Security of Blind Signatures Revisited

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Abstract. We revisit the definition of unforgeability of blind signatures as proposed by Pointcheval and Stern (Journal of Cryptology 2000). Surprisingly, we show that this established definition falls short in two ways of what one would intuitively expect from a secure blind signature scheme: It is not excluded that an adversary submits the same message m twice for signing, and then produces a signature for $m' \neq m$. The reason is that the forger only succeeds if *all* messages are distinct. Moreover, it is not excluded that an adversary performs k signing queries and produces signatures on $k+1$ messages as long as *each* of these signatures does not pass verification with probability 1.

Finally, we proposed a new definition, honest-user unforgeability, that covers these attacks. We give a simple and efficient transformation that transforms any unforgeable blind signature scheme (with deterministic verification) into an honest-user unforgeable one.

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1 Introduction

Blind signature schemes have been suggested by Chaum [Cha83, Cha84]. Roughly speaking, this widely-studied primitive allows a signer to interactively issue signatures for a user such that the signer learns nothing about the message being signed (*blindness*) while the user cannot compute any additional signature without the help of the signer (*unforgeability*). Typical applications of blind signatures include e-cash, where a bank signs coins withdrawn by users, and e-voting, where an authority signs public keys that voters later use to cast their votes. Another application of blind signature schemes are anonymous credentials, where the issuing authority blindly signs a key [Bra00, CG08]. Very recently, Microsoft introduced a new technology called U-Prove to "overcome the long standing dilema between identity assurance and privacy" [Bjo10, UP11]. Their technology uses as a central building block blind signatures [Bjo10, BP11].

There are two main security requirements for blind signature schemes. First, the scheme should be blind. That is, a malicious signer should not be able to link the final signatures output by the user to the individual interactions with the user. In other words, the signer cannot tell which session of the signing protocol corresponds to which message. Second, the scheme should be unforgeable. That is, an adversary, even if he can impersonate the user and interact freely with the signer, should not be able to produce signatures on messages except for those that the signer signed. It is the notion of unforgeability we are concerned with in this paper.

A formal definition of the unforgeability of blind signatures schemes (or generally interactive signature schemes) has been proposed by [PS00]. Roughly, their definition states that an adversary that interacts k times with the adversary cannot produce valid signatures on more than k different messages.[3] At this point, one may wonder why the definition of unforgeability does not just require that the adversary cannot output a signature for m unless there was an interaction with the signer in which m was queried. The reason is that in general, it is not well-defined which message is queried in a given interaction. The message is not sent in clear, and it might be even information-theoretically impossible to tell from an interaction which message is being signed.[4] Thus, in order to be able to tell which message is signed in a given interaction, we would have to add some kind of extractability to the security definition; this would be an additional requirement on the protocols and make them more complex.

Insecurity of unforgeable blind signatures schemes. Unfortunately, however, the definition of unforgeability might not cover all cases in which one would intuitively expect unforgeability to be sufficient. We illustrate this by the following toy protocol:

Consider the setting of an online video store such as Netflix. In our setting, we assume that the store is implemented via two entities, the content provider and the reseller. We assume that the contract between client and reseller is a flatrate that allows the client to download a fixed number of movies. For privacy reasons, we do not wish the reseller to know which movies the client actually watches. On the other hand, we wish to ensure that underage clients can only download

[3] There is also a variant called strong unforgeability which requires that the adversary cannot produce more than k different message/signature pairs. In particular, this means that the adversary wins even if he produces additional signatures for an already signed message. In this work, we focus on the weaker notion.

[4] This might be the case when signing a message m is implemented by signing an information-theoretically hiding commitment on m.

movies suitable for their age. To achieve this, we introduce another (trusted) entity, the parental control server whose job it is to work as a proxy between reseller and client and to ensure that the client only obtains appropriate movies. Then, to download a movie X, the client first sends X to the parental control server. If X is appropriate for the client, the parental control server then runs a blind signature scheme with the reseller to obtain a signature σ on X (the blind signature is used to protect the privacy of the client, there is no need for the reseller to know which movies the client watches). Then σ is sent to the client, and the client uses σ to download X from the content provider. (We assume that all communication is suitably authenticated.)

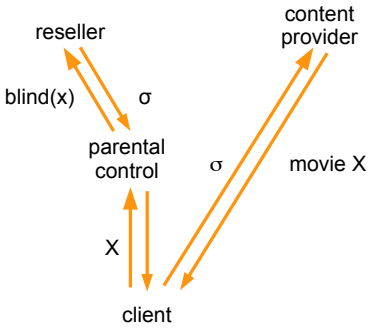

Fig. 1. Setting of an online video store.

At a first glance, it seems that this protocol is secure. In particular, the client will not be able to download a movie that is not approved by the parental control server. It turns out, however, that the client can cheat the parental control server: Assume the client twice requests a signature on some harmless movie X. He will then obtain two signatures σ_1, σ_2 on X from the parental control server. Then, given σ_1 and σ_2, the client might be able to compute a signature on an adult movie Y that has not been approved by the parental control server.

It seems that unforgeability should forbid the possibility of such an attack. But it does not. From the point of view of the signer, two signing queries have been performed, and finally signatures on two different messages X and Y have been produced. This does not violate the definition of unforgeability. In fact, we show in Section 4.2 that blind signature schemes exist that allow such attacks but that are still unforgeable.

What went wrong? The definition of unforgeability covers *only partially* the case that the user of the scheme is honest. It only ensures that the number of signed messages is not greater than the number of interactions with the signer. Only considering the number of messages but not their content is fine from the signer's point of view who is not allowed to know the messages anyway. It is not, however, fine from the user's point of view. If the user signs some messages $m_1, \ldots, m_k$ (by interacting with the signer), he expects that no signature on some different message m' can be computed from his signatures. We believe that settings in which the user is honest are natural, and that the definition of unforgeability should cover this case. We thus propose a new definition, honest-user unforgeability, which is a strengthening of unforgeability.

Definition 1 (Honest-user unforgeability – informal). *If an adversary performs k direct interactions with the signer, and requests signatures for the message $m_1, \ldots, m_n$ from the user (which produces these signatures by interacting with the signer), then the adversary cannot produce signatures for pairwise distinct messages $m_1^*, \ldots, m_{k+1}^*$ with $\{m_1^*, \ldots, m_{k+1}^*\} \cap \{m_1, \ldots, m_n\} = \varnothing$.*

Notice that this definition also covers the hybrid case in which the adversary interacts with an honest user and the signer simultaneously. Alternatively, one could also require that security in each of the setting individually: Security when there is no honest user (that is, the normal definition of unforgeability), and security when the adversary may not query the signer directly (we call this $\mathcal{S}+\mathcal{U}$-unforgeability). We show in Section 6 that requiring these variants of security individually leads to a strictly weaker security notion. Notice that $\mathcal{S}+\mathcal{U}$-unforgeability would be sufficient to solve the problem in our video store example. It seems, however, restrictive to assume that in all

protocols, there will always be only either queries from honest users or only from dishonest users but never from both in the same execution.

Achieving honest-user unforgeability. We show that any unforgeable blind signature scheme can be converted into a honest-user unforgeable blind signature scheme. The transformation is very simple and efficient: Instead of signing a message m, in the transformed scheme the user signs the message (m, r) where r is some randomness. Furthermore, we show that if a scheme is already strongly unforgeable, then it is strongly honest-user unforgeable (as long as the original scheme is *randomized* which holds for most signature schemes).

Insecurity with probabilistic verification. Most interactive and non-interactive signature schemes have a deterministic verification algorithm. In general, however, having a deterministic verification is not a necessity. Yet, when we allow a probabilistic verification algorithm (and this is usually not excluded), both the definition of unforgeability as well as the definition of honest-user unforgeability are subject to an attack: Consider again our video store example. Let λ denote the security parameter. Fix a polynomial $p = p(\lambda) > \lambda$. Assume that the parental control server and the client are malicious and collude. The parental control server interacts with the reseller λ times, and produces p "half-signatures" on movie names $X_1, \ldots, X_p$. Here, a half-signature means a signature that passes verification with probability $\frac{1}{2}$. Then the client can then download the movies $X_1, \ldots, X_n$ from the content provider. (If in some download request, a half-signature does not pass verification, the client just retries his request.) Thus the client got p movies, even if his flatrate only allows for downloading λ movies.

Can this happen? It seems that unforgeability would exclude this because $p > \lambda$ signatures were produced using λ queries to the signer. In the definition of unforgeability, however, the adversary succeeds if it outputs $p > \lambda$ signatures such that *all* signatures pass verification. However, the signatures that are produced are half-signatures: That is, the probability that all $p > \lambda$ signatures pass the verification simultaneously is negligible! Thus, producing more than λ half-signatures using λ queries would not be considered an attack by the definition of unforgeability. In Section 5, we show that blind signature schemes exist that allow such attacks but that satisfy the definition of unforgeability. The same applies to honest-user unforgeability as described so far; we thus need to augment the definition further.

There are two solutions to this problem. One is to explicitly require that the verification algorithm is deterministic. Since most schemes have deterministic verification, this is not a strong restriction. To cover the case of probabilistic verification, we propose an augmented definition of honest-user unforgeability in Section 5: This definition considers a list of signatures as a successful forgery if each of them would pass verification with noticeable probability (roughly speaking).

We do not propose a generic transformation that makes scheme with probabilistic verification secure according to our definition. Yet, since most schemes have a deterministic verification anyway; these schemes will automatically satisfy our augmented definition.

Related work. Many blind signature schemes have been proposed in the literature, these schemes differ in their round complexity, their underlying computational assumptions, and the model in which the proof of security is given. For example, some schemes rely on the random oracle heuristic [PS00, Abe01, BNPS03, Bol03, AO09], some constructions are secure in the standard model [CKW04, Oka06, HK07, KZ08, AFG+10, SU11], and some constructions are based on general assumptions [JLO97, Fis06, HKKL07, SU11].

Only a few works consider the security of blind signatures [JLO97, PS00, FS09] or their round complexity [FS10].

Notations. Before presenting our results we briefly recall some basic definitions. In what follows we denote by $\lambda \in \mathbb{N}$ the security parameter. Informally, we say that a function is *negligible* if it vanishes faster than the inverse of any polynomial. A function is non-negligible if it is not negligible. If S is a set, then $x \xleftarrow{\$} S$ indicates that x is chosen uniformly at random over S (which in particular assumes that S can be sampled efficiently).

2 Blind signatures

To define blind signatures formally we introduce the following notation for interactive executions between algorithms $\mathcal{X}$ and $\mathcal{Y}$. By $(a, b) \leftarrow \langle \mathcal{X}(x), \mathcal{Y}(y) \rangle$ we denote the joint execution of $\mathcal{X}$ and $\mathcal{Y}$, where x is the private input of $\mathcal{X}$ and y defines the private input of $\mathcal{Y}$. The private output of $\mathcal{X}$ equals a and the private output of $\mathcal{Y}$ is b. We write $\mathcal{Y}^{\langle \mathcal{X}(x), \cdot \rangle^{\infty}}(y)$ if $\mathcal{Y}$ can invoke an unbounded number of executions of the interactive protocol with $\mathcal{X}$ in arbitrarily interleaved order. Accordingly, $\mathcal{X}^{\langle \cdot, \mathcal{Y}(y_0) \rangle^1, \langle \cdot, \mathcal{Y}(y_1) \rangle^1}(x)$ can invoke arbitrarily ordered executions with $\mathcal{Y}(y_0)$ and $\mathcal{Y}(y_1)$, but interact with each algorithm only once.

The invoking oracle machine does not see the private output of the invoked machine. In the above definition this means that $\mathcal{Y}$ does not learn a and $\mathcal{X}$ does not learn b_0 (resp. b_1).

Definition 2 (Interactive signature scheme). *An interactive signature scheme consists of a tuple of efficient*[5] *algorithms* $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ *(the key-generation algorithm* KG*, the signer* $\mathcal{S}$*, the user* $\mathcal{U}$*, and the verification algorithm* Vf*) where*

Key Generation. $\mathsf{KG}(1^\lambda)$ *for parameter* λ *generates a key pair* (sk, pk)*.*

Signature Issuing. *The joint execution of algorithm* $\mathcal{S}(sk)$ *and algorithm* $\mathcal{U}(pk, m)$ *for message* $m \in \{0,1\}^*$ *generates an output* σ *of the user (and some possibly empty output out for the signer.),* $(out, \sigma) \leftarrow \langle \mathcal{S}(sk), \mathcal{U}(pk, m) \rangle$*.*

Verification. $\mathsf{Vf}(pk, m, \sigma)$ *outputs a bit.*

It is assumed that the scheme is complete*, i.e., with overwhelming probability in* $\lambda \in \mathbb{N}$ *the following holds:* $(sk, pk) \leftarrow \mathsf{KG}(1^\lambda)$*, any message* $m \in \{0,1\}^*$ *and any* σ *output by* $\mathcal{U}$ *in the joint execution of* $\mathcal{S}(sk)$ *and* $\mathcal{U}(pk, m)$ *we have* $\mathsf{Vf}(pk, m, \sigma) = 1$*.*

3 Security of blind signatures

Security of blind signature schemes is defined by unforgeability and blindness [JLO97, PS00].

[5] More precisely, KG and Vf run in polynomial-time in the total length of their inputs. The total running time of $\mathcal{S}$ is polynomial in the total length of its input (sk) plus the total length of its incoming messages. The total running time of $\mathcal{U}$ is polynomial in the total length of its input (pk, m). (But the running time of $\mathcal{U}$ may not depend on its incoming messages.) The asymmetry between the running time of $\mathcal{S}$ and $\mathcal{U}$ is necessary to ensure that (a) an interaction between $\mathcal{U}$ and $\mathcal{S}$ always runs in polynomial-time, and (b) that the running-time of $\mathcal{S}$ may depend on the length of the message m that only $\mathcal{U}$ has in its input.

Unforgeability. An efficient adversary $\mathcal{U}^*$ against unforgeability tries to generate $k+1$ valid message/signatures pairs with different messages after at most k completed interactions with the honest signer, where the number of executions is adaptively determined by $\mathcal{U}^*$ during the attack. To identify completed sessions we assume that the honest signer returns a special symbol ok when having sent the final protocol message in order to indicate a completed execution (from its point of view). We remark that this output is "atomically" connected to the final transmission to the user.

Definition 3 (Unforgeability). *An interactive signature scheme* $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ *is called* unforgeable *if for any efficient algorithm* $\mathcal{A}$*(the malicious user) the probability that experiment* $\mathsf{Unforge}^{\mathsf{BS}}_{\mathcal{A}}(\lambda)$ *evaluates to* 1 *is negligible (as a function of* λ*) where*

Experiment $\mathsf{Unforge}^{\mathsf{BS}}_{\mathcal{A}}(\lambda)$
$(sk, pk) \leftarrow \mathsf{KG}(1^\lambda)$
$((m_1^*, \sigma_1^*), \ldots, (m_{k+1}^*, \sigma_{k+1}^*)) \leftarrow \mathcal{A}^{\langle \mathcal{S}(sk), \cdot \rangle^\infty}(pk)$
Return 1 *iff*
$m_i^* \neq m_j^*$ *for* i, j *with* $i \neq j$*, and*
$\mathsf{Vf}(pk, m_i^*, \sigma_i^*) = 1$ *for all* i*, and*
$\mathcal{S}$ *has returned* ok *in at most* k *interactions.*

An interactive signature scheme is *strongly unforgeable* if the condition "$m_i^* \neq m_j^*$ for i, j with $i \neq j$" in the above definition is substituted by "$(m_i^*, \sigma_i^*) \neq (m_j^*, \sigma_j^*)$ for i, j with $i \neq j$".

Observe that the adversary $\mathcal{A}$ does not learn the private output *out* of the signer $\mathcal{S}(sk)$. We assume schemes in which it can be efficiently determined from the interaction between signer and adversary whether the signer outputs ok. If this is not the case, we need to augment the definition and explicitly give the adversary access to the output *out* since *out* might leak information that the adversary could use to produce forgeries.

Blindness. The blindness condition says that it should be infeasible for a malicious signer $\mathcal{S}^*$ to decide which of two messages m_0 and m_1 has been signed first in two executions with an honest user $\mathcal{U}$. This condition must hold, even if $\mathcal{S}^*$ is allowed to choose the public key maliciously [ANN06]. If one of these executions has returned $\perp$ then the signer is not informed about the other signature either.

Definition 4 (Blindness). *A blind signature scheme* $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ *is called* blind *if for any efficient algorithm* $\mathcal{S}^*$ *(working in modes* find*,* issue *and* guess*) the probability that the following experiment* $\mathsf{Blind}^{\mathsf{BS}}_{\mathcal{S}^*}(\lambda)$ *evaluates to* 1 *is negligibly close to* 1/2*, where*

Experiment $\mathsf{Blind}^{\mathsf{BS}}_{\mathcal{S}^*}(\lambda)$
$(pk, m_0, m_1, \mathsf{st}_{\mathsf{find}}) \leftarrow \mathcal{S}^*(\mathsf{find}, 1^\lambda)$
$b \xleftarrow{\$} \{0, 1\}$
$\mathsf{st}_{\mathsf{issue}} \leftarrow \mathcal{S}^{*\langle \cdot, \mathcal{U}(pk, m_b) \rangle^1, \langle \cdot, \mathcal{U}(pk, m_{1-b}) \rangle^1}(\mathsf{issue}, \mathsf{st}_{\mathsf{find}})$
and let σ_b, σ_{1-b} *denote the (possibly undefined) local outputs of* $\mathcal{U}(pk, m_b)$ *resp.* $\mathcal{U}(pk, m_{1-b})$*.*
set $(\sigma_0, \sigma_1) = (\perp, \perp)$ *if* $\sigma_0 = \perp$ *or* $\sigma_1 = \perp$
$b^* \leftarrow \mathcal{S}^*(\mathsf{guess}, \sigma_0, \sigma_1, \mathsf{st}_{\mathsf{issue}})$
return 1 *iff* $b = b^*$*.*

4 Honest-user unforgeability

In this section we introduce a stronger notion of unforgeability that we call *honest-user unforgeability*. In the traditional definition of unforgeability due to [JLO97, PS00], the adversary fulfills the role of the user. This means that the attacker may choose all messages that are exchanged during the signature issue protocol at will. In particular, the attacker may sample random message *without* fixing a specific message and a certain randomness for the user algorithm. Even if the adversary runs the honest user algorithm, due to the blindness, it is impossible to tell which message has been used. Thus, from a definitional perspective, one has to count the number of executions and produced signatures in order to determine the success condition for the attacker.

This, however, might not be sufficient. Consider an attacker that queries twice the same message m (through, say, some third party honestly implementing the user's algorithm) and is then able to compute a valid signature on some message $m' \neq m$. Since this adversary queried twice the same message, it *still* has to output three distinct messages in order to succeed in the unforgeability game.

In this section we show that giving the attacker, in addition to controlling the user, access to a protocol oracle (that takes as input a message and returns the signature and the user's transcript) yields a strictly stronger definition.

4.1 Defining honest-user unforgeability

Before proposing the new definition, we fix some notation. Let $\mathcal{P}(sk, pk, \cdot)$ be an oracle that on input a message m runs the signature issue protocol $\langle \mathcal{S}(sk), \mathcal{U}(pk, m) \rangle$ obtaining a signature σ. Let trans denote the transcript of the messages exchanges in that interaction. We assume that the transcript consists of all messages exchanged between the parties.[6] This oracle then returns (σ, trans).

Definition 5 (Honest-user unforgeability). *An interactive signature scheme* $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ *is* honest-user unforgeable *if* Vf *is deterministic and the following holds: For any efficient algorithm* $\mathcal{A}$ *the probability that experiment* $\mathsf{HUnforge}^{\mathsf{BS}}_{\mathcal{A}}(\lambda)$ *evaluates to* 1 *is negligible (as a function of* λ*) where*

Experiment $\mathsf{HUnforge}^{\mathsf{BS}}_{\mathcal{A}}(\lambda)$
$(sk, pk) \leftarrow \mathsf{KG}(1^\lambda)$
$((m_1^*, \sigma_1^*), \ldots, (m_{k+1}^*, \sigma_{k+1}^*)) \leftarrow \mathcal{A}^{\langle \mathcal{S}(sk), \cdot \rangle^\infty, \mathcal{P}(sk, pk, \cdot)}(pk)$
Let $m_1, \ldots, m_n$ *be the messages queried to* $\mathcal{P}(sk, pk, \cdot)$.
Return 1 *iff*
$m_i^* \neq m_j$ *for all* i, j
$m_i^* \neq m_j^*$ *for* i, j *with* $i \neq j$, *and*
$\mathsf{Vf}(pk, m_i^*, \sigma_i^*) = 1$ *for all* i, *and*
$\mathcal{S}$ *has returned* ok *in at most* k *interactions.*

(When counting the interactions in which $\mathcal{S}$ *returns* ok*, we do not count the interactions simulated by* $\mathcal{P}$*.)*

[6] The definition of honest-user unforgeability could be easily strengthened by including the randomness of $\mathcal{U}$ in trans. Our results also hold with respect to that strengthened definition. However, it is not clear that giving the honest-user's randomness to the adversary models any realistic attacks.

An interactive signature scheme is *strongly honest-user unforgeable* if the condition "$m_i^* \neq m_j$ for all i, j" in the above definition is substituted by "$(m_i^*, \sigma_i^*) \neq (m_j, \sigma_j)$ for all i, j" and if we change the condition "$m_i^* \neq m_j^*$ for i, j with $i \neq j$" to "$(m_i^*, \sigma_i^*) \neq (m_j^*, \sigma_j^*)$ for i, j with $i \neq j$".

Notice that we require Vf to be deterministic. When we drop this requirement, the definition does not behave as one would intuitively expect. We explain this problem in detail in Section 5.

4.2 Unforgeability does not imply honest-user unforgeability

We show that unforgeability does not imply honest-user unforgeability. The high-level idea of our counterexample is to change the verification algorithm of an interactive signature scheme such that it accepts a message m' if it obtains as input two distinct and valid signatures on some message $m \neq m'$ (in addition to accepting honestly generated signatures). More precisely, fix an unforgeable and blind signature scheme $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ that is strongly unforgeable. Fix some efficiently computable injective function $f \neq id$ on bitstrings (e.g., $f(m) := 0 \| m$). We construct a blind signature scheme $\mathsf{BS}_1 = (\mathsf{KG}_1, \langle \mathcal{S}_1, \mathcal{U}_1 \rangle, \mathsf{Vf}_1)$ as follows:

- $\mathsf{KG}_1 := \mathsf{KG}$, $\mathcal{S}_1 := \mathcal{S}$, and $\mathcal{U}_1 := \mathcal{U}$.
- $\mathsf{Vf}_1(pk, m, \sigma)$ executes the following steps:
  - Invoke $v := \mathsf{Vf}(pk, m, \sigma)$. If $v = 1$, return 1.
  - Otherwise, parse σ as (σ^1, σ^2). If parsing fails or $\sigma^1 = \sigma^2$, return 0.
  - Invoke $v_i := \mathsf{Vf}(pk, f(m), \sigma^i)$ for $i = 1, 2$. If $v_1 = v_2 = 1$, return 1. Otherwise return 0.

Lemma 6. *If* BS *is complete, strongly unforgeable, and blind, then* BS_1 *is complete, unforgeable, and blind.*

Blindness and completeness of BS_1 follow directly from the blindness and completeness of BS. The main idea behind unforgeability is the following: The only possibility for the adversary to forge a signature is to obtain two different signatures σ_1, σ_2 on the same message m. Then (σ_1, σ_2) is a valid signature on $f(m)$. However, since the underlying scheme BS is strongly unforgeable, the adversary can only get σ_1, σ_2 by performing two signing queries. Thus, using two queries, the adversary gets two signatures on the message m and one on $f(m)$. This is not sufficient to break the unforgeability of BS_1 since the adversary would need to get signatures on three different messages for that.

Proof (of Lemma 6). Assume for the sake of contradiction that BS_1 is not unforgeable. Then, there is an efficient adversary $\mathcal{A}$ that succeeds in the unforgeability game for BS_1 with non-negligible probability. This attacker, when given oracle access to the signer $\mathcal{S}_1$, returns a $(k+1)$-tuple $((m_1, \sigma_1), \ldots, (m_{k+1}, \sigma_{k+1}))$ of message/signature pairs, where $\mathsf{Vf}_1(pk, m_i, \sigma_i)$ for all i and $m_i \neq m_j$ for all $i \neq j$ and where $\mathcal{S}$ has returned ok at most k times. In the following, we call such a tuple *k-bad*. We now show how to build an algorithm $\mathcal{B}$ that wins the strong unforgeability game of BS.

The input of the algorithm $\mathcal{B}$ is the public key pk, it runs a black-box simulation of $\mathcal{A}$ on input pk, and answers all oracle queries with its own oracle by simply forwarding all messages. Eventually, $\mathcal{A}$ stops, outputting a tuple $F := ((m_1, \sigma_1), \ldots, (m_{k+1}, \sigma_{k+1}))$. Suppose in the following that $\mathcal{A}$ succeeds. Then the tuple F is k-bad. We will show how to efficiently construct from F $k+1$ distinct message/signature pairs (m_i^*, σ_i^*) that verify under $\mathsf{Vf}(pk, \cdot, \cdot)$. Now, consider a message/signature pair (m, σ) and observe that the verification algorithm Vf_1 outputs 1 if $\mathsf{Vf}(pk, m, \sigma) = 1$ *or* if $\sigma = (\sigma^1, \sigma^2)$ (where $\sigma^1 \neq \sigma^2$) and $\mathsf{Vf}(pk, f(m), \sigma^1) = \mathsf{Vf}(pk, f(m), \sigma^2) = 1$. We define two sets V_0 and V_1 where V_1 is the set that contains a message/signature pairs (m_i, σ_i) that verify under the

first condition, and the set V_0 contains all pairs (m_i, σ_i) (with $\sigma_i = (\sigma_i^1, \sigma_i^2)$) that verify under the second condition, i.e.,

$$V_1 := \{(m_i, \sigma_i) : \mathsf{Vf}(pk, m_i, \sigma_i) = 1\} \quad \text{and} \quad V_0 := \{(m_i, \sigma_i) : \mathsf{Vf}(pk, m_i, \sigma_i) = 0\}.$$

Clearly, since $\mathcal{A}$ succeeds and F is k-bad, all messages m_i are distinct and hence $|V_0| + |V_1| = k + 1$. Next, we define the set V_0' that consists of a message/signature pairs $(f(m_i), \sigma_i^1), (f(m_i), \sigma_i^2)$, i.e., all message/signature pairs that verify under the second condition. Formally,

$$V_0' := \{(f(m_i), \sigma_i^1), (f(m_i), \sigma_i^2) : (m_i, (\sigma_i^1, \sigma_i^2)) \in V_0\}.$$

Note that V_0 contains only elements (m_i, σ_i) with $\mathsf{Vf}'(m_i, \sigma_i) = 1$ and $\mathsf{Vf}(m_i, \sigma_i) = 0$. By definition of Vf' this implies that $\sigma_i = (\sigma_i^1, \sigma_i^2)$ with $\sigma_i^1 \neq \sigma_i^2$ and $\mathsf{Vf}(f(m_i), \sigma_i^1) = \mathsf{Vf}(f(m_i), \sigma_i^2) = 1$. Thus $|V_0'| = |V_0|$ and for all $(m, \sigma) \in V_0' \cup V_1$ we have that $\mathsf{Vf}(pk, m, \sigma) = 1$. We proceed to show that $|V_0' \cup V_1| \geq k + 1$ and we then let $\mathcal{B}$ output this set. First note that for any $(m_i, (\sigma_i^1, \sigma_i^2)) \in V_0$, at most one of $(f(m_i), \sigma_i^1)$, $(f(m_i), \sigma_i^2)$ is contained in V_1. Otherwise, V_1 would either contain two pairs (m, σ) with the same m, or $\sigma_i^1 = \sigma_i^2$. Furthermore, since f is injective, for any distinct $(m_i, (\sigma_i^1, \sigma_i^2)), (m_j, (\sigma_j^1, \sigma_j^2)) \in V_0$ we have $m_i \neq m_j$. Hence $(f(m_i), \sigma_i^a) \neq (f(m_j), \sigma_j^b)$ for any $a, b \in \{1, 2\}$ and $i \neq j$. Thus $|V_0' \setminus V_1| \geq |V_0|$ and therefore

$$\left|V_0' \cup V_1\right| = \left|(V_0' \setminus V_1) \dot\cup V_1\right| = \left|V_0' \setminus V_1\right| + |V_1| \geq |V_0| + |V_1| = k + 1.$$

The algorithm $\mathcal{B}$ then computes the set $V_0' \cup V_1$, and then picks distinct pairs

$$(m_1^*, \sigma_1^*), \ldots, (m_{k+1}^*, \sigma_{k+1}^*) \in V_0' \cup V_1$$

and outputs $(m_1^*, \sigma_1^*), \ldots, (m_{k+1}^*, \sigma_{k+1}^*)$.

Analysis. Obviously, $\mathcal{B}$ is efficient because $\mathcal{A}$ is efficient and because the overhead of handling all queries can be done efficiently. Since $\mathcal{A}$ outputs a k-bad tuple with non-negligible probability in the unforgeability game for BS_1, it follows that $\mathcal{B}$ outputs $k + 1$ distinct (m_i^*, σ_i^*) with $\mathsf{Vf}(m_i^*, \sigma_i^*) = 1$ in the unforgeability game for BS with at least the same probability. Thus, $\mathcal{B}$ breaks the strong unforgeability of BS. Since we assumed that BS is strongly unforgeable, we have a contradiction, thus our initial assumption that BS_1 is not unforgeable was false.

Before proving the next lemma, we need to define what a randomized (interactive) signature is. Roughly speaking, schemes that have this property output the same signature in two independent executions with same message only with negligible probability.

Definition 7 (Randomized signature scheme). *An interactive signature scheme* $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ *is* randomized *if with overwhelming probability in* $\lambda \in \mathbb{N}$ *the following holds: for any* (sk, pk) *in the range of* $\mathsf{KG}(1^\lambda)$*, any message* $m \in \{0,1\}^*$*, we have* $\sigma_1 \neq \sigma_2$ *where* $\sigma_1 \leftarrow \langle \mathcal{S}(sk), \mathcal{U}(pk, m) \rangle$ *and* $\sigma_2 \leftarrow \langle \mathcal{S}(sk), \mathcal{U}(pk, m) \rangle$*.*

Note that any scheme can easily be modified such that is satisfies this definition by letting the user algorithm pick some random value r, setting $m' \leftarrow m \| r$, and by including r in the signature.

Lemma 8. *If* BS *is complete and randomized, then* BS_1 *is not honest-user unforgeable.*

Proof. We construct an efficient adversary $\mathcal{A}$ against BS_1 as follows: Let $m \in \{0,1\}^*$ be such that $f(m) \neq m$. Recall that $f \neq id$, and therefore such a value m exists. Note that we can hardcode m directly into the adversary and therefore it is not necessary that m can be efficiently found.

The attacker $\mathcal{A}$ queries $\mathcal{P}$(the machine simulating $\langle \mathcal{S}_1, \mathcal{U}_1 \rangle$) twice, both times with the same message $f(m)$ and obtains the signatures σ_1 and σ_2. Since BS is randomized, and $\mathcal{S}_1 = \mathcal{S}$ and $\mathcal{U}_1 = \mathcal{U}$, with overwhelming probability $\sigma_1 \neq \sigma_2$. Since BS is complete, $\mathsf{Vf}(pk, f(m), \sigma_1) = \mathsf{Vf}(pk, f(m), \sigma_2) = 1$ with overwhelming probability. Hence with overwhelming probability, $\mathsf{Vf}_1(pk, m, \sigma) = 1$ for $\sigma := (\sigma_1, \sigma_2)$. The adversary $\mathcal{A}$ outputs (m, σ). Since $\mathcal{A}$ never queried $\mathcal{S}$, and because $\mathcal{A}$ only queries $f(m) \neq m$ from $\mathcal{P}$, this breaks the honest-user unforgeability of BS_1.

Theorem 9. *If complete, blind, and strongly unforgeable interactive signature schemes exist, then there are complete, blind, and unforgeable interactive signature schemes that are not honest-user unforgeable.*

Proof. If complete, blind, and strongly unforgeable interactive signature schemes exist, then there is a complete, blind, strongly unforgeable, and randomized interactive signature scheme BS (e.g., by applying the transformation from Section 7). From BS we construct BS_1 as described at the beginning of the section. By Lemmas 6 and 8, BS_1 is complete, blind, and unforgeable but not honest-user unforgeable.

4.3 Strong honest-user unforgeability

In this section we show that strong unforgeability implies strong honest-user unforgeability.

Lemma 10. *Assume that* BS *is complete,*[7] *randomized, and strongly unforgeable. Then* BS *is strongly honest-user unforgeable.*

Proof. Assume that BS is not strongly honest-user unforgeable. Then there is an adversary $\mathcal{A}$ in the strong honest-user unforgeability game for BS such that with non-negligible probability, the following holds:

(i) The adversary outputs a tuple $((m_1^*, \sigma_1^*), \ldots, (m_{k+1}^*, \sigma_{k+1}^*))$ for some k.
(ii) The signer $\mathcal{S}$ outputs ok at most k times.
(iii) For all $i \neq j$, we have $(m_i^*, \sigma_i^*) \neq (m_j^*, \sigma_j^*)$.
(iv) For all i, we have $\mathsf{Vf}(pk, m_i^*, \sigma_i^*) = 1$.
(v) Let $m_1, \ldots, m_n$ be the messages queried from the user $\mathcal{U}$ (which is part of the oracle $\mathcal{P}$), and let $\sigma_1, \ldots, \sigma_n$ be the corresponding answers. Then $(m_i, \sigma_i) \neq (m_j^*, \sigma_j^*)$ for all i, j.

Furthermore, since BS is complete, with overwhelming probability we have that

(vi) $\mathsf{Vf}(pk, m_i, \sigma_i) = 1$ for all i.

And since BS is randomized, with overwhelming probability we have that

[7] Completeness is actually necessary to show this lemma: For example, let BS' be a scheme derived from a complete and strongly unforgeable scheme BS in the following way: All machines except for the user are the same in BS and BS'. When the user $\mathcal{U}'$ should sign a message m, he signs $m+1$ instead. Since the user does not occur in the definition of strong unforgeability, the strong unforgeability of BS implies the strong unforgeability of BS'. Yet BS' is not strongly honest-user unforgeable: By performing a signature query for m from the user $\mathcal{U}'$, the adversary can get a valid signature for $m+1$.

(vi) $(m_i, \sigma_i) \neq (m_j, \sigma_j)$ for all $i \neq j$.

This implies that, with non-negligible probability, properties (i)–(vi) hold. Let $(\tilde{m}_1^*, \tilde{\sigma}_1^*), \ldots, (\tilde{m}_{k+n+1}^*, \tilde{\sigma}_{k+n+1}^*)$ be the sequence $(m_1^*, \sigma_1^*), \ldots, (m_{k+1}^*, \sigma_{k+1}^*), (m_1, \sigma_1), \ldots, (m_n, \sigma_n)$. From properties (iii), (v), and (vi), it follows that $(\tilde{m}_i^*, \tilde{\sigma}_i^*) \neq (\tilde{m}_j^*, \tilde{\sigma}_j^*)$ for all $i \neq j$. From (iv) and (vi), it follows that $\mathsf{Vf}(pk, \tilde{m}_i^*, \tilde{\sigma}_i^*) = 1$ for all i.

Let $\mathcal{B}$ be an adversary for the strong unforgeability game, constructed as follows: $\mathcal{B}$ simulates $\mathcal{A}$ and $\mathcal{U}$ in a black-box fashion. Whenever $\mathcal{A}$ queries $\mathcal{U}$, then $\mathcal{B}$ invokes the simulated user algorithm $\mathcal{U}$. If the simulated user $\mathcal{U}$ or the simulated $\mathcal{A}$ communicate with the signer, then $\mathcal{B}$ routes this communication to the external signer $\mathcal{S}$. Finally, $\mathcal{B}$ outputs $(\tilde{m}_1^*, \tilde{\sigma}_1^*), \ldots, (\tilde{m}_{k+n+1}^*, \tilde{\sigma}_{k+n+1}^*)$. Then we have that in the strong unforgeability game, with non-negligible probability, $\mathcal{B}$ outputs a tuple $(\tilde{m}_1^*, \tilde{\sigma}_1^*), \ldots, (\tilde{m}_{k+n+1}^*, \tilde{\sigma}_{k+n+1}^*)$ such that $(\tilde{m}_i^*, \tilde{\sigma}_i^*) \neq (\tilde{m}_j^*, \tilde{\sigma}_j^*)$ for all $i \neq j$ and $\mathsf{Vf}(pk, \tilde{m}_i^*, \tilde{\sigma}_i^*) = 1$ for all i and the signer outputs ok at most $k + n$ times (k times due to the invocations from $\mathcal{A}$, and n times due to the invocations from the simulated $\mathcal{U}$). This violates the strong unforgeability of BS, we have a contradiction, and thus BS is strongly honest-user unforgeable.

5 Probabilistic verification

In this section we show that, if we allow for a probabilistic verification algorithm, both the definition of honest-user unforgeability, as well as the usual definition of unforgeability will consider schemes to be secure that do not meet the intuitive notion of unforgeability.

One may argue that discussing problems in the definition of blind signature schemes in the case of probabilistic verification is not necessary because one can always just use schemes with deterministic verification. We disagree with this point of view: Without understanding why the definition is problematic in the case of probabilistic verification, there is no reason to restrict oneself to schemes with deterministic verification. Only the awareness of the problem allows us to circumvent it. We additionally give a definition that works in the case of probabilistic verification. This is less important than pointing out the flaws, since in most cases one can indeed use schemes with deterministic verification. But there might be (rare) cases where this is not possible (note that no generic transformation outside the random oracle model is known that makes the verification deterministic).

First, we give some intuition for our counterexample and formalize it afterwards. Assume an interactive signature scheme BS_3 that can distinguishes two kinds of signatures: A full-signature that will pass verification with probability 1, and a half-signature that passes verification with probability $\frac{1}{2}$. An honest interaction between the signer $\mathcal{S}_3$ and the user $\mathcal{U}_3$ will always produce a full-signature. A malicious user, however, may interact with the signer to get a half-signature for arbitrary messages. Furthermore, the malicious user may, by sending λ half-signatures to the signer (λ is the security parameter) and executing a special command, get two half-signatures instead of one. ("Buy $\lambda + 1$ signatures, get one free.") At the first glance, one would expect that such a scheme cannot be honest-user unforgeable or even unforgeable. But in fact, the adversary has essentially two options: First, he does not request λ half-signatures. Then he will not get a signature for free and thus will not win in the honest-user unforgeability game. Second, he does request λ half-signatures and then performs the extra query and thus gets $\lambda + 2$ half-signatures using $\lambda + 1$ queries. Then, to win, he needs that all $\lambda + 2$ signatures pass verification (since the definition of unforgeability/honest-user unforgeability requires that $\mathsf{Vf}_3(pk, m_i^*, \sigma_i^*)$ evaluates to 1 for all signatures (m_i^*, σ_i^*) output by the

adversary) However, since each half-signature passes verification with probability $\frac{1}{2}$, the probability that all signatures pass verification is negligible ($\leq 2^{-\lambda}$). Thus, the adversary does not win, and the scheme is honest-user unforgeable. Clearly, this is not what one would expect; so Definition 5 should not be applied to the case where the verification is probabilistic (and similarly the normal definition of unforgeability should not be applied either in that case).

More precisely, let $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ be a randomized, complete, blind, and honest-user unforgeable interactive signature scheme. Let Q be an efficiently decidable set such that the computation of arbitrarily many bitstrings $m \in Q$ and $m' \notin Q$ is efficiently feasible.

We define the scheme $\mathsf{BS}_3 = (\mathsf{KG}_3, \langle \mathcal{S}_3, \mathcal{U}_3 \rangle, \mathsf{Vf}_3)$ as follows:

- $\mathsf{KG}_3 := \mathsf{KG}$.
- $\mathcal{S}_3(sk)$ behaves like $\mathcal{S}(sk)$, except when the first message from the user is of the form ($\texttt{extrasig}$, $m_1^\circ, \ldots, m_\lambda^\circ, \sigma_1^\circ, \ldots, \sigma_\lambda^\circ, m_1', \ldots, m_q'$) where λ is the security parameter. Then $\mathcal{S}_3$ executes the following steps:
  - Check whether $m_1^\circ, \ldots, m_\lambda^\circ \in Q$ are pairwise distinct messages, and for all $i = 1, \ldots, q$ we have $m_i' \notin Q$, and for all $i = 1, \ldots, \lambda$ we have $\mathsf{Vf}(pk, 1 \| m_i^\circ, \sigma_i^\circ) = 1$.[8] If not, ignore the message.
  - If the check passes, run $\langle \mathcal{S}(sk), \mathcal{U}(pk, 1 \| m_i') \rangle$ for each $i = 1, \ldots, q$, resulting in signatures $\tilde{\sigma}_i$, and set $\sigma_i' := 1 \| \tilde{\sigma}_i$.
  - Then $\mathcal{S}_3$ sends $(\sigma_1', \ldots, \sigma_n')$ to the user, outputs ok and does not react to any further messages in this session.
- $\mathcal{U}_3(pk, m)$ runs $\sigma \leftarrow \mathcal{U}(pk, 0 \| m)$ and returns $0 \| \sigma$.
- $\mathsf{Vf}_3(pk, m, \sigma)$ performs the following steps:
  - If $\sigma = 0 \| \sigma'$ and $\mathsf{Vf}(pk, 0 \| m, \sigma') = 1$, Vf_3 returns 1.
  - If $\sigma = 1 \| \sigma'$ and $\mathsf{Vf}(pk, 1 \| m, \sigma) = 1$, Vf_3 returns 1 with probability $p := \frac{1}{2}$ and 0 with probability $1 - p$.
  - Otherwise, Vf_3 returns 0.

Lemma 11. *If* BS *is blind and complete, so is* BS_3.

Proof. Blindness and completeness of BS_3 follow directly from that of BS. The only difference between the schemes is that instead of a message m, a message $0 \| m$ is signed and 0 is prepended to the signatures (as long as the user is honest as is the case in the definitions of blindness and completeness).

Lemma 12. *If* BS *is honest-user unforgeable, so is* BS_3.

Proof. We first fix some notation. A pair (m, σ) is
- a *full-signature* if $\sigma = 0 \| \sigma'$ and $\mathsf{Vf}(pk, 0 \| m, \sigma') = 1$;
- a *half-signature* if $\sigma = 1 \| \sigma'$ and $\mathsf{Vf}(pk, 1 \| m, \sigma') = 1$;
- and a *non-signature* otherwise.

Note that if (m, σ) is a full-, half-, or non-signature, then $\mathsf{Vf}_3(pk, m, \sigma)$ is 1, $p = \frac{1}{2}$, or 0, respectively. An interaction between $\mathcal{A}$ and $\mathcal{S}_3$ that begins with a ($\texttt{extrasig}, \ldots$)-message passing the check in the definition of $\mathcal{S}_3$ is called an extra-query. Other interactions between $\mathcal{A}$ and $\mathcal{S}_3$ that lead to an output ok from $\mathcal{S}_3$ are called standard-queries.

[8] Without loss of generality, we assume that the public key pk can efficiently be computed from the secret key sk.

Fix an efficient adversary $\mathcal{A}$ against the honest-user unforgeability game for BS_3. Without loss of generality, we assume that the output of $\mathcal{A}$ is always of the form $((m_1^*, \sigma_1^*), \ldots, (m_{k+1}^*, \sigma_{k+1}^*))$ for some k. Let k_e denote the number of extra-queries and k_s the number of standard-queries performed by $\mathcal{A}$. Let $m_1, \ldots, m_n$ be the messages queried by $\mathcal{A}$ to the oracle $\mathcal{P}$ (which simulates $\langle \mathcal{S}_3, \mathcal{U}_3 \rangle$, and let $\sigma_1, \ldots, \sigma_n$ be the answers from $\mathcal{P}$. In an execution of the game, we distinguish the following cases (each case implicitly excludes all preceding cases):

(i) $k_e + k_s > k$, or for some i, (m_i^*, σ_i^*) is a non-signature, or for some $i \neq j$, $m_i^* = m_j^*$, or for some i, j, $m_i^* = m_j$.
(ii) For $h > \lambda$ different indices i, (m_i^*, σ_i^*) is a half-signature.
(iii) No extra-query was performed.
(iv) All other cases.

In case (i), by definition, the adversary does not win.

In case (ii), the probability that $\mathsf{Vf}_3(pk, m_i^*, \sigma_i^*) = 1$ for all i is upper-bounded by the probability that $\mathsf{Vf}_3(pk, m, \sigma) = 1$ for all half-signatures (m, σ) output by $\mathcal{A}$. That probability, in turn, is bounded by $p^h = 2^{-h} \leq 2^{-\lambda}$ because each invocation of $\mathsf{Vf}(pk, m, \sigma)$ succeeds with probability p for a half-signature (m, σ). Thus the adversary wins with negligible probability in case (ii).

Hence $\mathcal{A}$ only wins with non-negligible probability, if either case (iii) or (iv) occurs with non-negligible probability.

Assume that case (iii) happens with non-negligible probability and observe that any full- or half-signature on a message m can be efficiently transformed into a signature on $0\|m$ or $1\|m$, respectively (with respect to the original scheme BS). We construct an adversary $\mathcal{B}$ against the honest-user unforgeability game for the original scheme BS. $\mathcal{B}$ runs a black-box simulation of $\mathcal{A}$ and behaves as follows: Whenever $\mathcal{A}$ performs an extra-query, then $\mathcal{B}$ aborts. If $\mathcal{A}$ queries $\sigma_i \leftarrow \mathcal{P}(m_i)$, then $\mathcal{B}$ sets $m_i' = 0\|m_i$, sends m_i' to its own oracle $\mathcal{P}$ (which simulates $\langle \mathcal{S}, \mathcal{U} \rangle$); it then obtains a signature σ_i and returns $0\|\sigma_i$ to $\mathcal{A}$. Whenever $\mathcal{A}$ queries the signer directly, then $\mathcal{B}$ forwards all messages in both directions.

When $\mathcal{A}$ outputs $((m_1^*, b_1^*\|\sigma_1^*), \ldots, (m_{k+1}^*, b_{k+1}^*\|\sigma_{k+1}^*))$ with $b_i^* \in \{0, 1\}$, then the algorithm $\mathcal{B}$ outputs $((b_1^*\|m_1^*, \sigma_1^*), \ldots, (b_{k+1}^*\|m_{k+1}^*, \sigma_{k+1}^*))$. Obviously, if all m_i^* are distinct and different from all m_i, then all $b_i^*\|m_i^*$ are distinct and different from all $0\|m_i$. And if $\mathsf{Vf}'(m_i^*, b_i^*\|\sigma_i^*) = 1$ then $\mathsf{Vf}(b_i^*\|m_i^*, \sigma_i^*) = 1$. Thus, when (iii) occurs with non-negligible probability in the honest-user unforgeability game with $\mathcal{A}$ and BS_3, then $\mathcal{B}$ wins with non-negligible probability in the honest-user unforgeability game with BS. By assumption BS is honest-user unforgeable, so we have a contradiction. Thus our assumption that case (iii) occurs with non-negligible probability was false. Hence case (iii) occurs with negligible probability.

Now assume that case (iv) occurs with non-negligible probability. In this case, let Σ_f be the set of all full-signatures output by $\mathcal{A}$. Note that this is not the set of all $k+1$ signatures output by $\mathcal{A}$ because $\mathcal{A}$ may also output half-signatures.Let Σ_h denote the set of all half-signatures used in the first extra-query. More precisely, $(m, \sigma) \in \Sigma_h$ iff there the first extra-query was of the form $(\texttt{extrasig}, m_1^\circ, \ldots, m_\lambda^\circ, \sigma_1^\circ, \ldots, \sigma_\lambda^\circ, m', m'')$ with $(m, \sigma) = (m_i^\circ, \sigma_i^\circ)$ for some i. Let Σ_e denote the half-signatures returned by extra-queries, i.e., $(m', \sigma') \in \Sigma_e$ iff an extra-query $(\texttt{extrasig}, m_1^\circ, \ldots, m_\lambda^\circ, \sigma_1^\circ, \ldots, \sigma_\lambda^\circ, m_1', \ldots, m_q')$ was answered with $(\sigma_1', \ldots, \sigma_q')$ such that $(m', \sigma') = (m_i', \sigma_i')$ for some i. Let Σ_u be the set of all full- or non-signatures received from the user, i.e., $\Sigma_u = \{(m_1, \sigma_1), \ldots, (m_n, \sigma_n)\}$. Let ℓ be the number of half-signatures in the output of $\mathcal{A}$. We have $\ell \leq \lambda$ since otherwise we would be in case (ii).

Given a set Σ of pairs of messages and signatures , let Σ^* denote the set $\Sigma^* := \{(b\|m, \sigma') : (m, b\|\sigma') \in \Sigma, b \in \{0,1\}\}$.

Since the messages in Σ_f are distinct, and the messages in Σ_h are distinct, and Σ_f contains only full-signatures, and Σ_h contains only half-signatures, we have that all messages in $\Sigma_f^* \cup \Sigma_h^*$ are distinct, that $|\Sigma_f^* \cup \Sigma_h^*| = |\Sigma_f| + |\Sigma_h| \geq (k+1-\ell) + \lambda \geq k+1$, and that all $(m,\sigma) \in \Sigma_f^* \cup \Sigma_h^*$ satisfy $\mathsf{Vf}(pk, m, \sigma) = 1$.

Furthermore, the messages in Σ_h^* are different from those in Σ_u^* because Σ_h contains only half- and Σ_u only full-signatures or non-signatures. The messages in Σ_f^* are different from those in Σ_u^* because the messages in Σ_f are different from those in Σ_u (otherwise we would be in case (i)). The messages in Σ_h^* are different from those in Σ_e^* since by definition of extra-queries, the messages in Σ_h are in Q while the messages in Σ_e are not in Q. The messages in Σ_f^* are different from those in Σ_e^* because Σ_f contains only full- and Σ_e only half-signatures. Thus, the messages in $\Sigma_f^* \cup \Sigma_h^*$ are different from the messages in $\Sigma_u^* \cup \Sigma_e^*$.

Summarizing, in case (iv), we have $|\Sigma_f^* \cup \Sigma_h^*| \geq k+1$, the messages in $\Sigma_f^* \cup \Sigma_h^*$ are pairwise distinct and different from the messages in $\Sigma_u^* \cup \Sigma_e^*$, and all $(m,\sigma) \in \Sigma_f^* \cup \Sigma_h^*$ satisfy $\mathsf{Vf}(pk, m, \sigma) = 1$.

We then construct an adversary $\mathcal{B}$ against the original scheme BS. The attacker $\mathcal{B}$ simulates $\mathcal{A}$ with the following modifications. When $\mathcal{A}$ queries the oracle $\mathcal{P}$ on a message m_i, then $\mathcal{B}$ invokes its external oracle $\mathcal{P}$ (which simulates $\langle \mathcal{S}, \mathcal{U} \rangle$) on input $(0\|m_i)$, gets an answer σ_i', and returns $\sigma_i := 0\|\sigma_i'$ to $\mathcal{A}$. If $\mathcal{A}$ performs an extra-query $(\texttt{extrasig}, \ldots, m_1', \ldots, m_q')$, then $\mathcal{B}$ answers with $(\sigma_1', \ldots, \sigma_q') := (1\|\mathcal{U}(1\|m_1'), \ldots, 1\|\mathcal{U}(1\|m_q'))$ instead. Suppose that $\mathcal{A}$ outputs a message/signature sequence, then $\mathcal{B}$ computes the sets Σ_u^*, Σ_h^*, Σ_f^*, and Σ_e^* instead and outputs the message/signature pairs contained in the set $\Sigma_f^* \cup \Sigma_h^*$. Notice that $\mathcal{B}$ only queries messages from $\mathcal{U}$ that are in the set $\Sigma_u^* \cup \Sigma_e^*$. If (iv) occurs with non-negligible probability, then we have an adversary $\mathcal{B}$ that outputs at least $k+1$ message/signature pairs (m,σ) that are valid (i.e., $\mathsf{Vf}(pk, m, \sigma) = 1$), that are pairwise distinct, and that also differ from all message queried to $\mathcal{P}$ with non-negligible probability. Thus, $\mathcal{B}$ breaks the honest-user unforgeability of BS. Since by assumption, BS is honest-user unforgeable, our assumption that case (iv) occurs with non-negligible probability was false.

Summing up, we have shown that both case (iii) and case (iv) happen only with negligible probability. Since in cases (i) and (ii) the adversary $\mathcal{A}$ wins only with negligible probability, it follows that overall, $\mathcal{A}$ wins only with negligible probability. Since this holds for any adversary $\mathcal{A}$, BS_3 is honest-user unforgeable.

The following lemma shows that, although BS_3 is honest-user unforgeable (and thus also unforgeable), it should not be considered secure! Namely, an adversary can, given λ queries, produce $\lambda+1$ message/signature pairs, each of which passes verification with probability $\frac{1}{2}$. In particular in a setting where the machine which verifies the signatures is stateless and where the adversary may thus just resubmit a rejected signature, such signatures are as good as signatures that pass verification with probability 1. Thus, the adversary has essentially forged one signature.

Lemma 13. *We call (m,σ) a half-signature (with respect to some implicit public-key pk) if the probability that $\mathsf{Vf}(pk, m, \sigma) = 1$ is $1/2$. If BS is complete, then for any polynomial p, there is an adversary $\mathcal{A}$ that performs $\lambda+1$ interactions with $\mathcal{S}_3$ and does not query $\mathcal{P}$ and that, with overwhelming probability, outputs $p(\lambda)$ half-signatures $(m_1^*, \sigma_1^*), \ldots, (m_{p(\lambda)}^*, \sigma_{p(\lambda)}^*)$ such that all m_i^* are distinct.*

Proof. The adversary $\mathcal{A}$ that performs λ interactions with $\mathcal{S}_3$ and that never queries $\mathcal{P}$ works as follows. It picks λ distinct messages $m_1^\circ, \ldots, m_\lambda^\circ$ from Q and chooses $p(\lambda)$ additional distinct messages

$m'_j \notin Q$. It then queries the signer sequentially on the message $1\|m^\circ_i$ and obtains the corresponding signature σ°_i for $i = 1, \ldots, \lambda$. Since BS is complete, with overwhelming probability the $(m^\circ_i, \sigma^\circ_i)$ are half-signatures. Afterwards, the adversary $\mathcal{A}$ initiates another signature issue protocol session with the signer and sends as the first message: $(\texttt{extrasig}, m^\circ_1, \ldots, m^\circ_\lambda, \sigma^\circ_1, \ldots, \sigma^\circ_\lambda, m'_1, \ldots, m'_{p(\lambda)})$. The signer answers with signatures $\sigma'_1, \ldots, \sigma'_{p(\lambda)}$. Since BS is complete, with overwhelming probability the (m'_i, σ'_i) are half-signatures.

Finally, $\mathcal{A}$ stops, outputting $(m'_1, \sigma'_1), \ldots, (m'_{p(\lambda)}, \sigma'_{p(\lambda)})$.

Thus $\mathcal{A}$ outputs $p(\lambda)$ half-signatures while performing only $\lambda + 1$ queries.

5.1 Adapting the definition

We have shown that, if we allow for a probabilistic verification algorithm in the definition of honest-user unforgeability (and similarly in the definition of unforgeability), schemes that are intuitively insecure will be considered secure by the definition. There are two possible ways to cope with this problem.

The simplest solution is to require that the verification algorithm is deterministic. This is what we did in Section 4.1 (Definition 5). This choice is justified since almost all known blind signature schemes have a deterministic verification algorithm anyway. Thus restricting the verification algorithm to be deterministic may be preferable to getting a more complicated definition.[9]

In some cases, however, it might not be possible to make the verification deterministic. In such cases, it is necessary to strengthen the definition of honest-user unforgeability. Looking back at our counterexample, the problem was the following: If the adversary produces many signatures that each pass verification with non-negligible but not overwhelming probability, this is not considered an attack: The probability that all signatures pass verification simultaneously is negligible. In order to fix this problem, we thus need to change the definition in such a way that a signature that is accepted with non-negligible probability is always considered a successful forgery. More precisely, if a signature passes verification at least once when running the verification algorithm a polynomial number of times, then the signature is considered valid. This idea leads to the following definition:

Definition 14 (Honest-user unforgeability with probabilistic verification). *Given a probabilistic algorithm* Vf *and an integer* t*, we define* Vf^t *as follows:* $\mathsf{Vf}^t(pk, m, \sigma)$ *runs* $\mathsf{Vf}(pk, m, \sigma)$ t*-times. If one of the invocations of* Vf *returns* 1*,* Vf^t *returns* 1*. If all invocations of* Vf *return* 0*,* Vf^t *returns* 0*.*

A blind signature scheme $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ *is called* honest-user unforgeable (with probabilistic verification) *if the following holds: For any efficient algorithm* $\mathcal{A}$ *and any polynomial* p*, the probability that experiment* $\mathsf{HUnforge}^{\mathsf{BS}}_{\mathcal{A}}(\lambda)$ *evaluates to* 1 *is negligible (as a function of* λ*) where*

Experiment $\mathsf{HUnforge}^{\mathsf{BS}}_{\mathcal{A}}(\lambda)$
$(sk, pk) \leftarrow \mathsf{KG}(1^\lambda)$
$((m^*_1, \sigma^*_1), \ldots, (m^*_{k+1}, \sigma^*_{k+1})) \leftarrow \mathcal{A}^{\langle \mathcal{S}(sk), \cdot \rangle^\infty, \mathcal{P}(sk, pk, \cdot)}(pk)$
Let $m_1, \ldots, m_n$ *be the messages queried to* $\mathcal{P}(sk, pk, \cdot)$*.*
Return 1 *iff*
$m^*_i \neq m_j$ *for all* i, j

[9] Notice that one could weaken the requirement and only require that two invocations of the verification algorithm output the same value with overwhelming probability. This would allow for verification algorithms that essentially compute a deterministic function but have to solve problems in BPP during that computation.

$m_i^* \neq m_j^*$ *for* i, j *with* $i \neq j$*, and*
$\mathsf{Vf}^{p(\lambda)}(pk, m_i^*, \sigma_i^*) = 1$ *for all* i*, and*
$\mathcal{S}$ *has returned* ok *in at most* k *interactions.*

(When counting the interactions in which $\mathcal{S}$ *returns* ok*, we do not count the interactions simulated by* $\mathcal{P}$*.)*

Notice that the only difference to Definition 5 is that we additionally quantify over a polynomial p, and use $\mathsf{Vf}^{p(\lambda)}$ instead of Vf. If a signature is accepted with non-negligible probability, then there is a polynomial p such that $\mathsf{Vf}^{p(\lambda)}$ will accept that signature with overwhelming probability. (For our counterexample BS_3, one can choose $p(\lambda) := \lambda$ to show that it does not satisfy Definition 14.)

Notice that there is no obvious transformation for taking a signature scheme satisfying the regular unforgeability definition and constructing a scheme secure with respect to Definition 14 out of it. One obvious approach would be to include the randomness for verification in the message and thus to make the scheme deterministic. This might, however, make the scheme totally insecure because in this case a forger might include just the right randomness to get a signature accepted (if that signature would be accepted with negligible but non-zero probability otherwise). Another obvious approach would be to change the verification algorithm such that it verifies each signature p times (for a suitable polynomial p) and only accepts when all verifications succeed. This would make, e.g., half-signatures into signatures with negligible acceptance probability. But also this approach does not work in general: For any p, the adversary might be able to produce signatures that fails each individual verification with probability $1/2p$ and thus passes the overall verification with constant probability.

6 $\mathcal{S} + \mathcal{U}$-unforgeability

Reconsider the counterexample from Section 4.2 and think about it in another way: Instead of seeing the oracle $\mathcal{P}$ as an honest signer that runs the signature issue protocol with the honest user, think about it as a signing algorithm of a common (non-interactive and non-blind) signature scheme S. This intuition seems to be true as the oracle $\mathcal{P}$ takes as input a message, it then performs some internal computations using the private key, and finally outputs a signature for this message. The question now is whether the previously described attack can easily be prevented by requiring *in addition* to the interactive unforgeability, that the signature scheme S is unforgeable in the common meaning That is, for the forgery m^* w.r.t. S it must hold that $m^* \notin \{m_1, \ldots, m_n\}$, where $m_1, \ldots, m_n$ are the queries to the signing oracle Sig of S. We first give a formal definition:

Definition 15 ($\mathcal{S} + \mathcal{U}$-unforgeability). *Let* $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ *be an interactive signature scheme. We define* Sig *as the algorithm thatgets as input* (pk, sk, m) *and simulates* $(out, \sigma) \leftarrow \langle \mathcal{S}(sk), \mathcal{U}(pk, m) \rangle$ *and returns* σ*. The scheme* BS *is* $\mathcal{S} + \mathcal{U}$*-unforgeable (resp. strongly unforgeable), if* $(\mathsf{KG}, \mathsf{Sig}, \mathsf{Vf})$ *is unforgeable (resp. strongly unforgeable).*

Let us rephrase our question: If a scheme is interactively unforgeable and $\mathcal{S} + \mathcal{U}$-unforgeable, is it then automatically honest-user unforgeable? We then settle this question, somehow surprisingly, in the negative. The main intuition why this is not implied is that both properties are consider independent of each other. Thus, we construct the following counterexample where we can forge a signature if we combine malicious queries together with honest protocol executions.

Fix an interactive signature scheme $\mathsf{BS} = (\mathsf{KG}, \langle \mathcal{S}, \mathcal{U} \rangle, \mathsf{Vf})$ that is complete, randomized, blind, and strongly unforgeable. Fix some efficiently computable injective function $f \neq id$ on bitstrings (e.g., $f(m) := 0\|m$) and let g be a one-way function. We construct an interactive signature scheme $\mathsf{BS}_2 = (\mathsf{KG}_2, \langle \mathcal{S}_2, \mathcal{U}_2 \rangle, \mathsf{Vf}_2)$ as follows:

- $\mathsf{KG}_2(1^\lambda)$computes a key pair $(sk, pk) \leftarrow \mathsf{KG}(1^\lambda)$, it picks a random x in the domain of g, it sets $y := g(x)$, $sk_2 := (sk, x)$, and $pk_2 := (pk, y)$ and returns (sk_2, pk_2).
- $\mathcal{S}_2((sk, x))$ behaves like $\mathcal{S}(sk)$, except for the following extension: At any point in the interaction, the user may send a message `getx` (which is supposed never to be sent by the honest-user $\mathcal{U}$), whereupon $\mathcal{S}_2$ will return x. Thereafter, the interaction continues as with $\mathcal{S}$. (In other words, a malicious user may retrieve x for free.)
- $\mathcal{U}_2((pk, y), m)$ executes $\mathcal{U}(pk, m)$.
- $\mathsf{Vf}_2((pk, y), m, \sigma)$ executes the following steps:
  - Invoke $v := \mathsf{Vf}(pk, m, \sigma)$. If $v = 1$, return 1.
  - Otherwise, parse σ as (σ_1, σ_2, x'). If parsing fails or $\sigma_1 = \sigma_2$ or $f(x') \neq y$, return 0.
  - Invoke $v_i := \mathsf{Vf}(pk, f(m), \sigma_i)$ for $i = 1, 2$. If $v_1 = v_2 = 1$, return 1. Otherwise return 0.

Notice that the only change with respect to the counterexample from the previous section is that the secret key now contains a secret value x that is needed to "unlock" the possibility of producing additional signatures. This value x can be accessed easily by a malicious user, but an honest user will never get this value.

Lemma 16. *If* BS *is complete, strongly unforgeable, and blind, then* BS_2 *is complete, unforgeable and blind.*

The proof is analogous to that of Lemma 6 and is omitted.

Lemma 17. *If* BS *is strongly unforgeable, complete, and randomized, then* BS_2 *is strongly* $\mathcal{S} + \mathcal{U}$*-unforgeable.*

Proof. We define Sig_2 as the algorithm thatgets as input $(pk, sk, m)$and simulates $(out, \sigma) \leftarrow \langle \mathcal{S}_2(sk), \mathcal{U}_2(pk, m) \rangle$ and returns σ. Analogously, we define Sig simulating $\mathcal{S}$ and $\mathcal{U}$. By definition, to show that BS_2 is strongly $\mathcal{S} + \mathcal{U}$-unforgeable, we have to show that $(\mathsf{KG}_2, \mathsf{Sig}_2, \mathsf{Vf}_2)$ is strongly unforgeable.

Assume that this is not the case and that there is an adversary $\mathcal{A}$ that breaks the the strong unforgeability game for $(\mathsf{KG}_2, \mathsf{Sig}_2, \mathsf{Vf}_2)$. Note that since $\mathcal{U}_2$ never sends `getx`, Sig_2 never accesses x. Thus, in the strong unforgeability game, x is only used to produce $y = f(x)$. Since g is one-way, the probability that the signature $\sigma = (\sigma_1, \sigma_2, x')$ output by the adversary $\mathcal{A}$ contains x' such that $f(x') = y$ is negligible. On the other hand, if the signatures do not contain such an x', then Vf_2 coincides with Vf. But then, $\mathcal{A}$ breaks the unforgeability game for $(\mathsf{KG}, \mathsf{Sig}, \mathsf{Vf})$, which would imply that $(\mathsf{KG}, \mathsf{Sig}, \mathsf{Vf})$ is not strongly unforgeable.

However, since BS is strongly unforgeable, complete, and randomized, by Lemma 10, BS is strongly honest-user unforgeable which is easily seen to imply that BS is $\mathcal{S} + \mathcal{U}$-unforgeable. By definition, this contradicts the fact that $(\mathsf{KG}, \mathsf{Sig}, \mathsf{Vf})$ is not strongly unforgeable. Hence our assumption that $(\mathsf{KG}_2, \mathsf{Sig}_2, \mathsf{Vf}_2)$ is not strongly unforgeable was false.

Lemma 18. *If* BS *is complete and randomized, then* BS_2 *is not honest-user unforgeable.*

Proof. We construct an adversary $\mathcal{A}$ against BS_2 as follows: Let $m \in \{0,1\}^*$ be such that $f(m) \neq m$ and fix some m' with $m \neq m' \neq f(m)$. The adversary $\mathcal{A}$ queries $\mathcal{P}$ (the oracle simulating $\langle \mathcal{S}_2, \mathcal{U}_2 \rangle$) twice, both times with the same message $f(m)$. Call the resulting signatures σ_1 and σ_2. Since BS is randomized, and both $\mathcal{S}_1 = \mathcal{S}$ and $\mathcal{U}_1 = \mathcal{U}$ except for the different format of the public and secret key and for the fact that $\mathcal{S}_1$ additionally reacts to the message `getx`, with overwhelming probability, we have $\sigma_1 \neq \sigma_2$. Since BS is complete, with overwhelming probability, we have $\mathsf{Vf}(pk, f(m), \sigma_1) = \mathsf{Vf}(pk, f(m), \sigma_2) = 1$. Then the adversary $\mathcal{A}$ interacts with $\mathcal{S}_2$ directly to get a signature σ' for m'. Here $\mathcal{A}$ behaves like an honest $\mathcal{U}_2$, except that it additionally sends the message `getx` and learns x. Since BS is complete, with overwhelming probability, we have $\mathsf{Vf}(pk, m', \sigma') = 1$. Since $y = f(x)$ and $\mathsf{Vf}(pk, f(m), \sigma_1) = \mathsf{Vf}(pk, f(m), \sigma_2) = 1$ and $\sigma_1 \neq \sigma_2$, with overwhelming probability, we have $\mathsf{Vf}_2(pk_2, m, \sigma) = 1$ for $\sigma := (\sigma_1, \sigma_2, x)$. The adversary $\mathcal{A}$ outputs (m, σ) and (m', σ'). Since $\mathcal{A}$ queried $\mathcal{S}$ only once, and because $\mathcal{A}$ only queries $f(m) \neq m, m'$ from $\mathcal{U}$, this breaks the honest-user unforgeability of BS_2.

Theorem 19. *If complete, blind, and strongly unforgeable interactive signature schemes exist, then there are complete, blind, unforgeable, and strongly $\mathcal{S}+\mathcal{U}$-unforgeable interactive signature schemes that are not honest-user unforgeable.*

Proof. If complete, blind, and strongly unforgeable interactive signature schemes exist, then there is a complete, blind, strongly unforgeable, and randomized interactive signature scheme BS (e.g., by applying the transformation from Section 7). From BS we construct BS_2 as described at the beginning of this section. By Lemmas 16, 17, and 18, BS_2 is complete, blind, unforgeable, and strongly $\mathcal{S}+\mathcal{U}$-unforgeable, but not honest-user unforgeable.

7 From unforgeability to honest-user unforgeability

In this section we show how to turn any unforgeable interactive signature scheme into an honest-user unforgeable one. Our transformation is extremely efficient as it only adds some randomness to the message. Therefore, it not only adds a negligible overhead to original scheme, but it also preserves all underlying assumptions. The construction is formally defined in Construction 1 and depicted in Figure 2.

Construction 1 *Let $\mathsf{BS}' = (\mathsf{KG}', \langle \mathcal{S}', \mathcal{U}' \rangle, \mathsf{Vf}')$ be an interactive signature scheme and define the signature scheme BS through the following three procedures:*

Key Generation. *The algorithm $\mathsf{KG}(1^\lambda)$ runs $(sk', pk') \leftarrow \mathsf{KG}'(1^\lambda)$ and returns this key-pair.*

Signature Issue Protocol. *The interactive signature issue protocol for message $m \in \{0,1\}^*$ is described in Figure 2.*

Signature Verification. *The input of the verification algorithm Vf is a public key pk, a message m, and a signature $\sigma' = (\sigma, r)$. It sets $m' \leftarrow (m \| r)$ and returns the result of $\mathsf{Vf}'(pk, m\|r, \sigma)$.*

We first show that our transformation preserves completeness and blindness.

Lemma 20. *If BS' is a complete and blind interactive signature scheme, so is BS.*

Since the proof follows easily, we omit it here.

Now, we prove that our construction turns any unforgeable scheme into an honest-user unforgeable one.

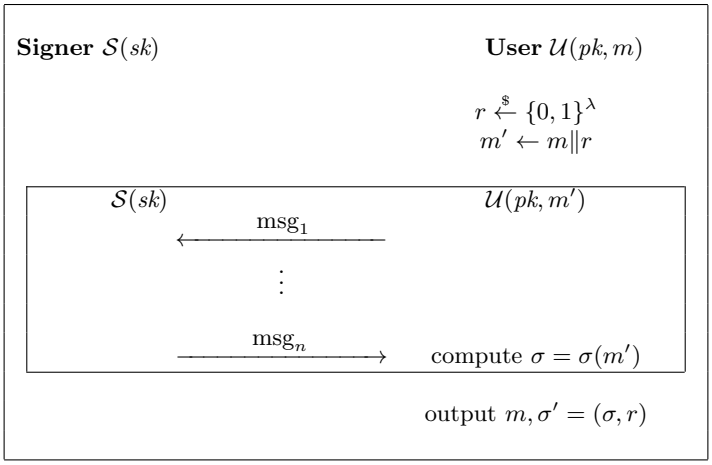

Fig. 2. Issue protocol of the blind signature scheme

Lemma 21. *If* BS' *is an unforgeable interactive signature scheme, then* BS *is an honest-user unforgeable interactive signature scheme.*

Proof. Assume for the sake of contradiction that BS is not honest-user unforgeable. Then there exists an efficient adversary $\mathcal{A}$ that wins the honest-user unforgeability game with non-negligible probability. We then show how to build an attacker $\mathcal{B}$ that breaks the unforgeability of BS'.

The input of the algorithm $\mathcal{B}$ is a public pk. It runs a black-box simulation of $\mathcal{A}$ and simulates the oracles as follows. Whenever $\mathcal{A}$ engages in an interactive signature issue protocol with the signer, i.e., when the algorithm $\mathcal{A}$ plays the role of the user, then $\mathcal{B}$ relays all messages between $\mathcal{A}$ and the signer. If $\mathcal{A}$ invokes the oracle $\mathcal{P}$ on a message m, then $\mathcal{B}$ picks a random $r \xleftarrow{\$} \{0,1\}^\lambda$, sets $m' \leftarrow m\|r$, and engages in an interactive signature issue protocol where $\mathcal{B}$ runs the honest user algorithm $\mathcal{U}'$. At the end of this protocol, the algorithm $\mathcal{B}$ obtains a signature σ on the message m'. It sets $\sigma' \leftarrow (\sigma, r)$, stores the pair (m', σ') in a list L and returns σ' together with the corresponding transcript trans to the attacker $\mathcal{A}$.

Eventually, the algorithm $\mathcal{A}$ stops, outputting a sequence of message/signature pairs (m_1^*, σ_1^*), $\ldots, (m_{k+1}^*, \sigma_{k+1}^*)$. In this case, $\mathcal{B}$ recovers all message/signature pairs $(m_1', \sigma_1'), \ldots, (m_n', \sigma_n')$ stored in L, it parses σ_i^* as (σ_i', r_i'), it sets $\widetilde{m}_i \leftarrow m_i^*\|r_i^*$ and $\widetilde{\sigma} \leftarrow \sigma_i'$ for all $i = 1, \ldots, k+1$ and outputs $(m_1', \sigma_1'), \ldots, (m_n', \sigma_n'), (\widetilde{m}_1, \widetilde{\sigma}_1), \ldots, (\widetilde{m}_{k+1}, \widetilde{\sigma}_{k+1})$.

Analysis. For the analysis first observe that $\mathcal{B}$ runs in polynomial time because $\mathcal{A}$ is efficient and because the handling of all queries can be done efficiently. Suppose that $\mathcal{A}$ succeeds with non-negligible probability. Then it outputs $(k+1)$ message/signature pairs that verify under Vf. Since $\mathcal{B}$ runs the honest user algorithm to compute the signatures $\sigma_1', \ldots, \sigma_n'$ it follows (from the completeness) that all message/signature pairs that $\mathcal{B}$ returns, verify with overwhelming probability. It is left to show that a) the algorithm $\mathcal{B}$ output one more message/signature pair (than queries to the signing oracle with output ok took place) and b) all messages are distinct.

The distinctness property follows immediately from the definition of the success probability in the honest-user unforgeability game and from the construction. More precisely, consider the messages $(m_1', \ldots, m_n')$ and $(\widetilde{m}_1, \ldots, \widetilde{m}_{k+1})$, where $m_i' = m_i\|r_i$ and $\widetilde{m}_j = m_j^*\|r_j^*$. According to our

assumption that $\mathcal{A}$ succeeds, it follows that all message pairs m_r^* and m_s^* (for all $r \neq s$) differ from each other. But then it follows easily that $\widetilde{m}_r^*$ and $\widetilde{m}_s^*$ are also distinct (for all $r \neq s$). Since the r_i are chosen randomly, the messages $(m'_1, \ldots, m'_n)$ also differ from each other with overwhelming probability. Now, consider the messages $(m_1, \ldots, m_n)$ that $\mathcal{A}$ send to the oracle $\mathcal{P}$. Note that all these messages must differ from the messages $(m_1^*, \ldots, m_{k+1}^*)$ returned by $\mathcal{A}$ by definition. This means, however, that $\widetilde{m}_r^*$ differs from m'_i for all i, r.

Finally we have to show that $\mathcal{B}$ returns one more message/signature pair (property (a)) than protocol executions with the signer $\mathcal{S}'$ took place (and that produced output ok). Since $\mathcal{A}$ wins the game, it follows that in at most k of the protocol executions that $\mathcal{B}$ forwarded between $\mathcal{A}$ and its external signer, the signer returned ok. $\mathcal{B}$ itself has executed n user instances to simulate the oracle $\mathcal{P}$. Since $\mathcal{A}$ outputs $k+1$ message signature pair (s.t. $m_i \neq m_j^*$ for all i, j) it follows that $\mathcal{B}$ has asked at most $n+k$ queries in which the signer $\mathcal{S}'$ returned ok, but $\mathcal{B}$ returned $n+k+1$ message/signature pairs. This, however, contradicts the assumption that BS is unforgeable.

Putting together the above results, we get the following theorem.

Theorem 22. *If complete, blind, and unforgeable interactive signature schemes exist, then there are complete, blind, unforgeable, and honest-user unforgeable interactive signature schemes.*

The proof of this theorem follows directly from Lemmas 20 and 21.

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