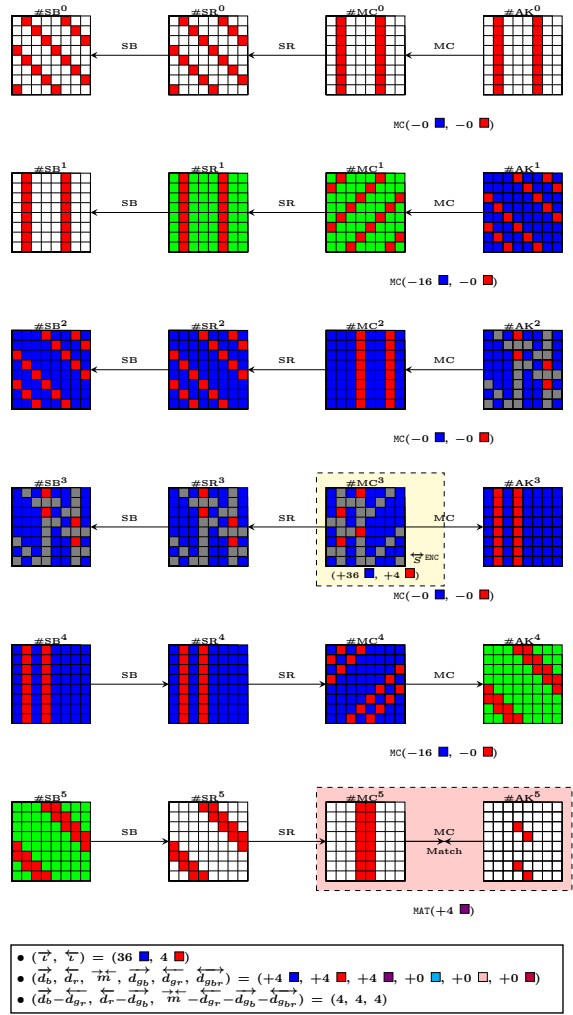


Fig. 10. An MITM collision attack of 6-round Whirlpool.

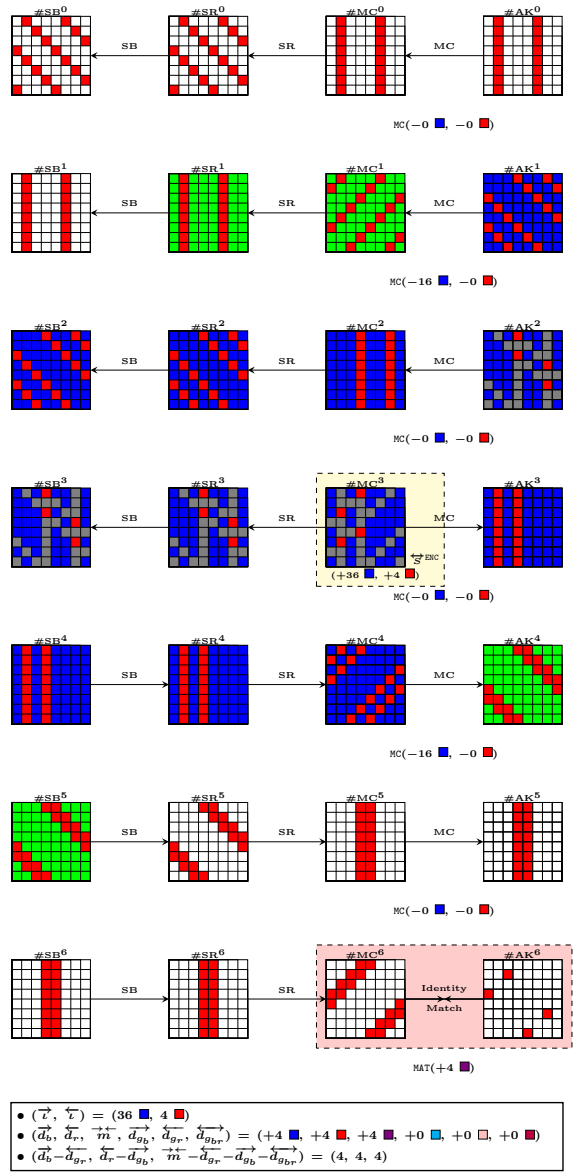


Fig. 11. An MITM collision attack of 6.5-round Whirlpool.

In Algorithm 4, we provide an efficient method to obtain blue neutral words satisfying the constraints (in Equation (14) and (15)) in a non-linear system, which has the time complexity and memory complexity both with $2^{8 \times 30} = 2^{240}$.

Algorithm 4: Compute forward neutral words (blue) for 6-round collision attack on Whirlpool

```

1 Initiate two tables  $T_{\text{blue}}^{\text{neutral}}$  and  $T_{\text{blue}}^{\text{neutral}}$ ;
  // The index list  $I$  corresponds with 12 constant cells in columns 0,
  2, 3, 5, 6 and 7 of #MC3
2 Set list  $I = [4, 6, 17, 20, 34, 39, 41, 46, 48, 53, 60, 63]$ ;
3 Fix 6 ■ constant cells  $(a_0, \dots, a_5)$  in Equation (14) to all zero;
4 Fix 12 ■ constant cells #MC3[ $I$ ] to all zero;
5 Fix 12 ■ constant cells in columns 1 and 3 of #MC3 to all zero;
6 for any 24 ■ blue cells in #SB4 =  $X \in (\mathbb{F}_2^8)^{24}$  do
7   Compute to 18 middle values denoted by  $m^+$ , which correspond with
    $(a_0, \dots, a_5)$  and #MC3[ $I$ ] when passing MC or MC-1;
8   Update to table with  $T_{\text{blue}}^{\text{neutral}}[m^+][0] += X$  (with size  $2^{8 \times 24}$ );
9 for left 24 ■ blue cells in #SB4 =  $Y \in (\mathbb{F}_2^8)^{24}$  do
10  Compute to 18 middle values denoted by  $m^-$ , which correspond with
    $(a_0, \dots, a_5)$  and #MC3[ $I$ ] when passing MC or MC-1;
11  Update to table with  $T_{\text{blue}}^{\text{neutral}}[m^-][1] += Y$  (with size  $2^{8 \times (48-18)} = 2^{8 \times 30}$ );
12 for each  $(x, y)$  in  $T_{\text{blue}}^{\text{neutral}}$  do
13   Compute to rest 10 constants cells  $(a_6, \dots, a_{15})$  in Equation (14);
14   Compute to 16 constants cells  $(b_0, \dots, b_{15})$  in Equation (15);
   // Each index has  $2^{8 \times (30-26)} = 2^{8 \times 4}$  values on average
15   Update to table with  $T_{\text{blue}}^{\text{neutral}}[(a_6, \dots, a_{15}, b_0, \dots, b_{15})] += (x, y)$ ;
16 return  $T_{\text{blue}}^{\text{neutral}}$ ;

```

Compute initial values backward neutral bytes (Red). As there is no DoF cost for backward neutral words, to obtain the corresponding initial values, one only needs to enumerate all possible values of 4 red cells in #MC³.

The MITM collision attack procedure for 6-round Whirlpool. The main attack procedure derived from Figure 10 is provided in Algorithm 5.

The attack complexity. The time complexity of the above MITM collision attack of 6-round Whirlpool is about 2^{240} . The table $T_{\text{blue}}^{\text{neutral}}$ dominates the memory complexity with $2^{30 \times 8} = 2^{240}$.

Appendix B Improved Pseudo-preimage Attack on 8-round AES-192

The attack configuration is as below:

Algorithm 5: MITM collision attack of 6-round Whirlpool

```

1 Let 10 ■ constant cells  $(a_6, \dots, a_{15})$  in Equation (14) and 16 ■ constant cells
    $(b_0, \dots, b_{15})$  in Equation (15) be  $C_G$ ;
2 Call Algorithm 4 to build  $T_{\text{blue}}^{\text{neutral}}$  (with size  $2^{8 \times 30}$ );
3 Initiate a table  $T_{\text{collision}}$ ;
4 for  $C_G \in (\mathbb{F}_2^8)^{26}$  do
    // For red neutral words
5   for 4 ■ red cells in  $\#MC^3 \in (\mathbb{F}_2^8)^4$  do
        // As there is no any ■ blue cell in matching states  $\#AK^5$ 
        // and  $\#MC^5$ , the same color match can be used here
6       Compute forward to the matching state  $\#MC^5$ ;
7       Compute backward to the matching state  $\#AK^5$ ;
8       if  $\#MC^5$  and  $\#AK^5$  pass the partial match then
            //  $2^{8 \times (26+4+4-4)} = 2^{8 \times 30}$  candidates expected
9           Lookup in table with  $T_{\text{blue}}^{\text{neutral}}[C_G]$ ;
10          Reconstruct the message  $X$  from neutral words and constants;
11          Insert  $X$  into  $T_{\text{collision}}$  indexed by the hash value of  $X$ ;
12          if The full collision is found then
                // One candidate expected
13             Output the collision and stop;

```

- The initial DoF for forward neutral words \vec{v} (■): 36 bytes (16 in $\#AK^4$, 12 in $\#SB^5[4..15]$ and eight in $\#K^3[0..7]$);
- Initial DoF for backward neutral words \overleftarrow{v} (in ■): 4 bytes ($\#SB^5[0..3]$);
- Consumed DoF for forward $\vec{\sigma}$: 32 bytes;
- Consumed DoF for backward $\overleftarrow{\sigma}$: zero bytes;
- Gussed bytes for blue, red, and both colors g_B, g_R, g_{BR} : $g_R = g_B = g_{BR} = 0$ bytes;
- Gussed byte equivalents for linearization: $d_{\mathcal{L}} = 0.5$ bytes.
- Matching DoF $d_{\mathcal{M}}$: 4 byte.

The remaining DoF are composed of $d_B - d_{\mathcal{L}} = 4 - 0.5 = 3.5$, $d_R - d_{\mathcal{L}} = 4 - 0.5 = 3.5$ and $d_{\mathcal{M}} = 4$.

Compute initial values forward neutral bytes (Blue). Note that the value of a byte in this phase represents the value computed from the blue parts.

Fix $\#K^0[23]$, $\#K^4[20..23]$, $\#K^5[0, 1, 2]$, $\#MC^0[1, 3, 8, 10]$, $\#MC^1[1, 3, 5, 7, 8, 10, 13, 15]$ be all zero, and the following constraints will be utilized by to get

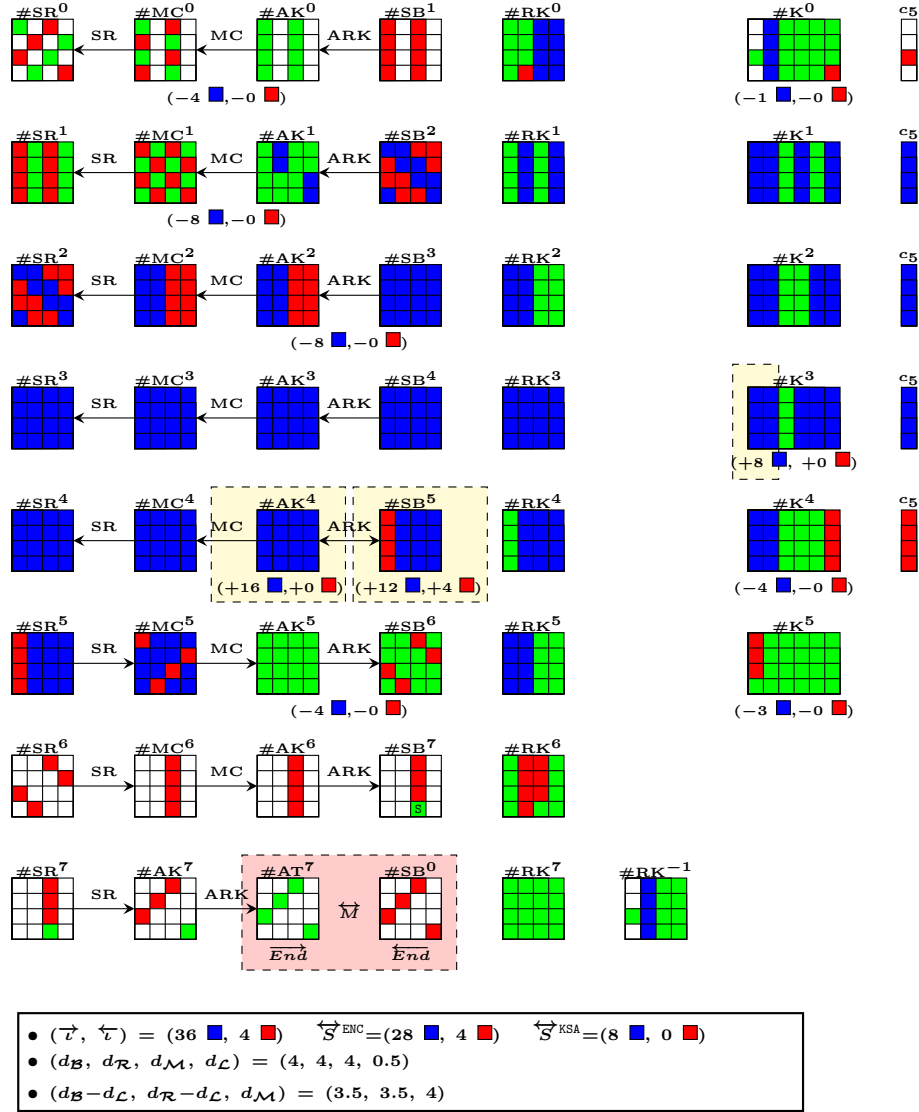


Fig. 12. An MITM pseudo-preimage attack of 8-round AES-192.

the initial values of blue neutral bytes.

$$\left\{ \begin{array}{l}
\bigoplus_{i=0}^5 \#K^3[4 \cdot i] \oplus S(\#K^3[21]) = 0, \\
\bigoplus_{i=0}^5 \#K^3[4 \cdot i + 1] \oplus S(\#K^3[22]) = 0, \\
\bigoplus_{i=0}^5 \#K^3[4 \cdot i + 2] \oplus S(\#K^3[23]) = 0, \\
\bigoplus_{i=0}^5 \#K^3[4 \cdot i + 3] \oplus S(\#K^3[20]) = 0, \\
\#K^3[0] \oplus S(\#K^3[21]) = 0, \\
\#K^3[1] \oplus S(\#K^3[22]) = 0, \\
\#K^3[2] \oplus S(\#K^3[23]) = 0 \\
\bigoplus_{i=2}^5 \#K^3[4 \cdot i + 3] = 0, \\
\text{MC}^{-1} \cdot \begin{pmatrix} \bigoplus_{i=1}^4 \#K^3[4 \cdot i] \\ \bigoplus_{i=1}^4 \#K^3[4 \cdot i + 1] \\ \bigoplus_{i=1}^4 \#K^3[4 \cdot i + 2] \\ \bigoplus_{i=1}^4 \#K^3[4 \cdot i + 3] \end{pmatrix} = \begin{pmatrix} * \\ 0 \\ * \\ 0 \end{pmatrix}, \\
\text{MC}^{-1} \cdot \begin{pmatrix} \#K^3[0] \oplus S(\#K^3[17] \oplus \#K^3[21]) \oplus S(\#K^3[13] \oplus \#K^3[21]) \\ \#K^3[1] \oplus S(\#K^3[18] \oplus \#K^3[22]) \oplus S(\#K^3[14] \oplus \#K^3[23]) \\ \#K^3[2] \oplus S(\#K^3[19] \oplus \#K^3[23]) \oplus S(\#K^3[15] \oplus \#K^3[23]) \\ \#K^3[3] \oplus S(\#K^3[16] \oplus \#K^3[20]) \oplus S(\#K^3[12] \oplus \#K^3[20]) \end{pmatrix} = \begin{pmatrix} 0 \\ * \\ 0 \\ * \end{pmatrix}.
\end{array} \right. \quad (16)$$

In the following, let $\#X[i] = \#SB^2[i] \oplus \#K^3[i]$ for all $i \in \{0, \dots, 15\}$.

$$\text{MC}^{-1} \cdot \begin{pmatrix} \#X[0] \oplus \#K^3[8] & \#X[4] \oplus \#K^3[12] & \#K^3[8] \oplus \#K^3[16] & \#K^3[12] \oplus \#K^3[20] \\ \#K^3[1] \oplus \#K^3[9] & \#X[5] \oplus \#K^3[13] & \#X[9] \oplus \#K^3[17] & \#K^3[13] \oplus \#K^3[21] \\ \#K^3[2] \oplus \#K^3[10] & \#K^3[6] \oplus \#K^3[14] & \#SB^2[10] \oplus \#K^4[10] \oplus \#K^4[18] & \#X[14] \oplus \#K^3[22] \\ \#X[3] \oplus \#K^3[11] & \#K^3[7] \oplus \#K^3[15] & \#K^3[11] \oplus \#K^3[19] & \#X[15] \oplus \#K^3[23] \end{pmatrix} = \begin{pmatrix} 0 & * & 0 & * \\ * & 0 & * & 0 \\ 0 & * & 0 & * \\ * & 0 & * & 0 \end{pmatrix}. \quad (17)$$

Algorithm 6: Compute forward neutral words (blue) for 8-round AES-192.

- 1 Initiate two tables $T_{\text{middle}}^{\text{neutral}}$ and $T_{\text{blue}}^{\text{neutral}}$;
 - 2 Fix $\#K^0[23]$, $\#K^4[20..23]$, $\#K^5[0, 1, 2]$, $\#MC^0[1, 3, 8, 10]$, and $\#MC^1[1, 3, 5, 7, 8, 10, 13, 15]$ to zeroes.
 - 3 **for** 12 ■ blue cells $\#K^3[0..11] \in (\mathbb{F}_2^8)^{12}$ **do**
 - 4 Compute to 18 middle values denoted by m^+ , which is the part of Equation (16) that involves $\#K^3[0..11]$;
 - 5 Update table with $T_{\text{middle}}^{\text{neutral}}[m^+] \stackrel{\pm}{=} \#K^3[0..11]$
 - 6 **for** 12 ■ blue cells $\#K^3[12..23] \in (\mathbb{F}_2^8)^{12}$ **do**
 - 7 Compute to 18 middle values denoted by m^- , which is part of Equation (16) that involves $\#K^3[0..11]$;
 - 8 Derive entries in $T_{\text{middle}}^{\text{neutral}}[m^-]$ to get $\#K^3[0..11]$;
 - 9 Derive 8 ■ blue bytes in $\#SB^2$ by solving 8 linear equations in Equation (17);
// Now we know all ■ blue cells in $\#SB^2$ and $\#K^3$.
 - 10 Compute 8 constants $(\#SB^5[0..3], \#SB^6[2, 7, 8, 13]) = (c_0, \dots, c_7)$;
 - 11 Update the table $T_{\text{blue}}^{\text{neutral}}[c_0, \dots, c_7] \stackrel{\pm}{=} (\text{■ in $\#K^3$ and $\#SB^2$ });
// $2^{8 \times 4}$ candidates expected$
 - 12 **return** $T_{\text{blue}}^{\text{neutral}}$;
-

Algorithm 6 generates the solution space of blue neutral words. The time complexity is upper bounded by $2^{8 \times 12} = 2^{96}$.

The MITM attack procedure for 8-round AES-192. We provide the main attack procedure derived from Figure 12 in Algorithm 7.

The attack complexity. The time complexity of the above MITM pseudo-preimage attack of 8-round AES-192 is about $2^{128 - 8 \times \min(3.5, 3.5, 4)} = 2^{100}$. The table T^+ dominates the memory complexity with approximately $2^{8 \times 4.5} = 2^{36}$. The pre-computation table $T_{\text{blue}}^{\text{neutral}}$ dominates the memory complexity with $2^{8 \times 12} = 2^{96}$.

Algorithm 7: MITM attack on 8 rounds of the AES-192 compression function.

```

// For constants
1 for  $(c_0, \dots, c_7) \in (\mathbb{F}_2^8)^8$  do
2   Initialize  $T^+$ .
   // For blue neutral words
3   for  $2^{8 \times 0.5}$  values  $c_8$  of the hint pool do
4     Lookup the table  $T_{\text{blue}}^{\text{neutral}}[c_0, \dots, c_7]$  to get candidates of ■ blue cells in
       #K3 and #SB2;
5     for the  $2^{8 \times 4}$  values in #K3 and #SB2 do
6       Compute  $m^+$  (the blue part of #AT7[2, 5, 8, 15]) and update the
         table  $T^+[m^+, c_8] \pm$  [blue neutral bytes];
         //  $2^{8 \times 0}$  entries for each index in  $T^+$ .

   // For red neutral words
7   for red bytes in #SB5  $\in (\mathbb{F}_2^8)^4$  and the  $2^{8 \times 0.5}$  values of  $c_8$  do
8     Compute  $m^-$  (the red part of #AT7[2, 5, 8, 15]  $\oplus$  #SB0[2, 5, 8, 15]), ,
       which equals  $m^+$ ;
9     Check for entries in  $T^+[m^-, c_8]$  to derive 1 blue neutral byte;
10    Compute the blue and red parts of #SB7[11];
11    Check if the red and blue parts of #SB7[11] really produce  $c_8$ ;
       //  $2^{8 \times (8 + 4.5 - 0.5)} = 2^{8 \times 12}$  candidates expected
12    Compute the full #AT7 and #SB0 in both colors to check the
       remaining 12 cells;
13    if The full match is found then
14      //  $2^{8 \times (12 - 12)} = 1$  candidate expected
       Output the preimage and stop;

```

Appendix C Improved Pseudo-preimage Attack on 9-round Rijndael-192/192

C.1 Improved Pseudo-preimage Attack on 9-round Rijndael-192/192

The attack configuration is as below:

- The initial DoF for forward neutral words \vec{v} (■): 2 bytes (2 in #AK³);
- Initial DoF for backward neutral words \overleftarrow{v} (in ■): 44 bytes (#K⁷ and #SR³);
- Consumed DoF for forward $\vec{\sigma}$: zero bytes;
- Consumed DoF for backward $\overleftarrow{\sigma}$: 42 bytes;
- Guessed bytes for blue, red, and both colors $g_{\mathcal{B}}, g_{\mathcal{R}}, g_{\mathcal{BR}}$: $g_{\mathcal{R}} = g_{\mathcal{BR}} = 0$ byte and $g_{\mathcal{B}} = g_{\mathcal{BR}} = 0$ bytes;
- Guessed byte equivalents for linearization: $d_{\mathcal{L}} = 0.5$ bytes.
- Matching DoF $d_{\mathcal{M}}$: 2 bytes between #AT⁸ and #SB⁰.

The remaining DoF are composed of

- $d_{\mathcal{B}} - g_{\mathcal{BR}} - d_{\mathcal{L}} = 2 - 0 - 0.5 = 1.5$,
- $d_{\mathcal{R}} - g_{\mathcal{BR}} - d_{\mathcal{L}} = 2 - 0 - 0.5 = 1.5$,
- $d_{\mathcal{M}} - g_{\mathcal{BR}} = 2 - 0 = 2$.

Compute initial values forward neutral bytes (Blue). We choose constants as follows:

- Fix 1 constant for the red parts of #AK³[19].
- Fix 1 constant for the red part of #SB⁴[19] that defines the red part of #MC⁴[7].
- Fix 6 constants for the red parts of #SB⁵[4, 5, 6, 7, 10, 11].
- Fix 12 constants for the red parts of #SB⁶[0, 3, 4, 5, 8, 9, 10, 17, 18, 19, 22, 23].
- Fix 4 constants for the red parts of #SB⁷[0, 5, 10, 15].
- Fix 1 constant for #K⁴[19].
- Fix 2 constants for #K⁸[0, 3].

Moreover, we will iterate over 20 values:

- Define 2 variables for the red parts of #MC⁰[0, 3] = (a_0, a_1) .
- Define 4 variables for the red parts of #MC¹[0, 15, 18, 21] = (a_2, a_3, a_4, a_5) .
- Define 2 variables for #AK³[16, 17] = (a_6, a_7) .
- Define 1 variable for the red part of #AK³[18] = a_8 .
- Define 9 variables for the red parts of #SB²[0, 1, 2, 14, 15, 16, 19, 20, 21] that define #MC²[0, 3, 6, 7, 16, 17, 18, 20, 21] = (a_9, \dots, a_{17}) .
- Define 2 variables for the red part of #MC⁵[5, 8] = (a_{18}, a_{19}) .

Algorithm 8 generates the solution space of red neutral words. The time complexity is upper bounded by $2^{8 \times 22.5} = 2^{180}$.

The MITM attack procedure for 9-round Rijndael-192/192. For forward neutral words (■ cells), we will iterate over #AK³[18, 19]. We provide the main attack procedure derived from Figure 13 in Algorithm 9.

The attack complexity. The time complexity of the above MITM pseudo-preimage attack of 9-round Rijndael-192/192 is about $2^{192 - 8 \times \min(1.5, 1.5, 2)} = 2^{180}$. The table $T_{\text{red}}^{\text{neutral}}$ needs memory for $2^{8 \times 22.5} = 2^{180}$ entries.

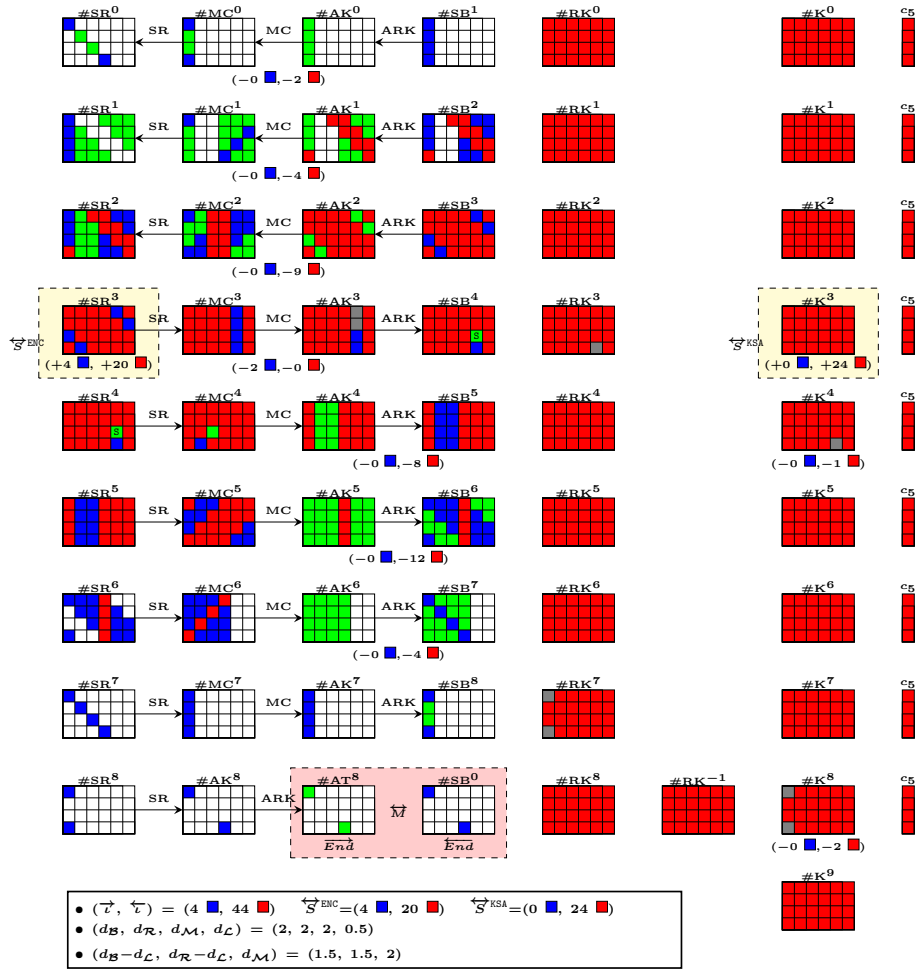


Fig. 13. An MITM pseudo-preimage attack of 9-round Rijndael-192/192.

Algorithm 8: Compute backward neutral words (red) for 9-round Rijndael-192/192.

```

1 Initiate a table  $T_{\text{red}}^{\text{neutral}}$ ;
2 for all values of  $\#K^7[0, 3]$  do
3   Derive  $\#K^7[20, 21]$  such that  $\#K^7[0, 3] \oplus S(\#K^7[21]), S(\#K^7[20])$  produce
   the expected constants of  $\#K^8[0, 3]$ .
   //  $2^{8 \times (2-0)}$  candidates expected
4 for all values of  $\#K^7[7, 11, 15]$  do
5   Derive  $\#K^7[19]$  such that  $\#K^7[7] \oplus \#K^7[11] \oplus \#K^7[15] \oplus \#K^7[19]$  yields the
   constant of  $\#K^4[19]$ .
   //  $2^{8 \times (3-0)}$  candidates expected
6 Construct  $\#K^7$  by combining the candidates with all possible values for the
   remaining 16 bytes. At this stage, we know the full blue part of  $\#K^4$  and can
   derive the red parts of all key words, including  $\#RK^{-1}[0, 15]$  and  $\#RK^8[0, 15]$ .
   //  $2^{8 \times (3+2+16)} = 2^{8 \times 21}$  candidates expected
7 Compute  $\#AK^4[4..7]$  from  $\#RK^4[4..7]$  and the constants of the red parts of
    $\#SB^5[4..7]$ . Derive  $\#MC^4[4..7]$  and keep only those values that yield the
   expected constant of the red part of  $\#MC^4[7]$ .
   //  $2^{8 \times (21-1)} = 2^{8 \times 20}$  candidates expected
8 Compute  $\#AK^6[0, 5, 10, 15]$  from  $\#RK^6[0, 5, 10, 15]$  and the constants of the red
   parts of  $\#SB^7[0, 5, 10, 15]$ . With the 12 fixed constants of the red parts of
    $\#MC^6[0, 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 15]$  (defined by the constants in  $\#SB^6$ ),
   derive the red parts of  $\#MC^6[0..15]$ ,  $\#AK^6[0..15]$ , and  $\#SB^7[0..15]$  column by
   column.
   //  $2^{8 \times 20}$  candidates expected
9 for those values  $2^{8 \times 20}$  candidates do
10   Compute backwards to  $\#SB^6[12..15]$ . Combine them with
    $\#RK^5[0, 3, 4, 5, 8, 9, 10, 12..15, 17, 18, 19, 22, 23]$  and the fixed red parts of
    $\#SB^6$ , derive  $\#AK^5[0, 3, 4, 5, 8, 9, 10, 12..15, 17, 18, 19, 22, 23]$ .
11   for all possible values of  $\#MC^5[5, 8] = (a_{18}, a_{19})$  do
12     Combine the knowledge of partial  $\#AK^5$  with the constants of
      $\#MC^5[1, 2, 4, 19, 22, 23]$  defined from  $\#SB^5[4, 5, 6, 7, 10, 11]$  to derive
     the full red part of  $\#MC^5$  column by column.
     //  $2^{8 \times (20+2)} = 2^{8 \times 22}$  candidates expected
13     Compute backwards to  $\#SR^4$ . Guess the 16 possible values of  $a_{20}$  of
      $\#SR^4[18]$  to compute to  $\#AK^3$ .
     //  $2^{8 \times (22+0.5)} = 2^{8 \times 22.5}$  candidates expected
14     Define  $\#AK^3[16, 17] = (a_6, a_7)$  and the red part of  $\#AK^3[18] = a_8$ .
15     Compute backwards to  $\#MC^2$ . Define the red parts of
      $\#MC^2[0, 3, 6, 7, 16, 17, 18, 20, 21] = (a_9, \dots, a_{17})$ .
16     Compute backwards to  $\#MC^1[0..3, 12..23]$ . Define the red parts of
      $\#MC^1[0, 15, 18, 21] = (a_2, a_3, a_4, a_5)$ .
17     Compute backwards to  $\#MC^0[0..3]$ . Define the red parts of
      $\#MC^0[0, 3] = (a_0, a_1)$ .
18     Update the table  $T_{\text{red}}^{\text{neutral}}[a_0, \dots, a_{19}, a_{20}] \stackrel{\pm}{=} (K^7, \#RK^8[0, 15],$ 
      $\#RK^{-1}[0, 15])$ ;
     // For each of the  $2^{8 \times (20+0.5)}$  buckets for each value of
      $(a_0, \dots, a_{19}, a_{20})$ ,  $2^{8 \times (22.5-20.5)} = 2^{8 \times 2}$  candidates expected
19 return  $T_{\text{red}}^{\text{neutral}}$ ;

```

Algorithm 9: MITM attack on 9 rounds of the Rijndael-192/192 compression function.

```

// For constants
1 for  $(a_0, \dots, a_{19}) \in (\mathbb{F}_2^8)^{20}$  combined with the 16 values  $a_{20}$  of the hint pool do
    // For red neutral words
2   for all  $2^{8 \times 2}$  red candidates in  $T_{red}^{neutral}[a_0, \dots, a_{19}, a_{20}]$  do
3     Retrieve the red parts of  $\#AT^8[0, 15] = \#RK^8[0, 15] \oplus \#RK^{-1}[0, 15]$ ;
        //  $2^{8 \times (20.5+2)} = 2^{8 \times 22.5}$  candidates expected
    // For blue neutral words
4   for  $\#AK^3[18, 19] \in (\mathbb{F}_2^8)^2$  do
        //  $2^{8 \times (20.5+2)} = 2^{8 \times 22.5}$  candidates expected
5     Compute forwards to  $\#AK^8[0, 15]$ ;
6     Compute backwards to  $\#SB^0[0, 15]$ ;
7     Check for the candidates whose red part of  $\#RK^8[0, 15] \oplus \#RK^{-1}[0, 15]$ 
        equals the red part of
         $\#RK^8[0, 15] \oplus \#RK^{-1}[0, 15] \oplus \#AT^8[0, 15] \oplus \#SB^0[0, 15]$ ;
8     if the two-byte partial match is passed then
9       Check if the red and red parts of  $\#SR^4[18]$  really produce  $a_{20}$ ;
        //  $2^{8 \times (22.5-0.5)} = 2^{8 \times 22}$  candidates expected
10      Compute the full  $\#AT^9$  and  $\#SB^0$  in both colors to check the
        remaining 22 cells;
11      if The full match is found then
12        //  $2^{8 \times (22-22)} = 1$  candidate expected
        Output the preimage and stop;

```

C.2 Improved Pseudo-preimage Attack on 10-round Rijndael-192/256

The attack configuration is as below:

- The initial DoF for forward neutral words \vec{v} (■): 53 bytes (29 in $\#K^4$ and 24 in $\#SR^4$);
- Initial DoF for backward neutral words \overleftarrow{v} (in ■): 3 bytes (3 in $\#K^4$);
- Consumed DoF for forward $\vec{\sigma}$: 50 bytes;
- Consumed DoF for backward $\overleftarrow{\sigma}$: zero bytes;
- Guessed bytes for blue, red, and both colors g_B, g_R, g_{BR} : $g_R = g_{BR} = 0$ bytes and $g_B = g_{BR} = 0$ bytes;
- Guessed byte equivalents for linearization: $d_{\mathcal{L}} = 1.5$ bytes;
- Matching DoF $d_{\mathcal{M}}$: 2 bytes between $\#AT^9$ and $\#SB^0$.

The remaining DoF are composed of

- $d_B - g_{BR} - d_{\mathcal{L}} = 3 - 0 - 3 \times 0.5 = 1.5$,
- $d_R - g_{BR} - d_{\mathcal{L}} = 3 - 0 - 3 \times 0.5 = 1.5$,
- $d_{\mathcal{M}} - g_{BR} = 2 - 0 = 2$.

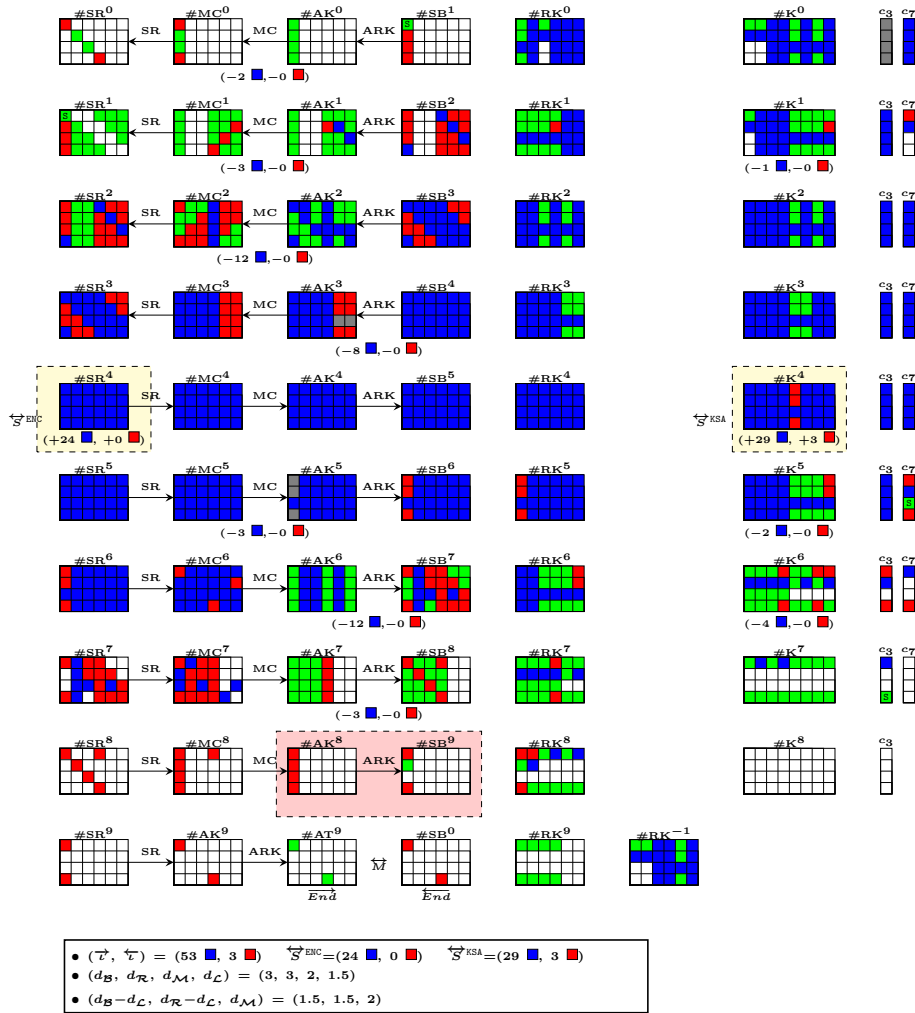


Fig. 14. An MITM pseudo-preimage attack of 10-round Rijndael-192/256.

Algorithm 10: Compute forward neutral words (blue) for 10-round Rijndael-192/256.

```

1 Initiate table  $T'$  and  $T_{\text{blue}}^{\text{neutral}}$ ;
2 for values of
   $\#K^4[0, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 18, 20, 21, 22, 23, 26, 29, 30, 31]$  do
3   Derive all round keys with the 7 fixed zeros in key schedule;
4   Compute  $\alpha_0, \dots, \alpha_4$  as:
5    $MC^{-1} \cdot \#RK^0[0, 1, 2, 3]^T = [\alpha_0, *, *, \alpha_1]^T$ ,
6    $MC^{-1} \cdot \#RK^1[12, 13, 14, 15]^T = [*, *, *, \alpha_2]^T$ ,
7    $MC^{-1} \cdot \#RK^1[16, 17, 18, 19]^T = [*, *, \alpha_3, *]^T$ ,
8    $MC^{-1} \cdot \#RK^1[20, 21, 22, 23]^T = [*, \alpha_4, *, *]^T$ ;
9    $T'[\alpha_0, \dots, \alpha_4, \#RK^2[1, 2, 6, 7, 11, 16, 20, 21], \#RK^1[0, 1, 2, 3]]^{\pm}$ 
      (blue parts of  $\#K^4$ );
      // At each of the  $2^{8 \times 17}$  indices of  $T'$ ,  $2^{8 \times 5}$  entries expected
10 for all blue parts in  $\#MC^2$ , all values of  $\#RK^1[0, 1, 2, 3]$  and all values of  $h_0$  do
11   Compute  $\beta_0, \dots, \beta_4$  as:
12    $MC^{-1} \cdot [\#SB^1[0], 0, 0, 0]^T = [\beta_0, *, *, \beta_1]^T$ ,
13    $MC^{-1} \cdot [\#SB^2[12], 0, 0, 0]^T = [*, *, *, \beta_2]^T$ ,
14    $MC^{-1} \cdot [0, \#SB^2[17], 0, 0]^T = [*, *, \beta_3, *]^T$ ,
15    $MC^{-1} \cdot [0, 0, \#SB^2[22], 0]^T = [*, \beta_4, *, *]^T$ ;
16   for all  $2^{8 \times 5}$  entries at
       $T'[\beta_0, \dots, \beta_4, \#AK^2[1, 2, 6, 7, 11, 16, 20, 21], \#RK^1[0, 1, 2, 3]]$  do
17     for all combinations of  $h_1$  and  $h_2$  do
18       Forward compute and record  $\#AK^5[0, 1, 3]$ ,
           $\#SB^7[0, 3, 8, 9, 12, \dots, 17, 19, 22, 23]$ , and  $\#SB^8[0, 5, 10]$  as
           $a_0, \dots, a_{17}$ ;
19        $T_{\text{blue}}^{\text{neutral}}[a_0, \dots, a_{17}, h_0, h_1, h_2]^{\pm}$  (blue parts of  $\#K^4, \#MC^2$ );
          // Under each of the  $2^{8 \times (18+1.5=19.5)}$  indices,  $2^{8 \times 3}$  entries
          expected
20 return  $T_{\text{blue}}^{\text{neutral}}$ ;

```

Compute initial values forward neutral bytes (Blue). We first fix the following 13 constants (for the blue parts) as all zeros:

- 13 in encryption states: $\#MC^0[0, 3]$, $\#MC^1[15, 18, 21]$, and $\#AK^3[16, \dots, 23]$,
- 7 in the key schedule: $\#K^1[29]$, $\#K^5[28, 29]$, and $\#K^6[12, 15, 24, 27]$.

Alternatively, we choose the 12 blue parts in $\#MC^2$ as the starting point so that the 12 costs from $\#AK^2$ to $\#MC^2$ can be bypassed and the XOR cost from $\#AK^3$ to $\#SB^4$ can be equivalently considered between $\#AK^2$ and $\#SB^3$. We remove $\#K^4[7, 12, 13, 24, 25, 27, 28]$ from S^{KSA} as they can be computed given the previous 7 bytes fixed as 0 in key schedule, which leaves 22 bytes to distribute initial DoF in $\#K^4$. Then we will iterate over 18 values:

- Denote 3 variables of $\#AK^5[0, 1, 3]$ by (a_0, a_1, a_2) .

- Denote 12 variables for the blue parts of $\#SB^7[0, 3, 8, 9, 12, \dots, 17, 19, 22, 23]$ by (a_3, \dots, a_{14}) .
- Denote 3 variables for the blue parts of $\#SB^8[0, 5, 10]$ by (a_{15}, a_{16}, a_{17}) .

Finally, 3 nibbles for the linearization hints are used in $\#SB^1[0]$, $c_7^5[2]$, and $c_3^7[3]$ (see in the key schedule in Figure 14), denote them by h_0, h_1, h_2 . Algorithm 10 generates the solution space of blue neutral words. The time complexity is upper bounded by $2^{8 \times 22.5} = 2^{180}$.

The MITM attack procedure for 10-round *Rijndael-192/256*. For backward neutral words (■ cells), we will iterate over $\#K^4[16, 17, 19]$. We provide the main attack procedure derived from Figure 14 in Algorithm 11.

Algorithm 11: MITM attack on 10 rounds of the Rijndael-192/256 compression function.

```

// For constants
1 for  $(a_0, \dots, a_{17}) \in (\mathbb{F}_2^8)^{18}$  combined with the  $16^3$  guesses of  $h_0, h_1, h_2$  do
    // For blue neutral words
    2 for all  $2^{8 \times 3}$  blue candidates do
    3     Retrieve the blue part of  $\#AT^9[0, 15] = \#RK^9[0, 15] \oplus \#RK^{-1}[0, 15]$ ;
        //  $2^{8 \times (19.5+3)} = 2^{8 \times 22.5}$  candidates expected
        // For red neutral words
    4 for  $\#K^4[16, 17, 19] \in (\mathbb{F}_2^8)^3$  do
        //  $2^{8 \times (19.5+3)} = 2^{8 \times 22.5}$  candidates expected
    5     Derive the red parts in the key schedule including  $\#RK^9[0, 15]$  and
         $\#RK^{-1}[0, 15]$ ;
    6     Derive  $\#SB^6[0, 1, 3]$  and compute forwards to  $\#AK^9[0, 15]$ ;
    7     Derive  $\#AK^3[16..23]$  and compute backwards to  $\#SB^0[0, 15]$ ;
    8     Check for the blue candidates in  $T_{\text{blue}}^{\text{neutral}}$  whose blue part of
         $\#RK^9[0, 15] \oplus \#RK^{-1}[0, 15]$  equals the red part of
         $\#RK^9[0, 15] \oplus \#RK^{-1}[0, 15] \oplus \#AT^9[0, 15] \oplus \#SB^0[0, 15]$ ;
    9     if the two-byte partial match is passed and the 3 hints are compatible
        then
        //  $2^{8 \times (23.5-1.5)} = 2^{8 \times 22}$  candidates expected
    10         Compute and check for a full match;
    11         if The full match is found then
        //  $2^{8 \times (22-22)} = 1$  candidate expected
    12             Output the preimage and stop;

```

The attack complexity. The time complexity of the above MITM pseudo-preimage attack of 10-round *Rijndael-192/256* is about $2^{192-8 \times \min(1.5, 1.5, 2)} = 2^{180}$.

Appendix D Improved Preimage Attacks of Whirlpool

D.1 Improved Preimage Attack of 6-round Whirlpool

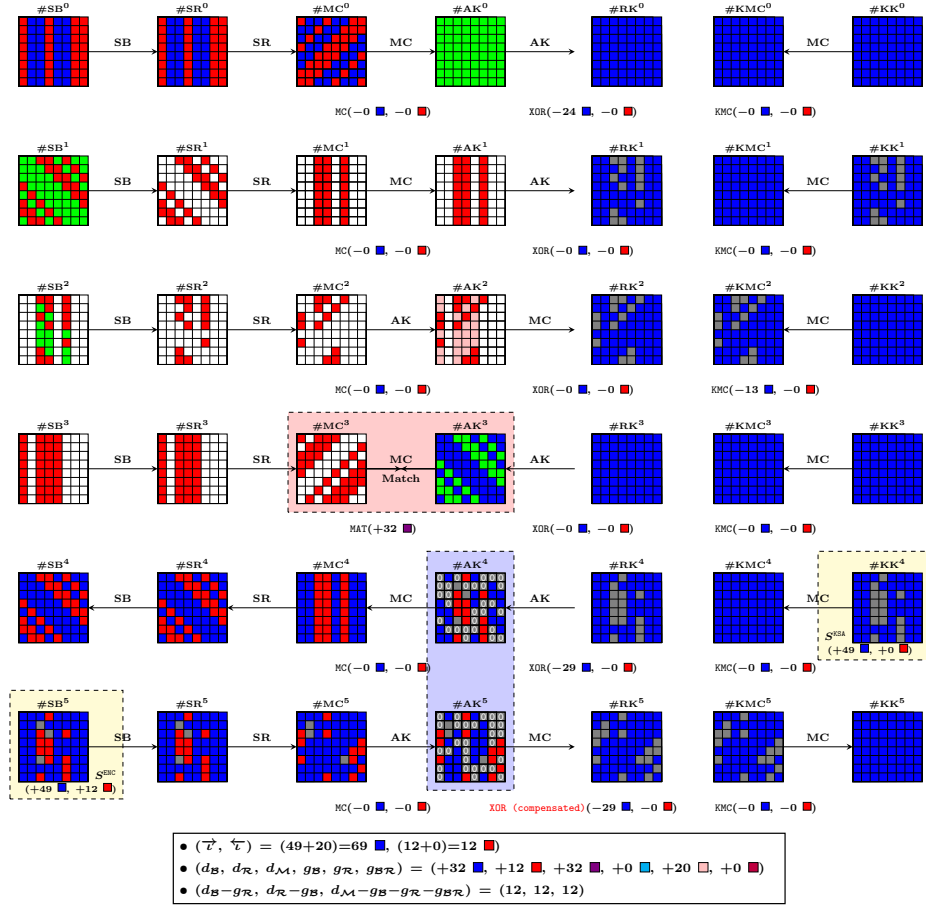


Fig. 15. An MITM pseudo-preimage attack of 6-round Whirlpool.

The improved result on 6-round Whirlpool is presented in Figure 15. By using the **SIM** technique (at Round 4 and 5), the XOR DoF costs can be compensated at Round 5 with 29 \blacksquare blue bytes DoF cost, which reduces the time complexity by a factor of 2^{24} compared to previous best MITM pseudo-preimage attack. It leads to the attack configuration as below:

- Initial DoF for forward neutral words \vec{v} (\blacksquare): 69 bytes (among 49 blue cells in $\#SB^5$, 29 out of 49 cells are *set to be equal* to the corresponding cells in $\#KK^4$, and thus can be canceled to zero constant \blacksquare in $\#AK^4$ and $\#AK^5$);

- Initial DoF for backward neutral words \overleftarrow{v} (in \blacksquare): 12 bytes (in $\#SB^5$);
- Consumed DoF for forward $\overrightarrow{\sigma}$: 37 bytes (13 for $\#KK^2 \xrightarrow{MC^{-1}} \#KMC^2$ and 24 for $\#AK^0 \oplus \#RK^0 = \#SB^1$);
- Consumed DoF for backward $\overleftarrow{\sigma}$: zero bytes;
- Guessed bytes for blue, red and both colors $g_B, g_R, g_{BR} : g_B = g_{BR} = 0$ byte and $g_R = 20$ bytes;
- Matching DoF d_M : 32 bytes between $\#MC^3$ and $\#AK^3$.

Then, the remaining DoF is $d_B - g_R = 12, d_R = 12, d_M - d_R = 12$.

Compute initial values for forward neutral words (Blue). To get the initial values of forward neutral words, one should focus on the constraints among states $\#KK^2$ and $\#KMC^2$, $\#RK^0$, $\#AK^0$ and $\#SB^1$ as below

- For $\#KK^2$ and $\#KMC^2$, there are 13 bytes MC DoF cost for forward neutral words, which applies the constraints in Equation (18) as below

$$\begin{pmatrix} - & - & b_3 & b_6 & - & b_{12} & - & - \\ - & - & b_4 & - & b_9 & - & - & - \\ b_0 & - & b_7 & - & - & - & - & - \\ b_1 & - & b_5 & - & - & - & - & - \\ b_2 & - & - & - & - & b_{10} & - & - \\ - & - & - & b_8 & b_{11} & - & - & - \end{pmatrix} = MC^{-1} \cdot \begin{pmatrix} \#KK_0^2 & \#KK_8^2 & \cdots & \#KK_{36}^2 \\ \#KK_1^2 & \#KK_9^2 & \cdots & \#KK_{37}^2 \\ \#KK_2^2 & \#KK_{10}^2 & \cdots & \#KK_{38}^2 \\ \#KK_3^2 & \#KK_{11}^2 & \cdots & \#KK_{39}^2 \\ \#KK_4^2 & \#KK_{12}^2 & \cdots & \#KK_{40}^2 \\ \#KK_5^2 & \#KK_{13}^2 & \cdots & \#KK_{41}^2 \\ \#KK_6^2 & \#KK_{14}^2 & \cdots & \#KK_{42}^2 \\ \#KK_7^2 & \#KK_{15}^2 & \cdots & \#KK_{43}^2 \end{pmatrix}, \quad (18)$$

where b_i ($0 \leq i \leq 12$) are the chosen constants for $\#KMC^2$.

- For $\#AK^0$, $\#RK^0$ and $\#SB^1$, there are 24 bytes XOR DoF cost for forward neutral words, which applies the constraints in Equation (19) as below

$$\begin{pmatrix} \#AK_0^0 & \#AK_8^0 & \cdots & \#AK_{56}^0 \\ \#AK_1^0 & \#AK_9^0 & \cdots & \#AK_{57}^0 \\ \#AK_2^0 & \#AK_{10}^0 & \cdots & \#AK_{58}^0 \\ \#AK_3^0 & \#AK_{11}^0 & \cdots & \#AK_{59}^0 \\ \#AK_4^0 & \#AK_{12}^0 & \cdots & \#AK_{60}^0 \\ \#AK_5^0 & \#AK_{13}^0 & \cdots & \#AK_{61}^0 \\ \#AK_6^0 & \#AK_{14}^0 & \cdots & \#AK_{62}^0 \\ \#AK_7^0 & \#AK_{15}^0 & \cdots & \#AK_{63}^0 \end{pmatrix} \oplus \begin{pmatrix} \#RK_0^0 & \#RK_8^0 & \cdots & \#RK_{56}^0 \\ \#RK_1^0 & \#RK_9^0 & \cdots & \#RK_{57}^0 \\ \#RK_2^0 & \#RK_{10}^0 & \cdots & \#RK_{58}^0 \\ \#RK_3^0 & \#RK_{11}^0 & \cdots & \#RK_{59}^0 \\ \#RK_4^0 & \#RK_{12}^0 & \cdots & \#RK_{60}^0 \\ \#RK_5^0 & \#RK_{13}^0 & \cdots & \#RK_{61}^0 \\ \#RK_6^0 & \#RK_{14}^0 & \cdots & \#RK_{62}^0 \\ \#RK_7^0 & \#RK_{15}^0 & \cdots & \#RK_{63}^0 \end{pmatrix} = \begin{pmatrix} - & - & c_6 & c_9 & - & c_{15} & - & - \\ - & - & - & c_{10} & c_{12} & - & c_{18} & - \\ - & - & - & - & c_{13} & c_{16} & - & c_{21} \\ c_0 & - & - & - & - & c_{17} & c_{19} & - \\ - & c_3 & - & - & - & - & c_{20} & c_{22} \\ c_1 & c_4 & - & c_{11} & - & - & - & c_{23} \\ - & c_5 & c_8 & - & c_{14} & - & - & - \end{pmatrix}, \quad (19)$$

where c_i ($0 \leq i \leq 23$) are the chosen constants for $\#SB^1$.

However, one still needs an efficient method to obtain blue neutral words satisfying the constraints (in Equation (18) and (19)) in a non-linear system. We provide Algorithm 12 to efficiently obtain the solution space of blue neutral words with the time complexity $2^{8 \times 49} = 2^{392}$ and memory complexity $2^{8 \times 36} = 2^{288}$.

Compute initial values backward neutral bytes (Red). As there is no DoF cost for backward neutral words, to obtain the corresponding initial values, one only needs to enumerate all possible values of 12 red cells in $\#SB^5$.

The MITM attack procedure for 6-round Whirlpool. The main attack procedure derived from Figure 15 is provided in Algorithm 13 as below.

Algorithm 12: Compute forward neutral words (blue) for 6-round Whirlpool

```
1 Initiate a table  $T_{\text{blue}}^{\text{neutral}}$ ;  
  // The index list  $I$  corresponds with constant cells in Equation (19)  
2 Set list  $I =$   
  [3, 5, 6, 12, 14, 15, 16, 21, 23, 24, 25, 30, 33, 34, 39, 40, 42, 43, 49, 51, 52, 58, 60, 61];  
3 Fix 15  $\blacksquare$  constant cells in  $\#\text{KK}^4$  to zero;  
4 for 49  $\blacksquare$  blue cells in  $\#\text{KK}^4 \in (\mathbb{F}_2^8)^{49}$  do  
5   Compute backwards to  $\#\text{KMC}^2$ ;  
6   if 13  $\blacksquare$  constants cells in  $\#\text{KMC}^2$  are all zero then  
7     //  $2^{8 \times (49-13)} = 2^{8 \times 36}$  candidates expected  
7     Compute backwards to  $\#\text{KK}^0$ ;  
7     //  $2^{8 \times (36-24)} = 2^{8 \times 12}$  candidates expected for each row  
8     Update to table with  $T_{\text{blue}}^{\text{neutral}}[\#\text{KK}^0[I]] += \#\text{KK}^4$ ;  
9 return  $T_{\text{blue}}^{\text{neutral}}$ ;
```

Algorithm 13: MITM attack on 6-round Whirlpool compression function

```
1 Fix 3  $\blacksquare$  constant cells in  $\#\text{AK}^5$  and 15  $\blacksquare$  constant cells in  $\#\text{KK}^4$  to all zero;  
2 Fix 13 constants  $(b_0, \dots, b_{12})$  in Equation (18) to all zero;  
3 Fix 4 constants  $(c_{20}, c_{21}, c_{22}, c_{23})$  in Equation (19) to all zero;  
4 Set list  $I =$   
  [3, 5, 6, 12, 14, 15, 16, 21, 23, 24, 25, 30, 33, 34, 39, 40, 42, 43, 49, 51, 52, 58, 60, 61];  
5 Call Algorithm 12 to build  $T_{\text{blue}}^{\text{neutral}}$ ;  
6 for  $C = (c_0, \dots, c_{19}, 0, 0, 0, 0) \in (\mathbb{F}_2^8)^{20}$  do  
7   // For red neutral words  
7   for 20  $\square$  guessed red cells in  $\#\text{AK}^2 \in (\mathbb{F}_2^8)^{20}$  do  
8     for 12  $\blacksquare$  red cells in  $\#\text{SB}^5 \in (\mathbb{F}_2^8)^{12}$  do  
9       Compute forwards and backwards to cells with red information in  
9        $\#\text{MC}^3, \#\text{AK}^3$  and store in a table  $T^-$  (with size  $2^{8 \times 32}$ );  
10      // For blue neutral words  
10      for 20  $\blacksquare$  blue cells in  $\#\text{AK}^5 \in (\mathbb{F}_2^8)^{20}$  do  
11        Compute forward to  $\#\text{AK}^0$ ;  
12        Search table with  $T_{\text{blue}}^{\text{neutral}}[C \oplus \#\text{AK}^0[I]]$  and obtain  $\#\text{KK}^4$ ;  
13        Compute backward to  $\#\text{AK}^3$  according to  $2^{8 \times 12}$   $\#\text{KK}^4$  on average;  
14        Check in the table  $T^-$ ;  
15        if  $\#\text{MC}^3$  and  $\#\text{AK}^3$  pass the partial match then  
16          //  $2^{8 \times (20+32+32-32)} = 2^{8 \times 52}$  candidates expected  
16          Compute to check 20  $\square$  guessed red cells in  $\#\text{AK}^2$ ;  
17          if Guess values are correct then  
18            //  $2^{8 \times (52-20)} = 2^{8 \times 32}$  candidates expected  
18            Compute to match the rest 32 cells;  
19            if The full match is found then  
20              //  $2^{8 \times (32-32)} = 1$  candidate expected  
20              Output the preimage and stop;
```

The attack complexity. The time complexity of the above MITM pseudo-preimage attack of 6-round Whirlpool is about $2^{512-8 \times \min(12,12,12)} = 2^{416}$. The table $T_{\text{blue}}^{\text{neutral}}$ dominates the memory complexity with $2^{36 \times 8} = 2^{288}$.

D.2 Preimage Attack of 7.75-round Whirlpool

The previous best preimage attacks on 7-round Whirlpool in [8], could not be extended by another partial round due to the AK operation in the final round. We can use the **SIM** technique at the penultimate round #AK⁶, so that more constant cells can be guaranteed to pass the final MC operation. More precisely, we want #AK⁶ to have *equally colored cells*¹⁰. Finally, for the first time, we obtain an MITM attack of 7.75-round Whirlpool, which is presented in Figure 16 and has the attack configuration as below

- Initial DoF for forward neutral words \vec{t} (■): 16 bytes (16 blue cells in #SB⁶ are set to be equal to the corresponding cells in #KK⁵, and thus can be canceled to zero constant ■ in #AK⁵ and #AK⁶);
- Initial DoF for backward neutral words \overleftarrow{t} (in ■): 48 bytes (in #SB⁶);
- Consumed DoF for forward $\vec{\sigma}$: 12 bytes (12 for #KK⁵ $\xrightarrow{\text{MC}^{-1}}$ #KMC⁵);
- Consumed DoF for backward $\overleftarrow{\sigma}$: 32 bytes (16 for #AK⁵ $\xrightarrow{\text{MC}^{-1}}$ #MC⁵ and 16 for #MC⁰ $\xrightarrow{\text{MC}}$ #AK⁰);
- Guessed bytes for blue, red and both colors $g_{\mathcal{B}}, g_{\mathcal{R}}, g_{\mathcal{BR}}$: $g_{\mathcal{R}} = g_{\mathcal{BR}} = 0$ byte and $g_{\mathcal{B}} = 12$ bytes;
- Matching DoF $d_{\mathcal{M}}$: 16 bytes between #MC³ and #AK³.

Then, the remaining DoF is $d_{\mathcal{B}} = 4, d_{\mathcal{R}} - g_{\mathcal{B}} = 4, d_{\mathcal{M}} - g_{\mathcal{B}} = 4$.

Compute initial values forward neutral bytes (Blue). Considering that the constraints on forward neutral words are linear, to obtain the corresponding initial values, one can choose and enumerate all possible values of 4 ■ blue cells in #KMC⁵ for forward neutral words.

Compute initial values for backward neutral words (Red). To get the initial values of backward neutral words, one should focus on the constraints among states #AK⁵ and #MC⁵, #MC⁰ and #AK⁰.

- For #MC⁰ and #AK⁰, there are also 16 bytes MC DoF cost for backward neutral words, which applies the constraints in Equation (20) as below

$$\begin{pmatrix} g_0 & - & - & - & - & - & - & g_{14} \\ g_1 & g_2 & - & - & - & - & - & - \\ - & g_3 & g_4 & - & - & - & - & - \\ - & - & g_5 & g_6 & - & - & - & - \\ - & - & - & g_7 & g_8 & - & - & - \\ - & - & - & - & g_9 & g_{10} & - & - \\ - & - & - & - & - & g_{11} & g_{12} & - \\ - & - & - & - & - & - & g_{13} & g_{15} \end{pmatrix} = \text{MC} \cdot \begin{pmatrix} \#MC_0^0 & \#MC_8^0 & \dots & \#MC_{16}^0 \\ \#MC_1^0 & \#MC_9^0 & \dots & \#MC_{17}^0 \\ \#MC_2^0 & \#MC_{10}^0 & \dots & \#MC_{18}^0 \\ \#MC_3^0 & \#MC_{11}^0 & \dots & \#MC_{19}^0 \\ \#MC_4^0 & \#MC_{12}^0 & \dots & \#MC_{20}^0 \\ \#MC_5^0 & \#MC_{13}^0 & \dots & \#MC_{21}^0 \\ \#MC_6^0 & \#MC_{14}^0 & \dots & \#MC_{22}^0 \\ \#MC_7^0 & \#MC_{15}^0 & \dots & \#MC_{23}^0 \end{pmatrix}, \quad (20)$$

where g_i ($0 \leq i \leq 15$) are the chosen constants for #MC⁵.

¹⁰ That is with only blue and gray cells or only red and gray cells.

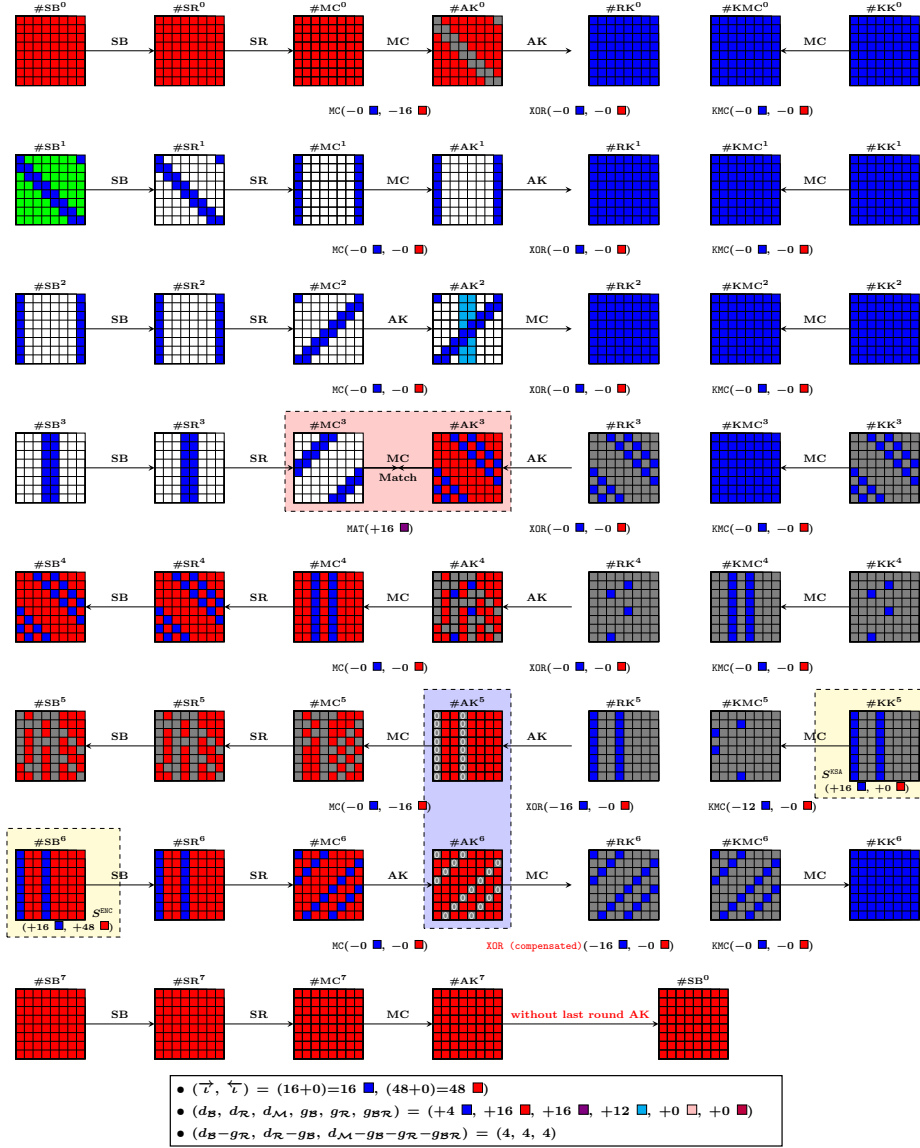


Fig. 16. An MITM pseudo-preimage attack of 7.75-round Whirlpool.

- For $\#AK^5$ and $\#MC^5$, there are 16 bytes MC DoF cost for backward neutral words, which applies the constraints in Equation (21) as below

$$\begin{pmatrix} - & - & e_2 & - & e_4 & - & - & - \\ - & e_0 & - & - & - & - & - & e_{13} \\ - & - & e_3 & - & - & - & e_{10} & - \\ - & e_1 & - & - & - & e_7 & - & e_{14} \\ - & - & - & - & e_5 & - & e_{11} & - \\ - & - & - & - & - & e_8 & - & e_{15} \\ - & - & - & - & e_6 & - & e_{12} & - \\ - & - & - & - & - & e_9 & - & - \end{pmatrix} = MC^{-1} \cdot \begin{pmatrix} \#AK_0^5 & \#AK_8^5 & \dots & \#AK_{56}^5 \\ \#AK_1^5 & \#AK_9^5 & \dots & \#AK_{57}^5 \\ \#AK_2^5 & \#AK_{10}^5 & \dots & \#AK_{58}^5 \\ \#AK_3^5 & \#AK_{11}^5 & \dots & \#AK_{59}^5 \\ \#AK_4^5 & \#AK_{12}^5 & \dots & \#AK_{60}^5 \\ \#AK_5^5 & \#AK_{13}^5 & \dots & \#AK_{61}^5 \\ \#AK_6^5 & \#AK_{14}^5 & \dots & \#AK_{62}^5 \\ \#AK_7^5 & \#AK_{15}^5 & \dots & \#AK_{63}^5 \end{pmatrix}, \quad (21)$$

where e_i ($0 \leq i \leq 15$) are the chosen constants for $\#MC^5$.

Algorithm 14: MITM attack on 7.75-round Whirlpool compression function

```

1 Fix 48 ■ constant cells in  $\#KMC^5$  (columns 1, 2, 4, 5, 6, 7) to all zero;
2 Let constant  $C_g = (g_0, \dots, g_{15})$  in Equation (20);
3 Let constant  $C_e = (e_0, \dots, e_{15})$  in Equation (21);
4 Let the remaining 12 ■ constant cells in  $\#KMC^5$  (columns 0, 3) be  $C_G$ ;
  // Enumerate  $C_e$  because starting from  $\#MC^5$  for red neutral words
5 for  $C_e \in (\mathbb{F}_2^8)^{16}$  do
  // For red neutral words
6 for 32 ■ red cells in  $\#MC^5 \in (\mathbb{F}_2^8)^{32}$  do
7   Initiate table  $T_{red}^{neutral}$ ;
8   Compute forwards to 16 constant cells in  $\#AK^0$ , i.e.,  $C_g$ ;
  //  $2^{8 \times (32-16)} = 2^{8 \times 16}$  candidates expected for each row
9   Update to table  $T_{red}^{neutral}[C_g] += \#MC^5$ ;
  // For blue neutral words
10 for  $C_G \in (\mathbb{F}_2^8)^{12}$  do
11   for  $C_g \in (\mathbb{F}_2^8)^{16}$  do
12     Search table with  $T_{red}^{neutral}[C_g]$  and obtain  $\#MC^5$  ( $2^{8 \times 16}$  on average);
13     Compute backwards to red cells in  $\#AK^3$  and store in a table  $T^-$ ;
14     for 4 ■ blue cells in  $\#KMC^5 \in (\mathbb{F}_2^8)^4$  do
15       for 12 ■ guessed blue cells in  $\#AK^2 \in (\mathbb{F}_2^8)^{12}$  do
16         Compute to the blue cells in  $\#MC^3$  and  $\#AK^3$  and check in
17          $T^-$ ;
18         if  $\#MC^3$  and  $\#AK^3$  pass the partial match then
19           //  $2^{8 \times (16+12+16+16+16-16)} = 2^{480}$  candidates expected
20           Compute to check 12 ■ guessed cells in  $\#AK^2$ ;
21           if Guess values are correct then
22             //  $2^{8 \times (60-12)} = 2^{8 \times 48}$  candidates expected
23             Compute to match the rest 48 cells;
24             if The full match is found then
25               //  $2^{8 \times (48-48)} = 1$  candidate expected
26               Output the preimage and stop;

```

The MITM attack procedure for 7.75-round *Whirlpool*. We provide the main attack procedure derived from Figure 16 in Algorithm 14.

The attack complexity. The time complexity of the above MITM pseudo-preimage attack of 7.75-round *Whirlpool* is about $2^{512-8 \times \min(4,4,4)} = 2^{480}$. The table $T_{\text{red}}^{\text{neutral}}$ dominates the memory complexity with $2^{32 \times 8} = 2^{256}$.

Appendix E Improved Preimage Attacks of Streebog

Streebog is a block-cipher based hash function in the Russian national standard GOST R 34.11-2012 [1] and has been an ISO/IEC standard [25] since 2018. It has two versions *Streebog*-512 and *Streebog*-256, which produce 512-bit and 256-bit hash values respectively. Similar to *Whirlpool*, *Streebog* adopts MP mode compression function g , defined as $g(N_i, H_i, M_i) = E_{\text{MR} \circ \text{TP} \circ \text{SB}(N_i \oplus H_i)}(M_i) \oplus M_i \oplus H_i$, where E is a 12-round AES-like block cipher with 8×8 -byte internal states, N_i is the counter and MR, TP, SB are operations in one round of E . Note that the compression function g of *Streebog* is adopted in the HAIFA framework [10], as shown in Figure 17. The initial state of E is $S_0 = M_i$, for each round, the state is updated by AddRoundKey (AK), SubBytes(SB), Transposition (TP) and MixRows (MR), i.e., $S_{j+1} = \text{MR} \circ \text{TP} \circ \text{SB}(S_j \oplus K_j)$, $j = 0, \dots, 11$. The output of the compression function g is finally computed by $S_{12} \oplus K_{12}$. Key state is initialized by $K_0 = \text{MR} \circ \text{TP} \circ \text{SB}(H_i \oplus N_i)$, then the round key is updated by $K_{j+1} = \text{MR} \circ \text{TP} \circ \text{SB}(K_j \oplus C_j)$, where $j = 0, \dots, 11$ and C_j is the round constant. Still for convenience, we transpose the row and column for the representation of *Streebog*. For more details, please refer to the original documents [1,15].

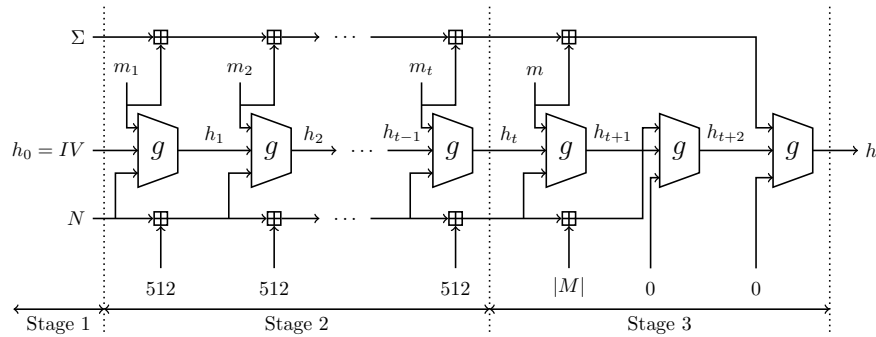


Fig. 17. *Streebog* hash function [26].

E.1 Improved Preimage Attack of 7.5-round Streebog-512

Using the **SIM** technique, we first search MITM attacks on 7.5-round compression function of **Streebog-512**, and our improved search result is given in Figure 18. Compared to the previous best 7.5-round pseudo-preimage attacks on **Streebog-512** found by Hua *et al.* [22] at ToSC 2022, we can improve the time complexity from 2^{441} to 2^{433} and memory complexity from 2^{192} to 2^{177} , our attack configuration is as below

- Initial DoF for forward neutral words \vec{t} (■): 10 bytes (10 blue cells in $\#SB^4$ are set to be equal to the corresponding cells in $\#KK^4$, and thus can be canceled to zero constant ■ in $\#AK^3$ and some fixed constants¹¹ ■ related to the round constants of **Streebog** in $\#AK^4$);
- Initial DoF for backward neutral words \overleftarrow{t} (in ■): 54 bytes (in $\#SB^4$);
- Consumed DoF for forward $\vec{\sigma}$: zero byte;
- Consumed DoF for backward $\overleftarrow{\sigma}$: 32 bytes (32 for $\#MC^5 \xrightarrow{MC} \#AK^5$);
- Guessed bytes for blue, red and both colors g_B, g_R, g_{BR} : $g_R = g_{BR} = 0$ byte and $g_B = 12$ bytes;
- Matching DoF d_M : 24 bytes between $\#MC^2$ and $\#AK^2$.

Then, the remaining DoF is $d_B = 10, d_R - g_B = 10, d_M - g_B = 12$.

Compute initial values forward neutral bytes (Blue). As there is no DoF cost for forward neutral words, to obtain the corresponding initial values, one only needs to enumerate all possible values of 10 blue cells in $\#SB^4$.

Algorithm 15: Compute backward neutral words (red) for 7.5-round **Streebog**

```

1 Initiate a table  $T_{red}^{neutral}$ ;
2 Fix 32 ■ constant cells in  $\#AK^5$  to all zero;
3 for 26 ■ cells in  $\#AK^4 \in (\mathbb{F}_2^8)^{26}$  in the four rightmost columns (columns 4-7) do
4   Compute forward to the four bottom rows (rows 4-7) of  $\#MC^5$ ;
5   With 32 ■ fixed constants in  $\#AK^5$ , derive the remaining red cells of  $\#AK^5$ ;
6   Derive the four topmost rows of  $\#MC^5$ ;
7   Compute backward to the four leftmost columns of  $\#AK^4$ ;
8   if 4 constants cells in  $\#AK^4$  in the leftmost columns are  $(a_0, \dots, a_3)$  then
9     //  $2^{8 \times (26-4)} = 2^{8 \times 22}$  candidates expected
9     Update table with  $T_{red}^{neutral} += \#AK^4$ ;
10 return  $T_{red}^{neutral}$ ;
```

¹¹ Because the round constant of the key schedule of **Streebog** is introduced at the input of the SB operation, which is different from **Whirlpool**. Thus, when considering forward computation by using the **SIM** technique, all zero constants will be changed to some fixed constants related to the round constants of **Streebog**, and this essentially has no impact on the attack.

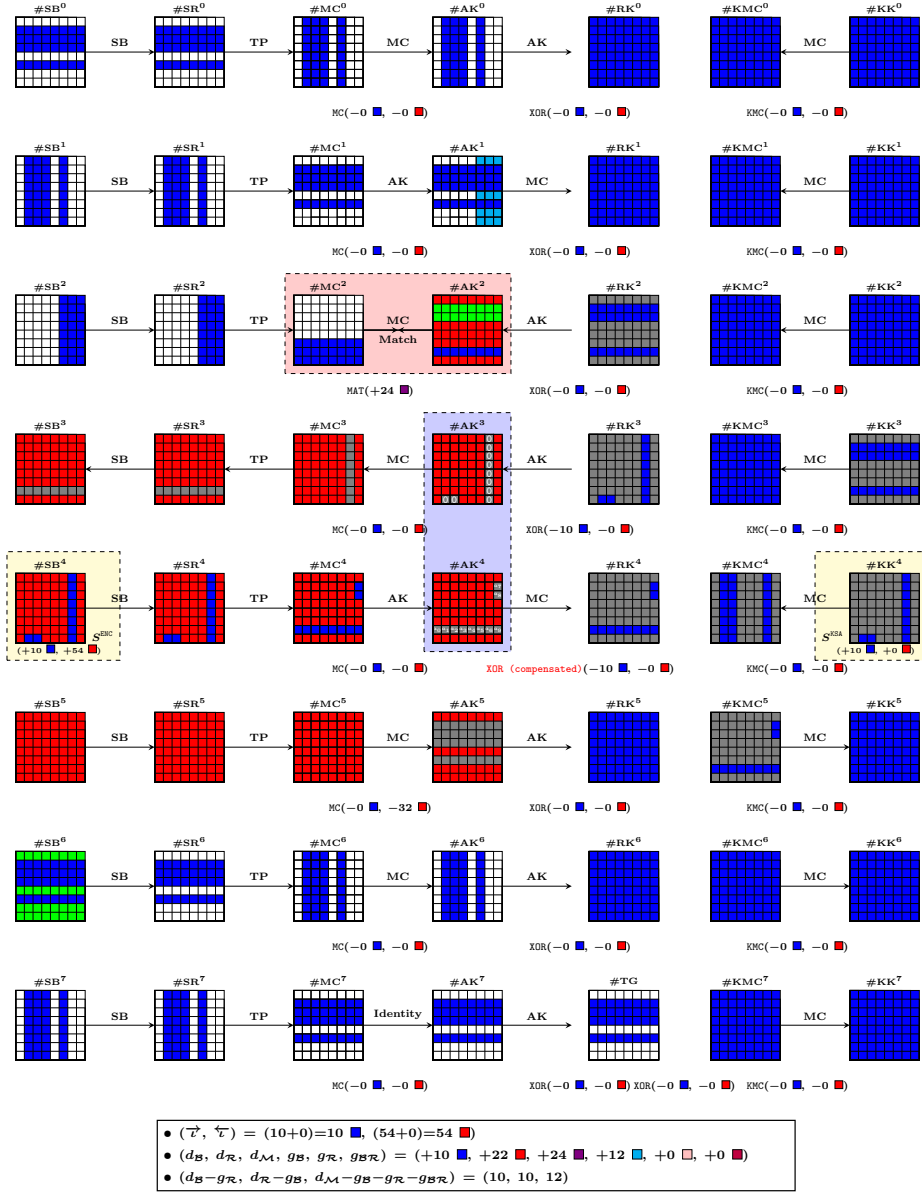


Fig. 18. An MITM attack on 7.5-round compression function of Streebog-512.

Compute initial values for backward neutral words (Red). To get the initial values of backward neutral words, we provide Algorithm 15 to obtain the solution space of red neutral words with the time complexity $2^{8 \times 26} = 2^{208}$ and memory complexity $2^{8 \times 22} = 2^{176}$.

The MITM attack procedure for 7.5-round *Streebog-512*. We provide the main attack procedure derived from Figure 18 in Algorithm 16 as below.

Algorithm 16: MITM attack on 7.5-round *Streebog* compression function

```

1 Fix 32 ■ constant cells in #AK5 to all zero;
2 Let 22 ■ constant cells in columns 0, 1 and 2 of #KK4 be all zero;
3 Let 32 ■ constant cells in columns 3, 4, 5 and 7 of #KK4 be CG;
4 Call Algorithm 15 to build Tredneutral (with size 28×22);
5 for CG ∈ (F28)32 do
    // For blue neutral words
6   for 10 ■ blue cells in #SB4 ∈ (F28)10 do
7     for 12 ■ guessed blue cells in #AK1 ∈ (F28)12 do
8       Compute forward to the matching state #MC2;
9       Compute backward to the matching state #AK2;
10      Store in a table T+ (with size 28×22);

    // For red neutral words
11   for #AK4 ∈ Tredneutral do
12     Compute backward to #AK2;
13     Check in the table T+;
14     if #MC2 and #AK2 pass the partial match then
15       // 28×(32+22+22-24) = 28×52 candidates expected
16       Compute to check 12 ■ guessed cells in #AK1;
17       if Guess values are correct then
18         // 28×(52-12) = 28×40 candidates expected
19         Compute forward and backward to match the rest 40 cells;
20         if The full match is found then
21           // 28×(40-40) = 1 candidate expected
22           Output the preimage and stop;

```

The attack complexity. According to Algorithm 16, the time complexity of the above MITM pseudo-preimage attack on the compression function of 7.5-round *Streebog* is about $2^{512-8 \times 10} + 2^{512-8 \times 10} + 2^{512-8 \times 12} \approx 2^{433}$. The tables T^+ and $T_{red}^{neutral}$ dominate the memory complexity with $2 \cdot 2^{8 \times 22} = 2^{177}$.

We use the conversion given by AlTawy *et al.* [3, Section 6], who employed Joux' multi-collision technique [27] and another MITM attack [33] on the HAIFA framework [10] in their work. By using the conversion, the attack on the com-

pression function of 7.5-round **Streebog** can be converted into a preimage attack on its hash function with the steps below:

1. Given an output $H(M)$ of **Streebog-512**, we produce 2^k preimages (Σ, h_{515}) for the last compression function and store them in a table T .
2. Using Joux’s multi-collisions [27], 2^{512} messages can be constructed with a length of 512 message blocks, which all lead to the same value of h_{512} . That is $M_i = m_1^j \| m_2^j \| \dots \| m_{512}^j$ ($i \in \{1, 2, \dots, 2^{512}\}, j \in \{1, 2\}$), then we have 2^{512} candidates for Σ_{M_i} .
3. Assume the message has 513 complete blocks, then m_{pad} and $|M|$ are known constants. By randomly choosing 2^{512-k} m_{513} , together with h_{512} produced in step 2 and the known values N_{513}, N_{514} , we can compute h_{515} . Then a right m_{513} is expected such that $h_{515} \in T$. Once we find a matching, Σ is known, so we can compute the sum $\Sigma_{M_i} = \Sigma - m_{\text{pad}} - m_{513}$.
4. We compute all the 2^{256} sums of all the 2^{256} 256-block message $\Sigma_{M_1} = m_1^j + m_2^j + \dots + m_{256}^j$ and store them in a table T_1 . Then, compute the sum of other 256-block messages $\Sigma_{M_2} = m_{256}^j + \dots + m_{512}^j$ and check if $\Sigma_{M_i} - \Sigma_{M_2}$ is in table T_1 . Once we find a matching, we know that the full 513-block message $M = m_1^j \| m_2^j \| \dots \| m_{512}^j \| m^{513}$ is the preimage of $H(M)$.

For 7.5-round **Streebog-512** hash function, $k = 40.25$ is an almost optimal solution, so the time complexity is about $2^{40.25} \cdot 2^{433} + 512 \times 2^{256} + 3 \times 2^{512-40.25} + 2^{256} \approx 2^{474.25}$ and the memory complexity is bounded by 2^{256} .

E.2 Improved Preimage Attack of 8.5-round **Streebog-512**

The previous best preimage attack on **Streebog-512** is also given by Hua *et al.* [22], which reaches 8.5-round and has the time complexity $2^{481}/2^{498.25}$ (compression function/hash function) and memory complexity 2^{288} . By using the **SIM** technique, obvious memory improvements can be achieved from 2^{288} to 2^{129} for the attack on compression function and to 2^{256} for the attack on hash function. Our better search results is provided in Figure 19, to gain the huge memory improvements, it firstly should be transformed into an equivalent configuration, which is given in Figure 20 and has the following attack configuration.

- Initial DoF for forward neutral words \overrightarrow{v} (■): 4 bytes (4 blue cells in $\#\text{AK}^4$ are *set to be equal to* the corresponding cells in $\#\text{KK}^5$, and thus can be some fixed constants ■ in $\#\text{AK}^3$ and some fixed constants ■ in $\#\text{AK}^5$, these fixed constants are related to the round constants of **Streebog**.);
- Initial DoF for backward neutral words \overleftarrow{v} (in ■): 32 bytes (in $\#\text{AK}^5$);
- Consumed DoF for forward $\overrightarrow{\sigma}$: zero byte (16 XOR DoF cost for $\#\text{RK}^3$ cancels blue cells in $\#\text{SB}^4$ can be compensated by the **SIM** technique);
- Consumed DoF for backward $\overleftarrow{\sigma}$: 16 bytes (16 for $\#\text{MC}^6 \xrightarrow{\text{MC}} \#\text{AK}^6$);
- Guessed bytes for blue, red and both colors $g_{\mathcal{B}}, g_{\mathcal{R}}, g_{\mathcal{BR}}$: $g_{\mathcal{R}} = g_{\mathcal{BR}} = 0$ byte and $g_{\mathcal{B}} = 12$ bytes;
- Matching DoF $d_{\mathcal{M}}$: 16 bytes between $\#\text{MC}^2$ and $\#\text{AK}^2$.

Then, the remaining DoF is $d_{\mathcal{B}} = 4, d_{\mathcal{R}} - g_{\mathcal{B}} = 4, d_{\mathcal{M}} - g_{\mathcal{B}} = 4$.

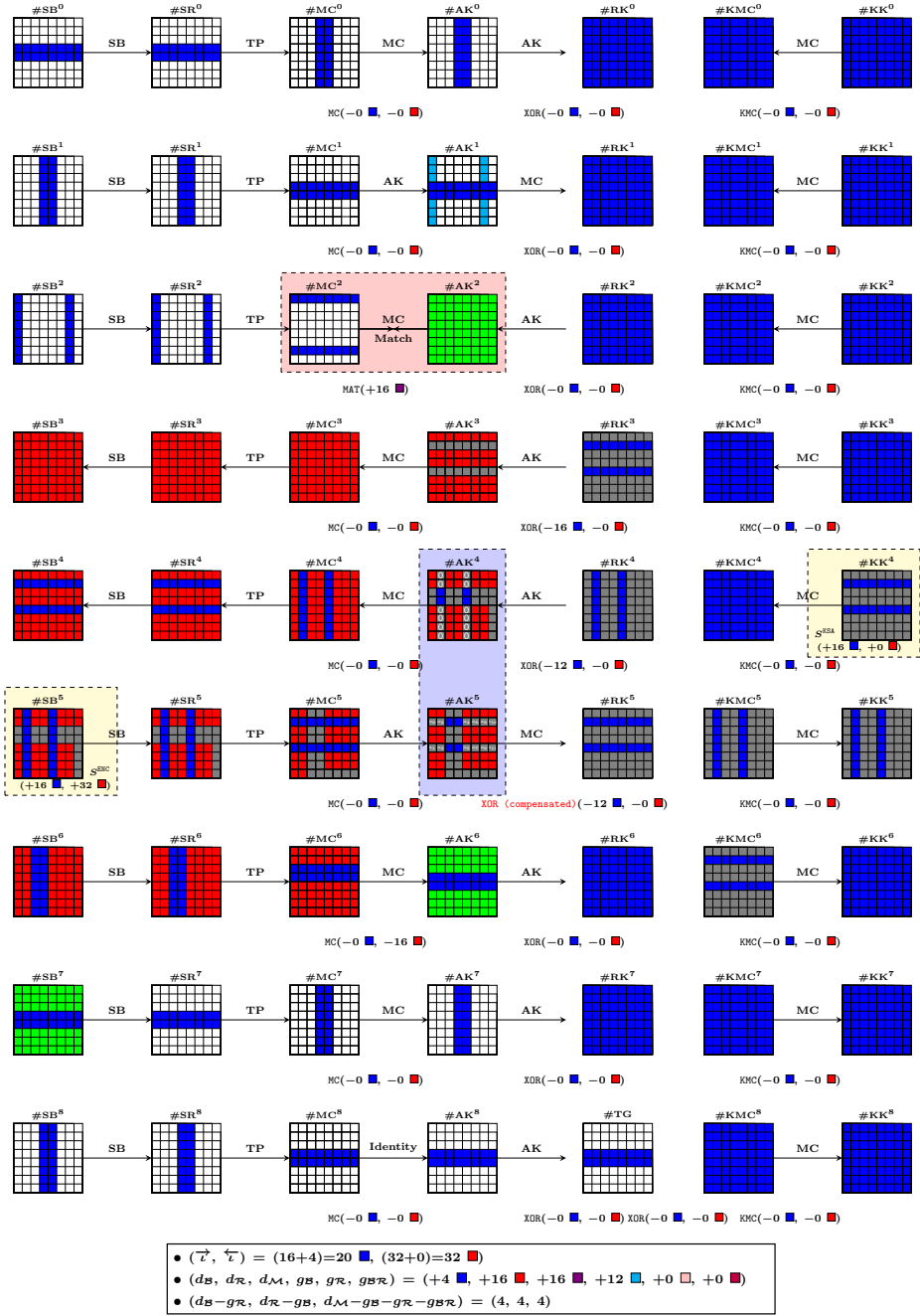


Fig. 19. An MITM attack on 8.5-round compression function of Streebog-512.

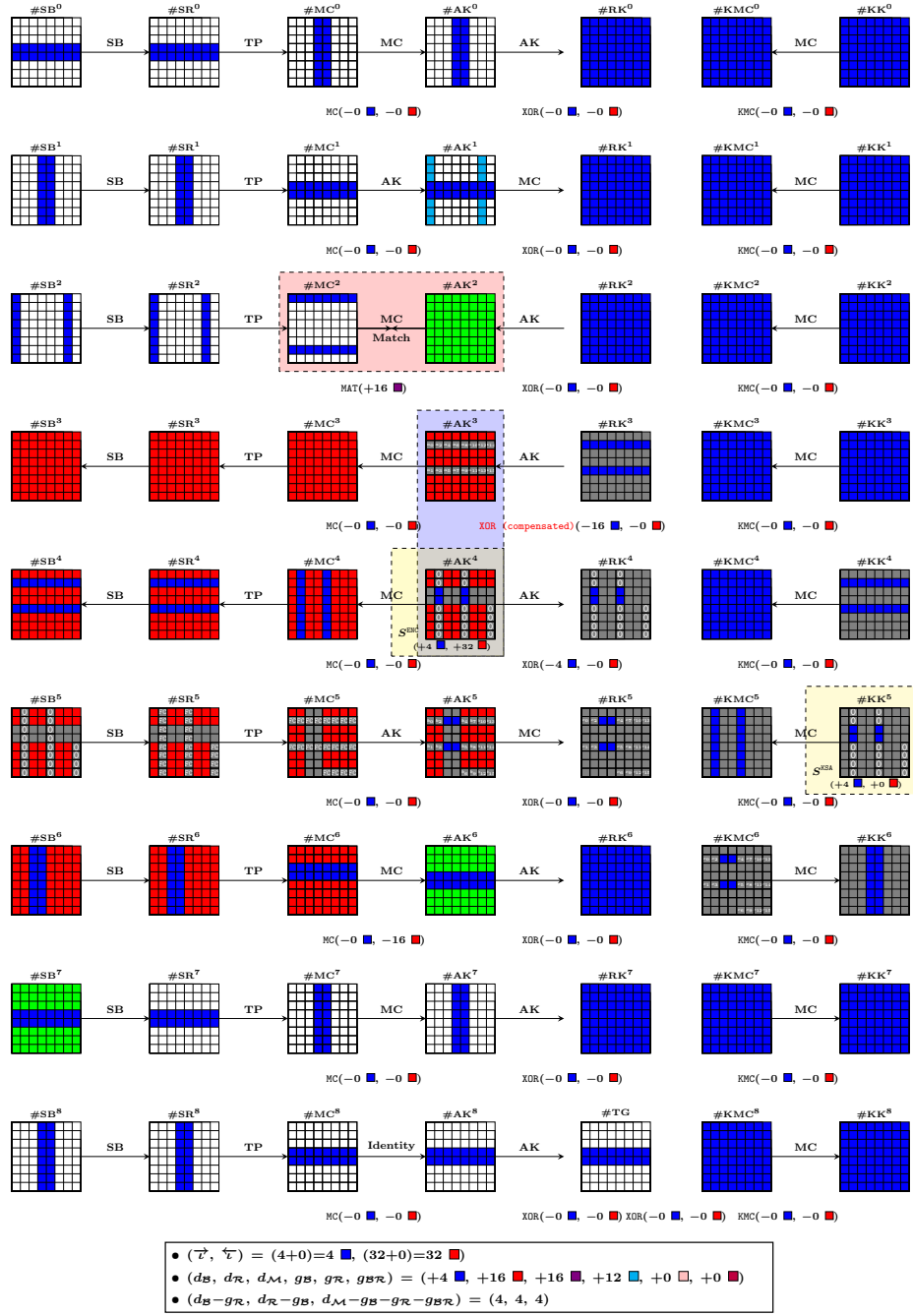


Fig. 20. Equivalent MITM attack on 8.5-round compression function of Streebog-512.

Compute initial values for forward neutral words (Blue). To obtain the initial values of forward neutral words, one should focus on the constraints among states $\#RK^3$ ($\#KK^4$), $\#AK^3$ and $\#SB^4$ as below

$$\begin{pmatrix} \#SB_0^4 & \#SB_8^4 & \dots & \#SB_{56}^4 \\ \#SB_1^4 & \#SB_9^4 & \dots & \#SB_{57}^4 \\ \#SB_2^4 & \#SB_{10}^4 & \dots & \#SB_{58}^4 \\ \#SB_3^4 & \#SB_{11}^4 & \dots & \#SB_{59}^4 \\ \#SB_4^4 & \#SB_{12}^4 & \dots & \#SB_{60}^4 \\ \#SB_5^4 & \#SB_{13}^4 & \dots & \#SB_{61}^4 \\ \#SB_6^4 & \#SB_{14}^4 & \dots & \#SB_{62}^4 \\ \#SB_7^4 & \#SB_{15}^4 & \dots & \#SB_{63}^4 \end{pmatrix} \oplus \begin{pmatrix} \#RK_0^3 & \#RK_8^3 & \dots & \#RK_{56}^3 \\ \#RK_1^3 & \#RK_9^3 & \dots & \#RK_{57}^3 \\ \#RK_2^3 & \#RK_{10}^3 & \dots & \#RK_{58}^3 \\ \#RK_3^3 & \#RK_{11}^3 & \dots & \#RK_{59}^3 \\ \#RK_4^3 & \#RK_{12}^3 & \dots & \#RK_{60}^3 \\ \#RK_5^3 & \#RK_{13}^3 & \dots & \#RK_{61}^3 \\ \#RK_6^3 & \#RK_{14}^3 & \dots & \#RK_{62}^3 \\ \#RK_7^3 & \#RK_{15}^3 & \dots & \#RK_{63}^3 \end{pmatrix} = \begin{pmatrix} a_0 & a_2 & a_4 & a_6 & a_8 & a_{10} & a_{12} & a_{14} \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ a_1 & a_3 & a_5 & a_7 & a_9 & a_{11} & a_{13} & a_{15} \\ - & - & - & - & - & - & - & - \end{pmatrix}, \quad (22)$$

where a_i ($0 \leq i \leq 15$) are the chosen constants for $\#AK^3$.

Thanks to the **SIM** technique, the conditions on Equation (22) can be naturally held if we properly set the states $\#AK^4$ and $\#KK^5$ as below

- Set 16 ■ zero constant cells as shown in $\#AK^4$ and $\#KK^5$;
- Set 4 ■ blue cells in $\#AK^4$ to be equal to the corresponding cells in $\#KK^5$;

Then for backward computation, as $\#AK^4$ and $\#KK^5$ follow almost the same operations in encryption and key schedule except the AddRoundConstant after passing the SB^{-1} operation in key schedule, thus it will cancel to 16 fixed constants ■ in $\#AK^3$, which is related to round constants of **Streebog**. Similarly, for forward computation¹², it will cancel to 16 fixed constant cells ■ in $\#AK^5$. We note that this usage of the **SIM** technique is different to previous results on **Whirlpool** and 7.5-round **Streebog**, which is asymmetric in terms of the DoF compensation and covers one more round to three rounds, *i.e.*, $\#AK^3$, $\#AK^4$ and $\#AK^5$. For $\#AK^3$ and $\#AK^4$, it can be used to efficiently obtain forward (blue) neutral words, and the 16 fixed constant cells ■ in $\#AK^5$ can be used to efficiently obtain backward (red) neutral words.

Compute initial values for backward neutral words (Red). To obtain the initial values of red neutral words, the constraints among $\#AK^6$ and $\#MC^6$ are set as below

$$\begin{pmatrix} - & - & - & - & - & - & - & - \\ e_0 & e_2 & e_4 & e_6 & e_8 & e_{10} & e_{12} & e_{14} \\ e_1 & e_3 & e_5 & e_7 & e_9 & e_{11} & e_{13} & e_{15} \\ - & - & - & - & - & - & - & - \end{pmatrix} = MC \cdot \begin{pmatrix} \#MC_0^6 & \#MC_8^6 & \dots & \#MC_{56}^6 \\ \#MC_1^6 & \#MC_9^6 & \dots & \#MC_{57}^6 \\ - & - & \dots & - \\ - & - & \dots & - \\ \#MC_4^6 & \#MC_{12}^6 & \dots & \#MC_{60}^6 \\ \#MC_5^6 & \#MC_{13}^6 & \dots & \#MC_{61}^6 \\ \#MC_6^6 & \#MC_{14}^6 & \dots & \#MC_{62}^6 \\ \#MC_7^6 & \#MC_{15}^6 & \dots & \#MC_{63}^6 \end{pmatrix}, \quad (23)$$

where e_i ($0 \leq i \leq 15$) are the chosen constants for $\#AK^6$. Then the following Algorithm 17 is provided to obtain the solution space of red neutral words with the time complexity $2^{8 \times 32} = 2^{256}$ and memory complexity $2^{8 \times 16} = 2^{128}$.

The MITM attack procedure for 8.5-round Streebog-512. We provide the main attack procedure derived from Figure 19 in Algorithm 18 as below.

¹² According to **Streebog**'s S-box π , $\pi(0) = FC$, thus it has ■ $\xrightarrow{\pi}$ ■ for $\#SB^5 \xrightarrow{SB} \#SR^5$.

Algorithm 17: Compute backward neutral words (red) for 8.5-round Streebog

```
1 Initiate a table  $T_{\text{red}}^{\text{neutral}}$ ;
2 Fix 16  $\blacksquare$  constants  $(e_0, \dots, e_{15})$  in Equation (23) to all zero;
3 Solve Equation (23) to get the solution space  $S_{\text{red}}$  (with dimension 32 bytes);
4 for each solution in  $S_{\text{red}}$  do
5   Compute backward to  $\#AK^5$ ;
6   if 16  $\blacksquare$  fixed constant cells in  $\#AK^5$  are matched then
7     //  $2^{8 \times (32-16)} = 2^{8 \times 16}$  candidates expected
7     Update to table with  $T_{\text{red}}^{\text{neutral}} += \#AK^5$ ;
8 return  $T_{\text{red}}^{\text{neutral}}$ ;
```

Algorithm 18: MITM attack on 8.5-round Streebog compression function

```
1 Fix 16  $\blacksquare$  constant cells in  $\#AK^3$ ;
2 Fix 16  $\blacksquare$  constant cells in  $\#AK^5$ ;
3 Fix 16  $\blacksquare$  constant cells in  $\#AK^4$ ;
4 Fix the rest 12  $\blacksquare$  constant cells in  $\#AK^4$  to all zero;
5 Fix 16  $\blacksquare$  constant cells in  $\#KK^5$ ;
6 Fix 16  $\blacksquare$  constants  $(e_0, \dots, e_{15})$  in Equation (23) to all zero;
7 Fix any 4  $\blacksquare$  constant cells in  $\#KK^5$  ( $\#RK^4$ ) be all zero;
8 Let the rest 44  $\blacksquare$  constant cells in  $\#KK^5$  be  $C_G$ ;
9 Call Algorithm 17 to build  $T_{\text{red}}^{\text{neutral}}$  (with size  $2^{8 \times 16}$ );
10 for  $C_G \in (\mathbb{F}_2^8)^{44}$  do
11   // For red neutral words
11   Compute backward to  $\#AK^2$  by  $T_{\text{red}}^{\text{neutral}}$  and store in a table  $T^-$ ;
12   // For blue neutral words
12   for 4  $\blacksquare$  blue cells in  $\#AK^4 \in (\mathbb{F}_2^8)^4$  do
13     Set 4  $\blacksquare$  corresponding blue cells in  $\#AK^5$  to be equal to that in  $\#AK^4$ ;
14     for 12  $\blacksquare$  guessed blue cells in  $\#MC^1 \in (\mathbb{F}_2^8)^{12}$  do
15       Compute to  $\#MC^2$  and  $\#AK^2$  and check in the table  $T^-$ ;
16       if  $\#MC^2$  and  $\#AK^2$  pass the partial match then
17         //  $2^{8 \times (44+16+12+4-16)} = 2^{8 \times 60}$  candidates expected
17         Compute to check 12  $\blacksquare$  guessed cells in  $\#AK^1$ ;
18         if Guess values are correct then
19           //  $2^{8 \times (60-12)} = 2^{8 \times 48}$  candidates expected
19           Compute forward and backward to match the rest 48 cells;
20           if The full match is found then
21             //  $2^{8 \times (48-48)} = 1$  candidate expected
21             Output the preimage and stop;
```

The attack complexity. According to Algorithm 18, the time complexity of the above MITM pseudo-preimage attack on the compression function of 8.5-round **Streebog** is about $2^{512-8 \times 4} + 2^{512-8 \times 4} + 2^{512-8 \times 4} \approx 2^{481}$. The table T^- and $T_{\text{red}}^{\text{neutral}}$ dominate the memory complexity with $2 \times 2^{8 \times 16} = 2^{129}$.

For 8.5-round **Streebog-512** hash function, same to the choice in [22], $k = 16.25$, so the time complexity is about $2^{16.25} \cdot 2^{481} + 512 \times 2^{256} + 3 \times 2^{512-16.25} + 2^{256} \approx 2^{498.25}$ and the memory complexity is bounded by 2^{256} .