Designated-ciphertext Searchable Encryption

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Abstract. Public-key encryption with keyword search (PEKS), proposed by Boneh *et al.*, allows users to search encrypted keywords without losing data privacy. Although extensive studies have been conducted on this topic, only a few have focused on insider keyword guessing attacks (IKGA) that can reveal a user's sensitive information. In particular, after receiving a trapdoor used to search ciphertext from a user, a malicious insider (*e.g.*, a server) can randomly encrypt possible keywords using a user's public key, and then test whether the trapdoor corresponds to the selected keyword. This paper introduces a new concept called *designated-ciphertext searchable encryption* (DCSE), which provides the same desired functionality as a PEKS scheme and prevents IKGA. Each trapdoor in DCSE is designated to a specific ciphertext, and thus malicious insiders cannot perform IKGA. We further propose a generic DCSE scheme that employs identity-based encryption and a key encapsulation mechanism. We provide formal proofs to demonstrate that the generic construction satisfies the security requirements. Moreover, we provide a lattice-based instantiation whose security is based on NTRU and ring-learning with errors assumptions; the proposed scheme is thus considered to be resistant to the quantum-computing attacks.

Keywords: Quantum-resistant · Searchable Encryption · Insider Keyword Guessing Attack · Designated-ciphertext · Lattices

1 Introduction

With the development of the 5G and Internet of Things (IoT), the importance of cloud storage is increasing. However, because the cloud providers cannot be easily trusted, to avoid data leakage or abuse, data owners need to ensure the privacy of sensitive data. A straightforward method is data encryption before upload to cloud servers. However, encrypted data loses its processing flexibility and cannot be used for useful operations, such as sorting or searching. Specifically, search functionality is important for cloud storage. If a data owner wants to search for some specific files among large encrypted data sets, it becomes necessary to download and decrypt all the data to search, which is impractical and resource-consuming. To resolve this issue, Song *et al.* [45] proposed the first searchable encryption (SE) that allows the ciphertext to be searched using the corresponding trapdoor. However, because their construction is based on a symmetric key primitive, only the owner of a particular secret key can generate any corresponding ciphertext and trapdoor. Hence, as with a symmetric cryptosystem, their work faces the key distribution problem when it is deployed in public cloud environments.

1.1 Public-key Encryption with Keyword Search

To circumvent the issue of the symmetric searchable encryption and allow multiple data owners to easily generate different ciphertexts for a single data receiver, Boneh *et al.* [10] proposed the first public-key encryption with keyword search (PEKS). The scheme, unlike Song *et al.*'s work [45], is built on a public-key cryptosystem. The PEKS scheme has three entities: data owner (Alice), data receiver (Bob), and cloud server. Consider the following scenario: Alice wants to store files that can be accessed and searched by Bob without any leakage of information to the cloud server. Therefore, in addition to encrypting files using Bob's public key pk, she also encrypts the related keywords of the files using a $PEKS$ algorithm that allows ciphertexts to be searched, *e.g.,* $Enc(\mathsf{pk}, \text{file}) \| PEKS(\mathsf{pk}, \text{“pkc”}) \| \cdots \| PEKS(\mathsf{pk}, \text{“crypto”})$; she uploads the encrypted keywords to the cloud server. If Bob would like to request the cloud server to search for any encrypted files containing the keyword “crypto,” he first generates a trapdoor for “crypto” by using his private key, and then sends the trapdoor to the cloud server. Using this trapdoor, the cloud server can test the value of all $PEKS$, determine which value is generated by the keyword “crypto”, and return the corresponding encrypted file to Bob.

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Compared with symmetric searchable encryption, PEKS is more suitable for purposes such as cloud services, IoT, and email. In the first two decades of the twenty-first century, numerous PEKS schemes suitable for different scenarios that provide notable functionality were proposed.

1.2 Motivation

Even though numerous PEKS schemes have been advanced, their security precautions are inadequate. For instance, as most of the proposed schemes assume the insider (*e.g.*, cloud server, mail server, or IoT gateway) to be trustworthy and thus do not consider possible attacks from insiders. In actual fact, because of the small number of commonly used keywords, a insider can guess some keywords from a trapdoor and obtain some useful information; this attack is called an *insider keyword guessing attack* (IKGA). More concretely, after receiving a trapdoor from the authorized data receiver, a malicious insider can encrypt possible keywords using the data receiver's public key. Then, the insider can test whether the trapdoor corresponds to the selected keywords.

On the other hand, Shor [44, 43] demonstrates the existence of quantum algorithms that can break some difficult assumptions in number theory (*i.e.,* the discrete logarithm assumption and the integer factoring assumption), the potential threat of quantum computers to modern cryptography is foreseeable. Arute *et al.* [8] recently proposed a 53-qubit quantum computer. Scholars believe that quantum computing will become mature in the twenty-first century. Although many studies on quantum-resistant PEKS [9, 47, 35, 48] to resist quantum computing attacks have been proposed, only Mao *et al.* [35] is IKGA secure. The security of this scheme is based on the learning-with-errors assumption, which has been proven to be as difficult as solving worst-case lattice problems [40]. However, this scheme not sufficiently practical because of size constraints; public keys and private keys would thus be as large as hundreds of megabytes.

Actually, Abdalla *et al.* [6] proved that an IND-ANON-ID-CPA secure identity-based encryption (IBE) scheme can obtain a secure PEKS scheme. However, how to support IKGA security based on it is still unknown. In this paper, we consider the following question:

Can we instantiate a cryptographic primitive that is quantum-resistant and supports search functionality as well as the strength against IKGA using IND-ANON-ID-CPA secure IBE scheme as a building block?

1.3 Our Results

To answer our question, we first introduce a new cryptographic primitive, called "designated-ciphertext searchable encryption" (DCSE), to provide the same functionality as a PEKS scheme with additional strength against IKGA.

In contrast to PEKS, each trapdoor in DCSE is designated to a ciphertext. Thus, an adversarial insider cannot adaptively select keywords to generate various ciphertexts and then test these ciphertexts with the trapdoor received. In addition, by combining a key encapsulation mechanism (KEM) with a pseudorandom generator, we use IBE to formalize a generic construction of our DCSE scheme under the standard model. Moreover, we implemented a lattice-based DCSE based on NTRU and ring-learning with errors (ring-LWE) assumptions; our implementation is more efficient, more secure, and more practical than other advanced schemes.

Designated-ciphertext Searchable Encryption. Attacks against conventional PEKS schemes succeed because insiders can adaptively generate ciphertext for any keyword. Therefore, our strategy, in DCSE, is to prevent insiders from producing valid ciphertexts themselves that can be effectively tested against trapdoor received from the authorized data receiver. Consider a scenario where Alice wants to encrypt some files and upload them to a cloud server, and she wants these encrypted files to be searchable within the cloud. In addition, she wants to avoid any IKGAs. Alice first executes $DCSE(\mathsf{pk}, w_i)$ for keywords $w_1, \cdots, w_n$ to generate the pairs of a ciphertext c_i and a tag v_i of the ciphertext that hides some private information, where pk is Bob's public key and the $DCSE$ is discussed in later passages. Similar to the $PEKS$ algorithm, the $DCSE$ output is searchable using the trapdoors generated from the data receiver. However, to resist IKGAs, $DCSE$ additionally generates a tag for the ciphertext, and only the specified data receiver (*i.e.,* Bob) can extract private information from the tag, which can be linked to a ciphertext. In addition to sending $Enc(\mathsf{pk}, \mathrm{file})\|c_1\|\cdots\|c_n)$ to the cloud server, Alice publicly sends the tags $(v_1, \cdots, v_n)$ to Bob. Bob first uses his private key to retrieve private information from the tag v_i, and then uses this information to generate a trapdoor t_i for the ciphertext corresponding to the tag. In other words, we can consider a trapdoor as having been *designated* to a ciphertext. Bob then submits (v_i, t_i) to the cloud server for searching. Using the trapdoor, the cloud server returns the matched encrypted files to Bob. Because the cloud server cannot randomly select a keyword to generate a ciphertext matching this trapdoor, the design provides no further method

to identify the keyword the data receiver is searching for. Furthermore, unlike PEKS, must test every ciphertext, the cloud server in the proposed scheme can quickly find all the matching ciphertexts by using the tag v_i as the index value.

For DCSE, we define two security models: indistinguishability under chosen-keyword attack (IND-CKA) and indistinguishability under insider-keyword-guessing attack (IND-IKGA). In particular, IND-CKA security and IND-IKGA security ensure that no adversary can retrieve any information about the keyword from the ciphertext and the trapdoor, even if that adversary can query a polynomial-time trapdoor oracle. Because the ability of a malicious insider exceeds that of a malicious outsider, we only consider IND-IKGA against DCSE.

Generic Formulation and Its Security. We present a generic formulation for DCSE with a pseudorandom generator $F(\cdot)$, an IND-ANON-ID-CPA secure IBE scheme $\mathcal{IBE} = (\mathit{KeyGen}, \mathit{Extract}, \mathit{Enc}, \mathit{Dec})$, and an IND-CCA2 secure KEM scheme $\mathcal{KEM} = (\mathit{KeyGen}, \mathit{Encaps}, \mathit{Decaps})$. The high-level idea is as follows: To encrypt a keyword w for an authorized data receiver, the data owner first uses KEM to generate a random key k and its corresponding encapsulation e using the data receiver's public key, that is $(k, e) \leftarrow \mathcal{KEM}.\mathit{Encaps}(\mathsf{pk})$. The data owner then defines the output of pseudorandom generator $f \leftarrow F(k\|w)$ as an identity and use it to encrypt another random message r, that is $\mathsf{ct} \leftarrow \mathcal{IBE}.\mathit{Enc}(\mathsf{pk}, f, r)$. Here, we can view the encapsulation e as a tag of the ciphertext ct. Only the authorized data receiver can decap e using her/his private key to obtain the private information, key k. That data receiver then generates the trapdoor t for the "identity" $f \leftarrow F(k\|w)$. In this way, the trapdoor actually links to a tag related to a ciphertext. Therefore, a malicious insider cannot randomly generate ciphertext to test the trapdoor.

Additionally, we provide rigorous proofs to demonstrate that this generic construction satisfies the criteria of IND-CKA and IND-IKGA security. Our main idea for proving security levels is a sequence of games, which slightly modify our origin protocol so that the challenge message contains no information of the keyword in the final game. Consequently, the strategy by which an adversary wins the games can only be guessed at, that is, the adversary cannot gain any advantage by attacking our construction.

Lattice-based Instantiation. We provide an instantiation utilizing two efficient and secure lattice-based constructions: the NTRU-based IBE [22] and the NTRU-based KEM [29]. The security of these constructions is based on the ring-LWE and NTRU assumptions that in turn makes our instantiation quantum-resistant. We also experimentally evaluated the performance of the instantiation on a modern laptop. Each encryption, trapdoor, test algorithm only required approximately 1, 0.3, 0.01 (ms), respectively. In comparison with other state-of-the-art schemes, our scheme is not only more efficient and practical, it also provides more robust security.

1.4 Paper Organization

The remainder of this paper is organized as follows. In Section 2, we describe the related works of PEKS. In Section 3, we introduce some notations and preliminaries. In Section 4, we introduce three cryptographic building blocks: pseudorandom generator, KEM scheme, and IBE scheme. In Section 5, we introduce a new notion, "designated-ciphertext searchable encryption," and define its system model and security requirements. In Section 8, we formalize DCSE from the IBE scheme and KEM scheme, and provide its security proofs. In Section 7, an efficient lattice-based DCSE is proposed. Finally, in Section 8, we provide the conclusion.

2 Related Works

Byun *et al.* [15] first reported IND-IKGA and indicated that Boneh *et al.*'s work [10] could not withstand such an attack. The schemes that are resist to IKGAs can be separated into the following three categories.

2.1 Designated-tester Public-key Encryption with Keyword Search

Some studies first utilized another server to perform the test algorithm, that is, decentralizing the power of the insider so that when the two insiders do not collude, the schemes can fend off IKGA. Rhee *et al.* [41] therefore proposed an enhanced security model for public key searchable cryptography called "trapdoor indistinguishability," to obtained the scenario of the IKGA. Then, the authors also proposed a secure public key searchable encryption scheme with a designated tester, and proved that the scheme is secure under the enhanced security model. Chen *et al.* [17, 18, 16] proposed "dual server" public key searchable encryption, which can withstand IKGA if the two servers do not collude with each other. Mao *et al.* [35] followed the idea of the designated tester to introduce the first lattice-based searchable encryption that protected against IKGA.

2.2 Public-key Authenticated Encryption with Keyword Search

By adding a test server or designating an additional server, the malicious insider cannot obtain the private information (*i.e.,* trapdoor) required for the test, so IKGA can be effectively avoided. However, adding another server may increase other communication overhead in an actual environment. Moreover, scholars do not presently know how to ensure that a designated server is trustworthy and will not collude with malicious insiders. Consequently, some studies have begun to investigate how to authenticate ciphertext let trapdoors only be valid for authenticated ciphertext.

Fang *et al.* [23, 24] proposed the first public key searchable encryption using a one-time signature and proved its security without random oracle. Huang and Li [28] introduced a new notion called "public-key authenticated encryption with keyword search (PAEKS)," to resist IKGAs. In their schemes, the data sender not only encrypts the keyword, but also authenticates it, and the trapdoor generated by the data receiver is only valid for the ciphertext authenticated by the data sender. Therefore, the malicious server cannot adaptively generate ciphertext to perform IKGA. However, Noroozi and Eslami [37] show that Huang and Li's scheme even insecure to outsider keyword guessing attacks; thus, they provide an improvement without adding the cost complexity.

Based on the concept of authenticating the ciphertext, Zhang *et al.* [48] proposed a forward secure lattice-based keyword search to protect against IKGAs. However, Liu *et al.* [32] recently demonstrated the security model in the work does not capture the IKGAs, and thus it is insecure. Additionally, Pakniat *et al.* [38] proposed the first certificateless PAEKS scheme.

Additionally, Qin *et al.* [39] and Li *et al.* [31] further consider the leakage of the information about the data receiver's query pattern. In other words, they ensure that only server can execute the test algorithm to avoid that an adversary can decide whether two ciphertexts share some identical keywords or not.

2.3 Witness-based Searchable Encryption

Ma *et al.* [34] introduced cryptographic primitive called "witness-based searchable encryption" in which the trapdoor is valid only when the ciphertext have a witness relation to the trapdoor. Chen *et al.* [19] further gave an improvement to reduce the complexity of the size of the trapdoor.

3 Preliminary

3.1 Notations

For simplicity and readability, we use the following notations throughout the paper. Let λ be the natural security parameter. We use standard notations, O and o, to classify the growth of functions. The notation $\mathbf{negl}(n)$ is denoted as an arbitrary function f is *negligible* in n, where $f(n) = o(n^{-c})$ for every fixed constant c. The notation $\mathbf{poly}(n)$ denotes an arbitrary function $f(n) = O(n^c)$ for some constant c. By $\mathbb{N}$ (resp. $\mathbb{Z}$ and $\mathbb{R}$) we denote the set of positive integers (resp. integers and reals). In addition, for a prime q, $\mathbb{Z}_q$ denotes a finite field (or Galois field) with order q. For a power-of-two n, $\mathcal{R} = \mathbb{Z}[x]/(x^n + 1)$ and $\mathcal{R}_q = \mathbb{Z}_q[x]/(x^n + 1)$. The PPT is short for probabilistic polynomial-time. For two string a, b, the concatenation of a and b is denoted as $a\|b$. Matrices are denoted by bold capital letters (*e.g.*, $\mathbf{X}$). For a vector x and a matrix $\mathbf{X}$, the Euclidean norm of x and $\mathbf{X}$ is denoted by $\|x\|$ and $\|\mathbf{X}\|$ respectively. For a finite set Q, $a \leftarrow Q$ denotes that a is sampled from Q with uniform distribution. For two vectors a, b, the inner product of a and b is denoted as $\langle a, b\rangle$.

3.2 Lattices

The formalization of our instantiation is based on the NTRU lattices. In this section, we first briefly introduce lattice theory, and then review some lattice hardness assumptions.

A m-dimension lattice Λ is an additive discrete subgroup of $\mathbb{R}^m$. Basically, a lattice is the set of all the integer combinations of some linearly independent vectors, called the basis of the lattice. The formal definition of a lattice is as follows.

Definition 1 (Lattice). *Let $\mathbf{B} = [b_1|\cdots|b_n] \in \mathbb{R}^{m\times n}$ be an $m \times n$ matrix, where $b_1, \cdots, b_n \in \mathbb{R}^m$ are n linear independent vectors. The m-dimensional lattice Λ generated by $\mathbf{B}$ is the set,*

$$\Lambda(\mathbf{B}) = \Lambda(b_1, \cdots, b_n) = \left\{ \sum_{i=1}^{n} b_i a_i | a_i \in \mathbb{Z} \right\}.$$

In addition, we call a lattice full-rank when $n = m$.

Hardness Assumptions. Regev introduced a new lattice hardness assumption, called learning with errors (LWE); he demonstrated that several worst-case lattice problems can be reduced to the LWE problem [40]. In addition, he proposed the first public-key cryptosystem based on the hardness of the LWE assumption.

Definition 2 (LWE Assumption). *Given $n, m \in \mathbb{N}$, q as a prime, a probability distribution χ over $\mathbb{Z}_q$. Suppose there exists an oracle $\mathcal{O}_s^n$ that outputs m samples of the form $(a, \langle a, s\rangle + e)$ where $a \in \mathbb{Z}_q^n$ and $e \in \chi$ are chosen freshly at random for each sample, and $s \leftarrow \mathbb{Z}_q^n$ is the same for every sample. The search-LWE assumption is to find the s. In addition, let $\mathcal{O}_r$ be an oracle that outputs samples $(a, b) \leftarrow (\mathbb{Z}_q^n \times \mathbb{Z}_q^n)$ uniformly at random. The decision-LWE assumption is to guess whether you are interacting with $\mathcal{O}_s^n$ or $\mathcal{O}_r$.*

After Regev's seminal work, many LWE-based cryptosystems have been proposed [25, 26, 7, 13, 14]. However, these cryptosystems encountered practical problems, because of their overly large key sizes and inefficiency. To solve the issue, in 2009, Lyubashevsky *et al.* introduced a new algebraic variant of the LWE assumption from [46], and called it ring-LWE [33]. The ring-LWE assumption is the LWE assumption specifically for polynomial rings over finite fields that can also be stated in "search" version and "decision" version that are defined as follows.

Definition 3 (Ring-LWE Assumption). *Given $n, m \in \mathbb{N}$, let q be a prime, a probability distribution χ over $\mathcal{R}_q$. Suppose there exists an oracle $\mathcal{O}_s$ that outputs m samples of the form $(a, \langle a, s\rangle + e)$ where $a \in \mathcal{R}_q$ and $e \in \chi$ is chosen freshly at random for each sample, and $s \leftarrow \mathcal{R}_q$ is the same for every sample. The search-Ring-LWE assumption is to find the s. In addition, let $\mathcal{O}_r$ be an oracle that outputs samples $(a, b) \leftarrow (\mathcal{R}_q \times \mathcal{R}_q)$ uniformly at random. The decision-Ring-LWE assumption is to guess whether the user is interacting with $\mathcal{O}_s$ or $\mathcal{O}_r$.*

Another lattice hardness assumption is the NTRU assumption, defined in [27].

Definition 4 (NTRU Assumption). *Let χ be a probability distribution over $\mathcal{R}_q$. The NTRU assumption is to distinguish the following two distributions. The first distribution sample is a polynomial $h = g/f$, where $f, g \leftarrow \chi$ and f is invertible, and the second distribution uniformly samples a polynomial h over $\mathcal{R}_q$.*

3.3 Public Key Encryption with Keyword Search

In this section we introduce the system model of the PEKS that was proposed by Boneh *et al.* [10]. A PEKS scheme consists of a set of four-tuple PPT algorithms $\mathcal{PEKS} = (KeyGen, PEKS, Trapdoor, Test)$, described as follows:

- $KeyGen(1^\lambda)$: Taking the security parameter λ as input, this algorithm outputs a master private key msk and a master public key mpk.
- $PEKS(\mathsf{mpk}, w)$: Taking the master public key mpk and a keyword w, this algorithm produces a searchable encryption c of w.
- $Trapdoor(\mathsf{msk}, w)$: Taking the master private key msk and a keyword w, this algorithm generate a trapdoor t.
- $Test(\mathsf{mpk}, c, t)$: Taking the master public key, a searchable encryption $s = PEKS(\mathsf{mpk}, w)$, and a trapdoor $t = Trapdoor(\mathsf{msk}, w')$, this algorithm output 1 if $w = w'$ and 0 otherwise.

Definition 5 (Correctness of PEKS). *Let λ be a security parameter. We say that a PEKS scheme is correct if:*

$$\mathbf{Pr}[Test(\mathsf{mpk}, PEKS(\mathsf{mpk}, w), Tradpoor(\mathsf{msk}, w)) = 1] = 1 - \mathbf{negl}(\lambda),$$

where $(\mathsf{msk}, \mathsf{mpk}) \leftarrow KeyGen(1^\lambda)$.

4 Cryptographic Building Blocks

In this section, we recall three crucial cryptographic primitives used as building blocks in our generic construction. They are the pseudorandom generator, IBE, and KEM.

4.1 Pseudorandom Generator

In our generic construction, we use a pseudorandom generator to generate a "pseudorandom." Informally, we say that a distribution $\mathcal{D}$ is pseudorandom if any polynomial-time distinguisher that can distinguish a string $s \leftarrow \mathcal{D}$ from a string s chosen randomly and uniformly does not exits. We recall the definition of the pseudorandom generator in [30] Definition 3.15.

Definition 6 (Pseudorandom Generator). *Let $F : \{0,1\}^n \to \{0,1\}^{n'}$ be a deterministic polynomial-time algorithm, where $n' = \mathbf{poly}(n)$ and $n' > n$. We say that F is a pseudorandom generator if it satisfies the following two conditions:*

- *Expansion: For every n, it holds that $n' > n$.*
- *Pseudorandomness: For all PPT distinguishers $\mathcal{D}$,*

$$|\mathbf{Pr}[\mathcal{D}(r) = 1] - \mathbf{Pr}[\mathcal{D}(F(s)) = 1]| \leq \mathbf{negl}(n),$$

where r is chosen randomly and uniformly from $\{0,1\}^{n'}$, the seed s is chosen randomly and uniformly from $\{0,1\}^n$, and the probabilities depend on the random coins used by $\mathcal{D}$ and the choice of r and s.

4.2 Identity-based Encryption (IBE)

IBE is an essential primitive of public key encryption, in which the public key of a user is information that can identify the user (such as e-mail address, name, and social security number). Its concept was first proposed by Shamir [42] as early as 1984. However, the first construction was realized by Cocks [20] based on the quadratic residuosity problem. Later, Boneh and Franklin [11, 12] proposed a more practical and secure IBE using the pairing technique.

An IBE scheme is a four-tuple of PPT algorithms $\mathcal{IBE} = (\mathit{Setup}, \mathit{Extract}, \mathit{Enc}, \mathit{Dec})$, described as follows:

- $\mathit{Setup}(1^\lambda)$: Taking the security parameter λ as input, this algorithm outputs a master private key msk and master public key mpk.
- $\mathit{Extract}(\mathsf{msk}, \mathsf{id})$: Taking the master private key msk and an identity id as input, this algorithm outputs the corresponding private key $\mathsf{sk}_{\mathsf{id}}$ for the identity.
- $\mathit{Enc}(\mathsf{mpk}, \mathsf{id}, m)$: Taking the master public key, mpk, an identity id, and a message m as input, this algorithm outputs a ciphertext ct encrypted by id.
- $\mathit{Dec}(\mathsf{sk}_{\mathsf{id}}, \mathsf{ct})$: Taking a private key $\mathsf{sk}_{\mathsf{id}}$, and a ciphertext ct as input, this algorithm outputs a decrypted message m'.

Definition 7 (Correctness of IBE).
We say that an IBE scheme, $\mathcal{IBE}$, is correct if

$$\mathbf{Pr}[\mathit{Dec}(\mathsf{sk}_{\mathsf{id}}, \mathit{Enc}(\mathsf{mpk}, \mathsf{id}, m)) = m] = 1 - \mathsf{negl}(\lambda),$$

where $(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathit{Setup}(1^\lambda)$ and $\mathsf{sk}_{\mathsf{id}} \leftarrow \mathit{Extract}(\mathsf{msk}, \mathsf{id})$.

Moreover, an IBE scheme must satisfy the indistinguishability under any adaptive chosen-identity chosen-plaintext attack (IND-ID-CPA), defined using the following game between a challenger $\mathcal{B}$ and an adversary $\mathcal{A}$.
Game IND-ID-CPA:

- **Setup.** In this stage, $\mathcal{B}$ runs the $\mathit{Setup}(1^\lambda)$ algorithm to generate the master public key mpk and master private key msk. Then, $\mathcal{B}$ keeps msk secret, and sends mpk to $\mathcal{A}$.
- **Phase 1.** $\mathcal{A}$ makes a polynomially bounded number of queries to the *Extract* oracle on any identity id, and $\mathcal{B}$ returns a private key $\mathsf{sk}_{\mathsf{id}}$ to $\mathcal{A}$.
- **Challenge.** In this stage, $\mathcal{A}$ sends two challenge messages, m_0, m_1, and a challenge identity, id^*, to $\mathcal{B}$, where id^* has never been queried to *Extract* oracle. After receiving the messages and identity, $\mathcal{B}$ chooses a random bit, $b \leftarrow \{0,1\}$, and generates the challenge ciphertext, $\mathsf{ct}^* \leftarrow \mathit{Enc}(\mathsf{mpk}, \mathsf{id}^*, m_b)$. Finally, $\mathcal{B}$ returns ct^* to $\mathcal{A}$.
- **Phase 2.** $\mathcal{A}$ can continue to ask for the *Extract* oracle the same as in **Phase 1**. The only restriction is that $\mathcal{A}$ cannot issue an *Extract* query on the challenge identity id^*.
- **Guess.** $\mathcal{A}$ outputs its guess b'. The adversary is said to win the game if $b' = b$. The advantage of $\mathcal{A}$ is as follows:

$$\mathbf{Adv}^{\text{IND-ID-CPA}}_{\mathcal{IBE},\mathcal{A}}(\lambda) = |\mathsf{Pr}[b' = b] - \tfrac{1}{2}|.$$

Definition 8 (IND-ID-CPA secure IBE).
We say that an IBE scheme $\mathcal{IBE}$ is IND-ID-CPA secure, if no PPT adversary $\mathcal{A}$ can win the aforementioned game with an advantage exceeding $\mathbf{negl}(\lambda)$.

Moreover, we say that an IBE scheme $\mathcal{IBE}$ is anonymous if it satisfies the following stronger notion of security:
Game IND-ANON-ID-CPA:

- **Setup.** In this stage, $\mathcal{B}$ runs the $Setup(1^\lambda)$ algorithm to generate the master public key mpk and master private key msk. Then $\mathcal{B}$ keeps msk secret, and sends mpk to $\mathcal{A}$.
- **Phase 1.** $\mathcal{A}$ makes a polynomially bounded number of queries to the *Extract* oracle on any identity id, and $\mathcal{B}$ returns a private key $\mathsf{sk}_{\mathsf{id}}$ to $\mathcal{A}$.
- **Challenge.** In this stage, $\mathcal{A}$ sends a challenge message m, and two challenge identities $\mathsf{id}_0, \mathsf{id}_1$ to $\mathcal{B}$, where id_0 and id_1 have never been queried to *Extract* oracle. After receiving the messages and identities, $\mathcal{B}$ chooses a random bit, $b \leftarrow \{0,1\}$, and generates the challenge ciphertext, $\mathsf{ct}^* \leftarrow Enc(\mathsf{mpk}, \mathsf{id}_b, m)$. Finally, $\mathcal{B}$ returns ct^* to $\mathcal{A}$.
- **Phase 2.** $\mathcal{A}$ can continue to ask for the *Extract* oracle, the same as in **Phase 1**. The only restriction is that $\mathcal{A}$ cannot issue an *Extract* query on the challenge identities id_0 and id_1.
- **Guess.** $\mathcal{A}$ outputs its guess b'.

We say that the adversary wins the game, if $b' = b$. The advantage of $\mathcal{A}$ is defined as follows:

$$\mathbf{Adv}^{\text{IND-ANON-ID-CPA}}_{\mathcal{IBE},\mathcal{A}}(\lambda) = |\mathbf{Pr}[b' = b] - \tfrac{1}{2}|.$$

Definition 9 (IND-ANON-ID-CPA secure IBE). *We say that an IBE scheme $\mathcal{IBE}$ is IND-ANON-ID-CPA secure if no PPT adversary $\mathcal{A}$ can win the aforementioned game with an advantage exceeding* $\mathbf{negl}(\lambda)$.

4.3 Key Encapsulation Mechanism (KEM)

KEM, first proposed by Cramer and Shoup [21], is a variant of the public key encryption. Rather than encrypting a message, KEM "encaps" a random value using public key, and outputs an encapsulation. With the corresponding private key, anyone can "decaps" the encapsulation to obtain the same random value.

A KEM scheme is a three-tuple of PPT algorithms, $\mathcal{KEM} = (KeyGen, Encaps, Decaps)$, described as follows.

- $KeyGen(1^\lambda)$: Taking the security parameter λ as input, this algorithm outputs a public key pk and a private key sk.
- $Encaps(\mathsf{pk})$: Taking the public key pk as input, this algorithm outputs a key k and an encapsulation e.
- $Decaps(\mathsf{sk}, e)$: Taking the private key sk and an encapsulation e as input, this algorithm outputs the corresponding key k, or an invalid symbol $\perp$.

Definition 10 (Correctness of KEM). *We say that a KEM scheme, $\mathcal{KEM}$, is correct, if*

$$\mathbf{Pr}[Decaps(\mathsf{sk}, e) = k : (k, e) \leftarrow Encaps(\mathsf{pk})] = 1 - \mathbf{negl}(\lambda),$$

where $(\mathsf{pk}, \mathsf{sk}) \leftarrow KeyGen(1^\lambda)$.

Indistinguishability under the adaptive chosen-ciphertext-attack (IND-CCA2) security of a KEM is defined using the following game between a challenger $\mathcal{B}$ and an adversary $\mathcal{A}$.
Game IND-CCA2:

- **KeyGen.** In this stage, $\mathcal{B}$ runs the $KeyGen(1^\lambda)$ algorithm to generate the public/private key pair $(\mathsf{pk}, \mathsf{sk})$. Then, $\mathcal{B}$ sends pk to $\mathcal{A}$.
- **Phase 1.** $\mathcal{A}$ makes a polynomially bounded number of queries to the *Decaps* oracle on any encapsulation e; $\mathcal{B}$ returns a key k or invalid symbol $\perp$ to $\mathcal{A}$.
- **Challenge.** In this stage, $\mathcal{B}$ chooses a random bit $b \leftarrow \{0,1\}$. Then, $\mathcal{B}$ generates $(e^*, k_0^*) \leftarrow Encaps(\mathsf{pk})$, and randomly chooses k_1^* from the key space $\mathcal{K}$. Finally, $\mathcal{B}$ returns the challenge ciphertext (e^*, k_b^*) to $\mathcal{A}$.
- **Phase 2.** $\mathcal{A}$ can continue to ask for the *Decaps* oracle, same as in **Phase 1**. The only restriction is that $\mathcal{A}$ cannot issue a *Decaps* query on e^*.
- **Guess.** $\mathcal{A}$ outputs its guess b'.

We say that the adversary wins the game, if $b' = b$. The advantage of $\mathcal{A}$ is defined as

$$\mathbf{Adv}^{\text{IND-CCA2}}_{\mathcal{KEM},\mathcal{A}}(\lambda) = |\mathbf{Pr}[b' = b] - \tfrac{1}{2}|.$$

Definition 11 (IND-CCA2 of KEM). *We say that a KEM scheme $\mathcal{KEM}$ is IND-CCA2 secure, if there is no PPT adversary $\mathcal{A}$ that can win the aforementioned game with an advantage exceeding* $\mathbf{negl}(\lambda)$.

5 Designated-ciphertext Searchable Encryption (DCSE)

In this section, we formalize the system model of a DCSE scheme and its security models.

5.1 System Model

We extend the system model of Boneh *et al.*'s work [10]. In DCSE, the trapdoor is linked not only to a keyword, but also to a ciphertext. More specifically, each ciphertext has a corresponding tag that the data receiver uses along with the keyword to generate a trapdoor for the search.

Let λ be a security parameter, $\mathcal{W}$ be a keyword space, $\mathcal{C}$ be a ciphertext space, and $\mathcal{V}$ be a tag space. A DCSE scheme is a four-tuple of PPT algorithms $\mathcal{DCSE} = (\mathit{KeyGen}, \mathit{DCSE}, \mathit{Trapdoor}, \mathit{Test})$, described as follows.

- $\mathit{KeyGen}(1^\lambda)$: Taking the security parameter λ as input, this algorithm outputs a public key pk and a private key sk.
- $\mathit{DCSE}(\mathsf{pk}, w)$: Taking a public key pk, and a keyword $w \in \mathcal{W}$, this algorithm outputs a searchable ciphertext $c \in \mathcal{C}$ and a tag $v \in \mathcal{V}$ of the ciphertext.
- $\mathit{Trapdoor}(\mathsf{sk}, w', v')$: Taking a private key sk, a keyword $w' \in \mathcal{W}$, and a tag $v' \in \mathcal{V}$ of the ciphertext, this algorithm outputs a trapdoor t.
- $\mathit{Test}(c, t)$: Taking a searchable ciphertext c, and a trapdoor t as input, this algorithm outputs 1, if t and c share the same keyword and the t is actually generated from the tag corresponding to c. Otherwise, it output 0.

Definition 12 (Correctness of DCSE). *Let λ be a security parameter, $\mathcal{W}$ be a keyword space, $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathit{KeyGen}(1^\lambda)$, and (c, v) be a pair of a searchable ciphertext and its corresponding tag generated from $\mathit{DCSE}(\mathsf{pk}, w)$, where $w \in \mathcal{W}$. We say that a DCSE scheme is correct if:*

$$\mathbf{Pr}[\mathit{Test}(c, \mathit{Trapdoor}(\mathsf{sk}, w, v)) = 1] = 1 - \mathbf{negl}(\lambda).$$

5.2 Security Models

We require that the proposed DCSE scheme satisfy the following two security requirements: indistinguishability under chosen-keyword attack (IND-CKA) and indistinguishability under insider keyword guessing attack (IND-IKGA), which are modeled through the following two games executed by an adversary $\mathcal{A}$ and a challenger $\mathcal{B}$. Note that because the ability of a malicious insider exceeds that of a malicious outsider, we only consider the IND-IKGA here.

Indistinguishability under Chosen-keyword Attack. The IND-CKA security ensures that the adversary cannot obtain any information on the keyword from a ciphertext and its corresponding tag.

Game IND-CKA:

- **KeyGen.** In this stage, $\mathcal{B}$ runs the $\mathit{KeyGen}(1^\lambda)$ algorithm to generate the user's public key pk and private key sk. Then, $\mathcal{B}$ sends sk to $\mathcal{A}$.
- **Phase 1.** $\mathcal{A}$ makes a polynomially bounded number of queries to the *Trapdoor* oracle. When $\mathcal{A}$ issues such a query on (w, v), $\mathcal{B}$ returns a trapdoor t to $\mathcal{A}$ using *Trapdoor* algorithm with the private key sk.
- **Challenge.** $\mathcal{A}$ sends two challenge keywords $w_0, w_1 \in \mathcal{W}$, where w_0, w_1 have not been queried in **Phase 1**. $\mathcal{B}$ chooses a random bit $b \leftarrow \{0, 1\}$, generates the challenge ciphertext c^* and the corresponding challenge tag v^* from $\mathit{DCSE}(\mathsf{pk}, w_b)$, and returns (c^*, v^*) to $\mathcal{A}$.
- **Phase 2.** $\mathcal{A}$ can continue to ask for the *Trapdoor* oracle, same as in **Phase 1**. The only restriction is that $\mathcal{A}$ cannot issue a *Trapdoor* query on w_0 or w_1.
- **Guess.** $\mathcal{A}$ outputs its guess b'.

We say that the adversary wins the game, if $b' = b$. The advantage of $\mathcal{A}$ wins this game is defined as

$$\mathbf{Adv}^{\text{IND-CKA}}_{\mathcal{DCSE},\mathcal{A}}(\lambda) = |\mathbf{Pr}[b' = b] - \tfrac{1}{2}|.$$

Definition 13 (IND-CKA of DCSE). *We say that a DCSE scheme $\mathcal{DCSE}$ is IND-CKA secure if there is no PPT adversary $\mathcal{A}$ that can win the aforementioned game with an advantage exceeding* $\mathbf{negl}(\lambda)$.

Indistinguishability under Insider-keyword-guessing Attack. The IND-IKGA security ensures that the adversary cannot obtain any information about the keyword from a trapdoor.

Game IND-IKGA:

- **KeyGen.** In this stage, $\mathcal{B}$ runs the $KeyGen(1^\lambda)$ algorithm to generate the user's public key pk and private key sk. Then, $\mathcal{B}$ sends pk to $\mathcal{A}$.
- **Phase 1.** $\mathcal{A}$ makes a polynomially bounded number of queries to the *Trapdoor* oracle. When $\mathcal{A}$ issues such a query on (w, v), $\mathcal{B}$ returns a trapdoor t to $\mathcal{A}$ using *Trapdoor* algorithm with private key sk.
- **Challenge.** $\mathcal{A}$ sends two challenge keywords $w_0, w_1 \in \mathcal{W}$, where w_0, w_1 have not been queried in **Phase 1**. $\mathcal{B}$ first randomly chooses a tag v^* from $\mathcal{V}$. Then, it chooses a random bit $b \leftarrow \{0,1\}$ and generates the challenge trapdoor $t^* \leftarrow Trapdoor(\mathsf{sk}, w_b, v^*)$. Finally, $\mathcal{B}$ returns t^* to $\mathcal{A}$.
- **Phase 2.** $\mathcal{A}$ can continue to ask for the *Trapdoor* oracle, same as in **Phase 1**. The only restriction is that $\mathcal{A}$ cannot issue a *Trapdoor* query on w_0 or w_1.
- **Guess.** $\mathcal{A}$ outputs its guess b'.

We say that the adversary wins the game, if $b' = b$. The advantage of $\mathcal{A}$ wins this game is defined as

$$\mathbf{Adv}^{\text{IND-IKGA}}_{\mathcal{DCSE},\mathcal{A}}(\lambda) = |\mathbf{Pr}[b' = b] - \tfrac{1}{2}|.$$

Definition 14 (IND-IKGA of DCSE). *We say that a DCSE scheme $\mathcal{DCSE}$ is IND-IKGA secure, if there is no PPT adversary $\mathcal{A}$, that can win the aforementioned game with an advantage exceeding* $\mathsf{negl}(\lambda)$.

6 Efficient Generic Construction of DCSE

In this section, we first propose a generic construction of DCSE from an IND-ANON-ID-CPA secure IBE scheme and an IND-CCA2 secure KEM scheme. Then, we present rigorous proofs to demonstrate that this construction satisfies the correctness and security requirements defined in Section 5.

6.1 Generic Construction

To construct a DCSE scheme $\mathcal{DCSE}$, we first set the following parameters. Let $\mathcal{IBE} = (Setup, Extract, Enc, Dec)$ be an IND-ANON-ID-CPA IBE scheme, and $\mathcal{KEM} = (KeyGen, Encap, Decaps)$ be an IND-CCA2 secure KEM. Let $\mathcal{W}$ and $\mathcal{C}$ be the keyword space and ciphertext space of $\mathcal{DCSE}$, respectively, and let $\mathcal{K}$ be the key space of $\mathcal{KEM}$. Let $F : \mathcal{X} \to \mathcal{Y}$ be a pseudorandom generator with appropriate domain $\mathcal{X}$ and range $\mathcal{Y}$. Here, the domain $\mathcal{X}$ includes the set of any keyword $w \in \mathcal{W}$ concatenating any key $k \in \mathcal{K}$. That is, $\mathcal{X} = \{w \| k \mid w \in \mathcal{W} \wedge k \in \mathcal{K}\}$. Furthermore, let the range $\mathcal{Y}$ include an appropriate length of randomness used by the algorithm $\mathcal{IBE}.Extract$. In addition, let H be a collision-resistant hash function defined on $\{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$. We then present a generic construction of DCSE from Algorithm 1 to Algorithm 4.

Algorithm 1 $KeyGen(1^\lambda)$

Input: a security parameter λ
Output: user's key pair $(\mathsf{pk}, \mathsf{sk})$
1: $(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathcal{KEM}.KeyGen(1^\lambda)$
2: $(\mathsf{pk}_2, \mathsf{sk}_2) \leftarrow \mathcal{IBE}.Setup(1^\lambda)$
3: Set public key $\mathsf{pk} = (\mathsf{pk}_1, \mathsf{pk}_2)$, private key $\mathsf{sk} = (\mathsf{sk}_1, \mathsf{sk}_2)$
4: Output a key pair $(\mathsf{pk}, \mathsf{sk})$

In this construction, the data receiver's public key and private key are generated from the $\mathcal{IBE}.KeyGen(1^\lambda)$ and $\mathcal{KEM}.KeyGen(1^\lambda)$. To generate a searchable ciphertext c for a keyword w, the data owner first use KEM to randomly generate a key k and its corresponding encapsulation e using the public key of data receiver, $(e, k) \leftarrow \mathcal{KEM}.Encaps(\mathsf{pk})$. He then runs $f \leftarrow \mathcal{F}(w \| k)$ to obtain a pseudorandom which can be considered as an "identity". Next, he chooses a random value r and encrypts it using identity f, that is $\mathsf{ct} \leftarrow \mathcal{IBE}.Enc(\mathsf{pk}, f, r)$, and computes a hash value $h = H(\mathsf{ct}, r)$. Finally, he outputs a searchable ciphertext $c = (\mathsf{ct}, h)$ and a tag $v = e$. Here, we note that

Algorithm 2 $DCSE(\mathsf{pk}, w)$

Input: data receiver's public key $\mathsf{pk} = (\mathsf{pk}_1, \mathsf{pk}_2)$ and a keyword $w \in \mathcal{W}$
Output: a ciphertext c and the tag v of the ciphertext
1: $(e, k) \leftarrow \mathcal{KEM}.Encaps(\mathsf{pk}_1)$
2: Randomly choose $r \leftarrow \{0,1\}^*$
3: $f \leftarrow F(w\|k)$
4: $\mathsf{ct} \leftarrow \mathcal{IBE}.Enc(\mathsf{pk}_2, f, r)$
5: Compute $h = H(\mathsf{ct}, r)$
6: Output a ciphertext $c = (\mathsf{ct}, h)$ and tag $v = e$

if data owner wants to encrypt different keywords for the same data receiver, he can re-use the same key k without re-running the encapsulation algorithm to reduce the computation cost.

To generate a trapdoor to search a ciphertext encrypted by a keyword w, the data receiver first obtains the key hidden in the tag, $k \leftarrow \mathcal{KEM}.Decaps(\mathsf{sk}, v)$, and computes "identity" $f \leftarrow F(w\|k)$. He then generates a trapdoor t for the identity f, $t \leftarrow \mathcal{IBE}.Extract(\mathsf{sk}, f)$, and sends it to the server.

Algorithm 3 $Trapdoor(\mathsf{sk}, w, v)$

Input: user's private key $\mathsf{sk} = (\mathsf{sk}_1, \mathsf{sk}_2)$, a keyword $w \in \mathcal{W}$, and its corresponding tag $v = e$
Output: a trapdoor t for keyword w and tag v
1: $k \leftarrow \mathcal{KEM}.Decaps(v, \mathsf{sk}_1)$
2: **if** $k = \perp$ **then**
3: Set trapdoor t to be an invalid symbol $\perp$
4: **else**
5: $f \leftarrow F(w\|k)$
6: Set trapdoor $t \leftarrow \mathcal{IBE}.Extract(\mathsf{sk}_2, f)$
7: **end if**
8: Output a trapdoor t

After receiving the trapdoor, because the ciphertext is actually encrypted by an identity, the server first decrypts the ciphertext to obtain the plaintext $r \leftarrow \mathcal{IBE}.Dec(t, \mathsf{ct})$. The can then check whether $H(\mathsf{ct}, r) = h$. If it matches, output 1. Otherwise, output 0. In a real-world scenario, the data receiver could not only sends the trapdoor but also sends an additional tag that she or he uses. The server can then use the tag as an index to quickly find any ciphertext that might need to be tested.

Algorithm 4 $Test(c, t)$

Input: a ciphertext $c = (\mathsf{ct}, h)$, and a trapdoor t
Output: 1 if t matches c or 0 otherwise
1: **if** $t = \perp$ **then**
2: Output 0
3: **else**
4: $r \leftarrow \mathcal{IBE}.Dec(t, \mathsf{ct})$
5: Output 1 if $H(\mathsf{ct}, \mathsf{r}) = h$ and 0, otherwise
6: **end if**

6.2 Correctness and Security Proofs

Theorem 1. *The proposed construction is correct, according to Definition 12.*

Proof (Proof of Theorem 1). Let $(c = (\mathsf{ct}, h), v) \leftarrow DCSE(\mathsf{pk}, w)$ be a valid ciphertext and its corresponding tag, and let $t \leftarrow Trapdoor(\mathsf{sk}, w, v)$ be a valid trapdoor, where $(\mathsf{pk}, \mathsf{sk}) \leftarrow KeyGen(1^\lambda)$. Because t is actually the private key of identity $F(w\|k)$ in the IBE scheme, and ct is a ciphertext that encrypts a random value r using identity $F(w\|k)$. With the correctness of the IBE scheme (Definition 7), one can obtain $r \leftarrow \mathcal{IBE}.Dec(t, \mathsf{ct})$ with overwhelming probability. Therefore, $H(\mathsf{ct}, r) = h$; thus, we have $Test(c, t) = 1$.

In the following, we prove that the proposed generic construction is IND-CKE secure and IND-IKGA secure. At a high level, our strategy is to use a series of games: we gradually modify the structure of the challenge phase so that the challenge does not contain any keywords in the final game. Therefore, the advantages of an attacker for winning the IND-CKE and IND-IKGA games are no higher than mere speculation.

Theorem 2. *The proposed scheme $\mathcal{DCSE}$ is IND-CKA secure if the underlying KEM scheme $\mathcal{KEM}$ is IND-CCA2 secure, the IBE scheme $\mathcal{IBE}$ is IND-ANON-ID-CPA secure.*

Proof (Proof of Theorem 2). We prove Theorem 2 using a sequence of games, defined as follows.

- $\mathbf{Game}_0$: This is the original IND-CKA game, as shown in Section 5.2.
- $\mathbf{Game}_1$: We now make a minor change to the aforementioned game. Rather than obtain k from $\mathcal{KEM}.Encaps(\mathsf{pk}_1)$, we choose k' from the range of the output of $\mathcal{KEM}.Encaps(\mathsf{pk}_1)$ randomly.
- $\mathbf{Game}_2$: We now transform $\mathbf{Game}_1$ into $\mathbf{Game}_2$. In this game, let $f = F(w\|k')$; we substitute the value $\mathsf{ct} \leftarrow \mathcal{IBE}.Enc(f, r)$ with $\mathsf{ct} \leftarrow \mathcal{IBE}.Enc(f', r)$, where f' is chosen randomly from $\mathcal{Y}$, and $\mathcal{Y}$ is the output range of F.

Let $\mathbf{Adv}_i$ denote the adversary's advantage for winning in $\mathbf{Game}_i$. We have the following claims.

Claim. For all the PPT algorithms $\mathcal{A}_{01}$, $|\mathbf{Adv}_0 - \mathbf{Adv}_1|$ is negligible, if the underlying KEM scheme $\mathcal{KEM}$ is IND-CCA2 secure.

Proof (Proof of Claim 6.2). Suppose that there exists an adversary $\mathcal{A}_{01}$ such that $|\mathbf{Adv}_0 - \mathbf{Adv}_1|$ is non-negligible, then, there exists another challenger $\mathcal{B}_{01}$ that can win the IND-CCA2 game in the underlying KEM scheme $\mathcal{KEM}$ with non-negligible advantage.

- **KeyGen.** $\mathcal{B}_{01}$ first invokes the IND-CCA2 game of $\mathcal{KEM}$ to obtain pk_1. Next, $\mathcal{B}_{01}$ computes $(\mathsf{pk}_2, \mathsf{sk}_2) \leftarrow \mathcal{IBE}.Setup(1^\lambda)$. Finally, $\mathcal{B}_{01}$ sets the public key $\mathsf{pk} = (\mathsf{pk}_1, \mathsf{pk}_2)$, and sends pk to $\mathcal{A}_{01}$.
- **Phase 1.** In this phase, $\mathcal{A}_{01}$ can make polynomially many *Trapdoor* queries with (pk, w, v), and $\mathcal{B}_{01}$ responds as follows. $\mathcal{B}_{01}$ first invokes $\mathcal{KEM}.Decaps$ oracle on v. The oracle returns an invalid symbol $\perp$ or a valid key k. If the oracle returns $\perp$, $\mathcal{B}_{01}$ also responds with $\perp$ to $\mathcal{A}_{01}$. Otherwise, $\mathcal{B}_{01}$ computes $f \leftarrow F(w\|k)$ and $t \leftarrow \mathcal{IBE}.Extract(\mathsf{sk}_2, f)$. Finally, t is returned to $\mathcal{A}_{01}$.
- **Challenge.** $\mathcal{A}_{01}$ sends two challenge keywords $w_0, w_1 \in \mathcal{W}$, where w_0, w_1 have not been queried in **Phase 1**. After receiving these challenge keywords, $\mathcal{B}_{01}$ chooses a random bit $b \leftarrow \{0,1\}$, and runs the following steps:
  - Invoke the Challenge phase of the IND-CCA2 game to obtain the challenge ciphertext (e^*, k^*).
  - Pick $r^* \leftarrow \{0,1\}^*$
  - Compute $f^* \leftarrow F(w_b\|k^*)$.
  - Compute $\mathsf{ct}^* \leftarrow \mathcal{IBE}.Enc(f^*, r^*)$
  - Compute $h^* = H(\mathsf{ct}^*, r^*)$.
  - Set $v^* = e^*$.

  Then, $\mathcal{B}_{01}$ returns $(c^* = (\mathsf{ct}^*, h^*), v^*)$ to $\mathcal{A}_{01}$.
- **Phase 2.** $\mathcal{A}_{01}$ can continue to make *Trapdoor* queries, same as in **Phase 1**. The only restriction is that $\mathcal{A}_{01}$ cannot make a *Trapdoor* query on w_0 or w_1.
- **Guess.** $\mathcal{A}_{01}$ outputs its guess b'. Then $\mathcal{B}_{01}$ outputs b'.

Note that, if k^* is a valid key, $\mathcal{B}_{01}$ gives the view of $\mathbf{Game_0}$ to $\mathcal{A}_{01}$; if k^* is a random element, then $\mathcal{B}_{01}$ gives the view of $\mathbf{Game}_1$ to $\mathcal{A}_{01}$. If $|\mathbf{Adv}_0 - \mathbf{Adv}_1|$ is non-negligible, $\mathcal{B}_{01}$ must also have non-negligible advantage against the IND-CCA2 game of the underlying KEM scheme. Therefore,

$$|\mathbf{Adv}_0 - \mathbf{Adv}_1| \leq \mathbf{nege}(\lambda).$$

Claim. For all the PPT algorithms, $\mathcal{A}_{12}$, $|\mathbf{Adv}_1 - \mathbf{Adv}_2|$ is negligible, if the underlying IBE scheme $\mathcal{IBE}$ is IND-ANON-ID-CPA.

Proof (Proof of Claim 6.2). Suppose that there is an adversary $\mathcal{A}_{12}$ such that $|\mathbf{Adv}_1 - \mathbf{Adv}_2|$ is non-negligible, then, there exists another challenger $\mathcal{B}_{12}$ that can win the IND-ANON-ID-CPA game of the underlying IBE scheme $\mathcal{IBE}$ with non-negligible advantage. $\mathcal{B}_{12}$ constructs a hybrid game interacting with an adversary $\mathcal{A}_{12}$ as follows:

- **KeyGen.** $\mathcal{B}_{12}$ first invokes the IND-ANON-ID-CPA game of $\mathcal{IBE}$ to obtain pk_2; then, $\mathcal{B}_{12}$ computes $(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathcal{KEM}.KeyGen(1^\lambda)$. Finally, $\mathcal{B}_{12}$ sets the public key $\mathsf{pk} = (\mathsf{pk}_1, \mathsf{pk}_2)$, and sends pk to $\mathcal{A}_{12}$.

- **Phase 1.** In this phase, $\mathcal{A}_{12}$ is able to make polynomially many *Trapdoor* queries with the (pk, w, v), and $\mathcal{B}_{12}$ responds as follows. $\mathcal{B}_{12}$ first obtains $k \leftarrow \mathcal{KEM}.Decaps(v, \mathsf{sk}_1)$. If k is an invalid symbol $\perp$, $\mathcal{B}_{12}$ returns $\perp$ to $\mathcal{A}_{12}$. Otherwise, $\mathcal{B}_{12}$ invokes $\mathcal{IBE}.Extract$ oracle on $F(k\|w)$ to obtain a trapdoor t. Finally, $\mathcal{B}_{12}$ sends t to $\mathcal{A}_{12}$.
- **Challenge.** $\mathcal{A}_{12}$ sends two challenge keywords w_0, w_1, where w_0, w_1 have not been queried in **Phase 1**. $\mathcal{B}_{12}$ chooses a random bit $b \leftarrow \{0,1\}$, and performs the following steps:
  - Compute $(e^*, k^*) \leftarrow \mathcal{KEM}.Encaps(\mathsf{pk}_1)$.
  - Randomly choose k'^* from the range of the output of $\mathcal{KEM}.Encaps(\mathsf{pk}_1)$.
  - Randomly choose $f' \leftarrow \mathcal{Y}$.
  - Pick $r^* \leftarrow \{0,1\}^*$.
  - Invoke the Challenge phase of the IND-ANON-ID-CPA game using $F(w_b\|k'^*, r^*)$ and (f', r^*) to obtain the challenge ciphertext ct^*.
  - Compute $h^* = H(\mathsf{ct}^*, r^*)$.
  - Set $v^* = e^*$.

  Then, $\mathcal{B}_{12}$ returns $(c^* = (\mathsf{ct}^*, h^*), v^*)$ to $\mathcal{A}_{12}$.
- **Phase 2.** $\mathcal{A}_{12}$ can continue to make *Trapdoor* queries, similar to **Phase 1**. The only restriction is that $\mathcal{A}_{12}$ cannot make a *Trapdoor* query on w_0 or w_1.
- **Guess.** $\mathcal{A}_{12}$ outputs its guess b'. Then, $\mathcal{B}_{12}$ outputs b'.

Note that if ct^* is generated from $(F(w_b\|k'^*), r^*)$, $\mathcal{B}_{12}$ gives the view of $\mathbf{Game}_1$ to $\mathcal{A}_{12}$; if ct^* is generated from (f', r^*), then $\mathcal{B}_{12}$ gives the view of $\mathbf{Game}_2$ to $\mathcal{A}_{12}$. If $|\mathbf{Adv}_1 - \mathbf{Adv}_2|$ is non-negligible, $\mathcal{B}_{12}$ must also have non-negligible advantage in the IND-ANON-ID-CPA game of the underlying IBE scheme. Therefore,

$$\mathbf{Adv}_1 - \mathbf{Adv}_2 \leq \mathbf{negl}(\lambda).$$

Claim. $\mathbf{Adv}_2 = 0$.

Proof (Proof of Claim 6.2). The proof of Claim 6.2 is intuitive. Because the ciphertext c^* is irrelevant to the keywords w_0, w_1, the ciphertext reveals nothing about the information of the keywords. The adversary $\mathcal{A}_2$ can only return b' by guessing. Therefore,

$$\mathbf{Adv}_2 = 0.$$

Combining Claim 6.2, Claim 6.2, and Claim 6.2, we can conclude that the advantages of the adversary of the three adjacent games are negligibly close, and thus $|\mathbf{Adv}_0 - \mathbf{Adv}_2|$ is negligibly close to 0. This completes the proof of Theorem 2.

Theorem 3. *The proposed scheme is IND-IKGA secure, if the underlying KEM scheme $\mathcal{KEM}$ is IND-CCA2 secure, and pseudorandom generator F satisfies pseudorandomness.*

Proof (Proof of Theorem 3). We prove Theorem 3 through a sequence of games, defined as follows.

- $\mathbf{Game}_0$: This is the original IND-IKGA game, as shown in Section 5.2.
- $\mathbf{Game}_1$: This game is identical to $\mathbf{Game}_0$, except that k is randomly chosen from the output range of $\mathcal{KEM}.Encaps(\mathsf{pk}_1)$, rather than being computed from $\mathcal{KEM}.Encaps(\mathsf{pk}_1)$.
- $\mathbf{Game}_2$ This game is the same as $\mathbf{Game}_1$, except that f is chosen randomly from $\mathcal{Y}$, instead of being computed from $F(w_b\|k)$.

Let Adv_i denote the adversary's advantage in $\mathbf{Game}_i$. We have the following claims.

Claim. For all the PPT algorithms, $\mathcal{A}_{01}$, $|\mathbf{Adv}_0 - \mathbf{Adv}_1|$ is negligible, if the underlying KEM scheme $\mathcal{KEM}$ is IND-CCA2 secure.

Proof (Proof of Claim 6.2). Suppose that there exists an adversary $\mathcal{A}_{01}$ such that $|\mathbf{Adv}_0 - \mathbf{Adv}_1|$ is non-negligible, then, there exists another challenger $\mathcal{B}_{01}$ that can win the IND-CCA2 game of the underlying KEM scheme $\mathcal{KEM}$ with non-negligible advantage.

- **KeyGen.** $\mathcal{B}_{01}$ first invokes the IND-CCA2 game of $\mathcal{KEM}$ to obtain pk_1. Next, $\mathcal{B}_{01}$ computes $(\mathsf{pk}_2, \mathsf{sk}_2) \leftarrow \mathcal{IBE}.Setup(1^\lambda)$. Finally, $\mathcal{B}_{01}$ sets the public key $\mathsf{pk} = (\mathsf{pk}_1, \mathsf{pk}_2)$, and sends pk to $\mathcal{A}_{01}$.
- **Phase 1.** In this phase, $\mathcal{A}_{01}$ can make polynomially many *Trapdoor* queries with (pk, w, v), and $\mathcal{B}_{01}$ responses as follows. $\mathcal{B}_{01}$ first invokes $\mathcal{KEM}.Decaps$ oracle on v. The oracle returns an invalid symbol $\perp$ or a valid key k. If the oracle returns $\perp$, $\mathcal{B}_{01}$ also responses $\perp$ to $\mathcal{A}_{01}$. Otherwise, $\mathcal{B}_{01}$ computes $f = F(w\|k)$ and $t \leftarrow \mathcal{IBE}.Extract(\mathsf{sk}_2, f)$. Finally, t is returned to $\mathcal{A}_{01}$.

- **Challenge.** $\mathcal{A}_{01}$ sends two challenge keywords $w_0, w_1 \in \mathcal{W}$, where w_0, w_1 have not been queried in **Phase 1**. $\mathcal{B}_{01}$ chooses a random bit $b \leftarrow \{0,1\}$, and runs the following steps:
  - Invoke the Challenge phase of the IND-CCA2 game to obtain the challenge (e^*, k^*).
  - Compute $f^* \leftarrow F(w_b \| k^*)$.
  - Compute $t^* \leftarrow \mathcal{IBE}.Extract(\mathsf{sk}_2, f)$.

  Then, $\mathcal{B}_{01}$ returns t^* to $\mathcal{A}_{01}$.
- **Phase 2.** $\mathcal{A}_{01}$ can continue to make *Trapdoor* queries, same as in **Phase 1**. The only restriction is that $\mathcal{A}_{01}$ cannot make a *Trapdoor* query on w_0 or w_1.
- **Guess.** $\mathcal{A}_{01}$ outputs its guess b'. Then, $\mathcal{B}_{01}$ outputs b'.

Note that if k^* is a valid key, $\mathcal{B}_{01}$ gives the view of $\mathbf{Game}_0$ to $\mathcal{A}_{01}$; if k^* is a random element, then, $\mathcal{B}_{01}$ gives the view of $\mathbf{Game}_1$ to $\mathcal{A}_{01}$. If $|\mathbf{Adv}_0 - \mathbf{Adv}_1|$ is non-negligible, $\mathcal{B}_{01}$ must also have non-negligible advantage in the IND-CCA2 game. Therefore,

$$|\mathbf{Adv}_0 - \mathbf{Adv}_1| \leq \mathbf{negl}(\lambda).$$

Claim. For all the PPT algorithms, $\mathcal{A}_{12}$, $|\mathbf{Adv}_1 - \mathbf{Adv}_2|$ is negligible, if F is a secure pseudorandom generator.

Proof (Proof of Claim 6.2). We prove the claim by describing a PPT reduction algorithm $\mathcal{B}_{12}$ that plays a pseudorandom generator security game. Given a challenge string $T \in \mathcal{Y}$ and the description of a pseudorandom generator F, $\mathcal{B}_{12}$ constructs a hybrid game, interacting with an adversary $\mathcal{A}_{12}$ as follows.

- **KeyGen.** $\mathcal{B}_{12}$ chooses the public parameters, as described in Section 6.1, except that, instead of choosing a proper pseudorandom generator from the pseudorandom generator family, $\mathcal{B}_{12}$ sets F as the public parameter. Then, $\mathcal{B}_{12}$ generates the key pair $(\mathsf{pk}, \mathsf{sk}) \leftarrow KeyGen(1^\lambda)$, and sends pk to $\mathcal{A}_{12}$. Note that $\mathcal{B}_{12}$ has full control of the private key sk.
- **Phase 1.** In this phase, $\mathcal{A}_{12}$ can make polynomially many *Trapdoor* queries using (pk, w, v). Due to the knowledge of sk, $\mathcal{B}_{12}$ answers the queries by simply running the *Trapdoor* algorithm.
- **Challenge.** $\mathcal{A}_{12}$ sends two challenge keywords $w_0, w_1 \in \mathcal{W}$, where w_0, w_1 have not been queried in **Phase 1**. $\mathcal{B}_{12}$ chooses a random bit $b \leftarrow \{0,1\}$, and runs the following steps:
  - Set $f^* = T$.
  - Compute $t^* \leftarrow \mathcal{IBE}.Extract(\mathsf{sk}_2, f^*)$.

  Then, $\mathcal{B}_{12}$ returns t^* to $\mathcal{A}_{12}$.

Note that, if T is generated from F, $\mathcal{B}_{12}$ provides the view of $\mathbf{Game}_1$ to $\mathcal{A}_{12}$; if T is a random string sampled from $\mathcal{Y}$, then $\mathcal{B}_{12}$ provides the view of $\mathbf{Game}_2$ to $\mathcal{A}_{12}$. If $|Adv_1 - Adv_2|$ is non-negligible, $\mathcal{B}_{12}$ must also have non-negligible advantage against the pseudorandom generator security game. Therefore,

$$|\mathbf{Adv}_1 - \mathbf{Adv}_2| \leq \mathbf{negl}(\lambda).$$

Claim. $\mathbf{Adv}_2 = 0$.

Proof (Proof of Claim 6.2). The proof of Claim 6.2 is intuitive. Because the trapdoor t^* is irrelevant to the keywords, w_0 and w_1, the trapdoor reveals nothing about the information of the keywords. The adversary $\mathcal{A}_2$ can only return b' by guessing. Therefore,

$$\mathbf{Adv}_2 = 0.$$

Combining Claim 6.2, Claim 6.2, and Claim 6.2, we can conclude that the advantages of the adversary of three adjacent games are negligibly close, and, thus, $|\mathbf{Adv}_2 - \mathbf{Adv}_0|$ is negligibly close to 0. This completes the proof of Theorem 3.

7 Efficient Instantiation and Comparison

In this section, we first propose an DCSE instantiation based on NTRU lattices. Then, we compare different aspects in our instantiation with other state-of-the-art schemes.

Table 1: Comparison with related schemes on the basis of security properties

Schemes	Quantum-resistance	IKGA security
[10]	✗	✗
[9]	✓	✗
[47]	✓	✗
[48]	✓	✗
[35]	✓	✓
Ours	✓	✓

Table 2: Comparison with related schemes on the basis of Key size, Trapdoor size, and Ciphertext size (in bytes). Note that $|ID|$ refers to the length of user identity.

Schemes	PK	SK	Trapdoor	Ciphertext
[10]	0.38	0.19	0.38	0.57
[9]	27.2	35	27	52
[47]	$\|ID\|$	560128	113	113
[48]	3657.05	139325.1	142.86	14.28
[35]	3657.42	139325.1	71.42	57.14
Ours	31.88	59.98	38.98	23

Table 3: Time taken (operations per second) by different operations of KeyGen (key generation), Encryption (PEKS in [10, 9] and DCSE in our scheme), Extract, and Test.

Scheme	KeyGen	Encryption	Extract	Test
[10]	84.88	186.48	17.41	100908.17
[9]	0.10	349.28	67.42	174.64
Ours	26.56	3224.35	739.06	63451.77

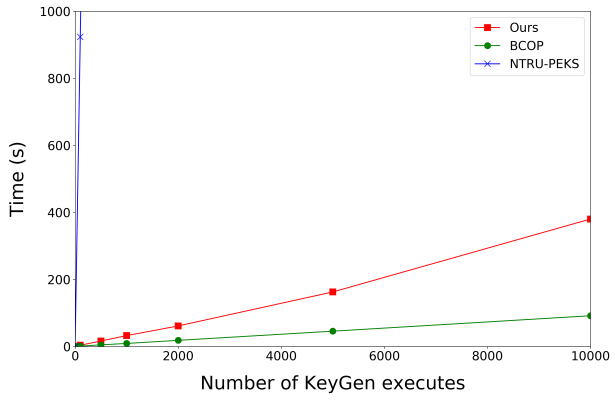

Fig. 1: Time taken by the key generation algorithm.

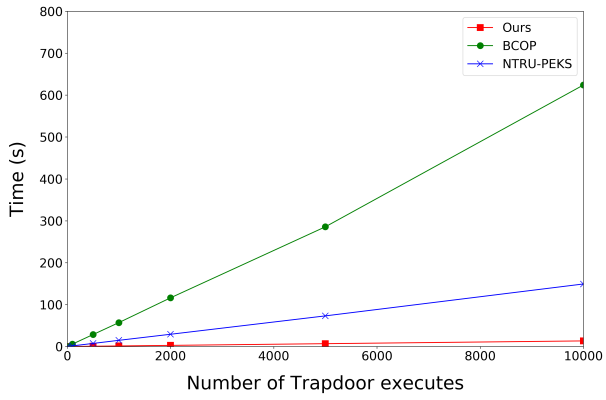

Fig. 2: Time taken by the extract algorithm.

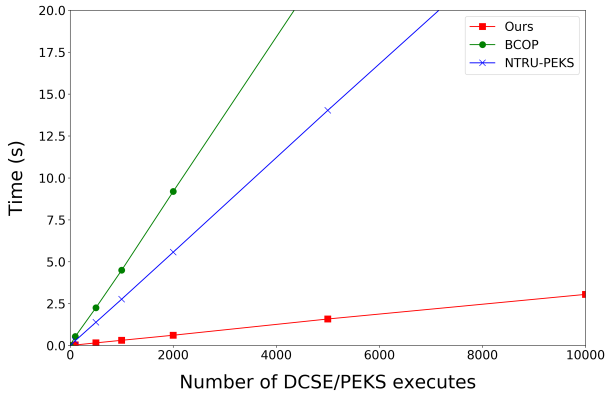

Fig. 3: Time taken by DCSE / PEKS algorithm.

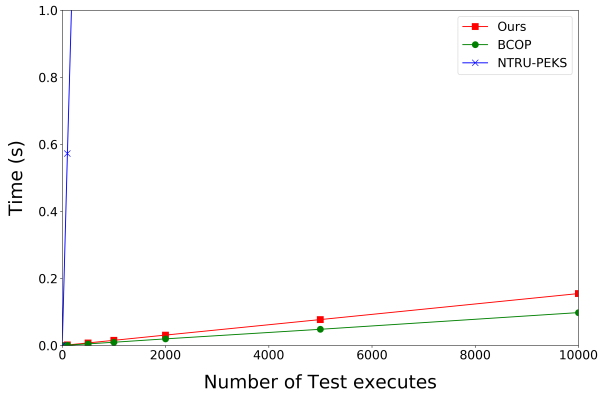

Fig. 4: Time taken by the test algorithm.

7.1 Efficient Instantiation

Our instantiation utilizes the IBE of Ducas *et al.* [22] and KEM of Hülsing *et al.* [29] (hereafter, referred to as DLP-IBE and HRSS-KEM, respectively).

The DLP-IBE is the first lattice-based IBE scheme with practical parameters. Its security is based on the NTRU and Ring-LWE assumptions. In addition, Behnia *et al.* have also proven that the DLP-IBE is IND-ANON-ID-CPA secure [9]. The first implementation of the DLP-IBE was provided by Ducas [3], written in C++ based on the NTL library [2]. Although this implementation is very efficient, it is merely a proof of concept (PoC) without any optimization. To improve efficiency, McCarthy *et al.* propose a practical implementation of the DLP-IBE, written in ANSI C, using the number theoretic transform (NTT) optimizations [36].

The HRSS-KEM is a candidate cryptographic KEM in the Round 2 of the National Institute for Standards and Technology's Post-Quantum Project [1]. In the work, Hülsing *et al.* first provide a OW-CPA secure NTRU-based encryption scheme with optimized parameters; then, they transform the scheme into a IND-CCA2 secure NTRU-based KEM under quantum-accessible random oracle model.

For concrete instantiation, we use SHA256 as a secure hash function, and symmetric encryption AES-256 as a pseudorandom generator. We used open source project software for DLP-IBE [5] and HRSS-KEM [4] to test the feasibility of our DCSE scheme on an Intel Core i7-8700 3.2-GHz CPU with 10G of RAM. For the DLP-IBE, we selected parameters $n = 1024, q \approx 2^{27}$ for 192-bit security level, and for HRSS-KEM, we selected parameters $n = 701, p = 3, q = 8192$ for 128-bit security level.

7.2 Comparison

To compare the proposed scheme with other state-of-the-art schemes, we set the parameters as follows. For the pairing-based PEKS scheme in [10], we chose the 160-bit group order and 2048-bit group elements $\mathcal{G}, \mathcal{G}_T$. For the NTRU-based PEKS scheme in [9], we chose $n = 1024, q = 2^{27}$ for 192-bit security level. For the LWE-based PEKS schemes in [47, 35, 48], we adopted the same secure parameter as in [47], that is $n = 256$, dimension $m = 9753$, and prime $q = 4093$. In addition, we set the number of distinct keywords $k = 1$ and unusual keywords $k' = 1$ for [35], and the security level $l = 10$ for [48].

Table 1 compares of our scheme with other schemes on the basis of its security properties. Only Mao *et al.*'s work [35] is quantum-resistant and IKGA secure. However, as illustrated in Table 2, the scheme's overly large key sizes make it impractical. We also note that Mao *et al.*'s scheme require another server to execute test algorithm; thus, the computation overhead is increased. Furthermore, compared with Mao's schemes [35], our public and private key sizes is 1/115 times smaller.

In Table 3, we further compare our instantiation with other two practical PEKS schemes [10, 9] on the basis of efficiency. Compared with [10], although our instantiation is 0.31x and 0.62x slower than that of the KeyGen and Test algorithms, respectively, our instantiation is 17x and 42x faster than those of the Encrypt and Extract algorithms, respectively. As for [9], our instantiation is 245x, 9x, 11x, and 363x faster than those of the KeyGen, Encrypt, Extract, and Test algorithms, respectively. Additionally, we carefully experimented with the time required for the algorithms under different execution times (100, 500, 1000, 2000, 5000, 10000), the results are presented in Figures 1 to 4.

8 Conclusions

This paper proposed a new cryptographic primitive, DCSE, to counter IKGA in public key searchable encryption. We first provided a generic formulation of DCSE using an IND-ANON-ID-CPA secure IBE and an IND-CCA2 secure KEM, and then proved its security in the standard model. Furthermore, we provided a quantum-resistant instantiation from NTRU lattices utilizing the DLP-IBE and HRSS-KEM. In conclusion, this paper provides a novel solution to IKGA in a searchable encryption. In addition to yielding interesting theoretical results, the proposed scheme is notably more efficient and safe compared with other state-of-the-art schemes.

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