W.E. JOHNSON'S 1913 PAPER AND THE QUESTION OF HIS KNOWLEDGE OF PARETO

BY

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ABSTRACT

In 1913, the Cambridge logician W.E. Johnson published a famous article on demand theory in the *Economic Journal*. Although Johnson's treatment of the subject strongly resembles the analysis set forth by Pareto in the *Manual of Political Economy*, Johnson does not cite the Italian economist. This has aroused a long-standing debate about Johnson's actual acquaintance with Pareto's works, but the debated point has never been thoroughly investigated. The present paper addresses the question of Johnson's knowledge of Pareto both from an analytical and historical viewpoint, by examining Johnson's life in the Cambridge environment and his available unpublished papers. Even though the new evidence gathered gives some weight to the thesis that Johnson could not have been unaware of Pareto's *Manual*, it cannot exclude the possibility that the logician wrote his paper autonomously.

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I. INTRODUCTION

In *December* 1913, William Ernest Johnson (1858-1931), a Cambridge logician, published a famous article on demand theory in the *Economic Journal* (*EJ*) entitled "The Pure Theory of Utility Curves" (Johnson 1913). Although Johnson's treatment of the subject was in some ways original, in others it strongly resembled the analysis set forth by Vilfredo Pareto in earlier contributions, particularly in the *Manual of Political Economy* (Italian edition 1906; French edition 1909). Despite this resemblance, Johnson did not cite Pareto. This failure to acknowledge Pareto's precedence aroused resentment and some suspicion of plagiarism among the Italian Paretians. In the end, however, the Paretian economists of the period generally assumed that Johnson was unfamiliar with Pareto's works and had obtained his results independently. For example, in 1916, Luigi Amoroso published a review of Johnson's 1913 paper in the *Giornale degli Economisti*, in which he wrote:

> *From Johnson's article it comes out that Johnson does not know Pareto's work* (Amoroso 1916, p. 410, my translation).

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Amoroso attributed Johnson's ignorance of Pareto's work to a more general ignorance on the part of English economists:

> *Johnson's lacuna, more than particular to him, is a lacuna of the area in which he lives* (Amoroso 1916, p. 410, my translation).

However, the doubts concerning Johnson's familiarity with Pareto's works did not disappear. For instance, in their classic paper on demand theory, J.R. Hicks and R.G.D. Allen noted: "Johnson's work *does not appear* to spring directly from Pareto" (Hicks and Allen 1934, p. 54, italics added). In his *History of Economic Analysis*, J.A. Schumpeter observed in a somewhat more doubtful way:

> This important paper [that of Johnson] contains several results that should secure for its author a place in any history of our science. But, having *apparently* been written in ignorance of Pareto's work, it aroused not unnatural resentment on the part of Italian economists because of its failure to acknowledge Pareto's priority in most essentials (Schumpeter 1954, p. 1063, note 5, italics added).

In their anthology of precursors in mathematical economics, W.J. Baumol and S.M. Goldfeld instead affirm quite peremptorily that "Johnson arrived at his conclusion independently" (Baumol and Goldfeld, 1968, p. 96). On the contrary, M. Blaug and P. Stuges (1983, p. 191) and, more recently, L. Bruni (2002, p. 104) leave open the question of Johnson's familiarity with Pareto.

How do things stand? What are, precisely, the similarities and differences between the ordinal utility theory expounded by Pareto in the *Manual* and the analysis presented by Johnson in his 1913 article? And who was this Cambridge logician who made such an important but also peculiar contribution to economic theory? Are there any primary sources (correspondence, diaries, other papers) which can be useful to clear up the long-standing

question of Johnson's familiarity with Pareto's work?

In order to address these questions, section two illustrates the most significant aspects of Johnson's life, especially those that could be particularly relevant to this study. Section three briefly presents Johnson's contributions to economics before 1913. Section four touches on the relationship between Johnson's lectures on mathematical economics at the University of Cambridge and his *EJ* article. Sections five and six present a detailed examination of Johnson's controversial paper and an analytical comparison with the French edition of Pareto's *Manual*, the edition the Cambridge Library has owned since 1910. Section seven explores the influence of Johnson's article on subsequent economic theory. Section eight considers the personal and intellectual relationships between Johnson and other individuals who may have played some role in the story. This part of the paper is largely based on a research carried out in the Cambridge archives and provides some new evidence regarding Johnson's knowledge of Pareto's work. In section nine, the evidence drawn from the initial overview of Johnson's life, the analytical comparison between Johnson's 1913 article and Pareto's *Manuel*, and the study of Johnson's personal relationships, is summed up. Even though the detailed analysis of all the relevant sources and the assessment of all the parts of the puzzle give a certain weight to the thesis that Johnson could not have been unaware of Pareto's *Manuel*, it cannot exclude the possibility that the eccentric logician may have written his 1913 paper autonomously. The result of my inquiry is therefore negative, in the sense that the question of Johnson's acquaintance with Pareto's work seems destined to remain unsolved.

II. JOHNSON, A BIOGRAPHICAL OVERVIEW

W.E. Johnson was an exemplary son of Cambridge where he spent almost all his life. He was born there on 23 June 1858, the fifth child of the headmaster of Llandaff House, a

Cambridge academy. In 1879 he won a Mathematical scholarship at King's College and was eleventh wrangler in the mathematical Tripos of 1882. Johnson was also interested in moral science, and particularly in logic. In 1883 he was placed in the first class in Moral Sciences Tripos and in 1885 obtained the M.A. degree.

Since Johnson was not well-to-do, for some years he had to earn his living as a tutor in mathematics. In 1884 he competed for a fellowship at King's College, submitting a dissertation entitled "Essay on Symbolic Logic" but failed to win[1]. At first he lectured on Psychology and Education in various Cambridge colleges and, from the Easter Term 1887, began teaching at the University for the Moral Sciences Tripos although his position was not permanent. From 1887 to 1900 he lectured on Logic and then on Psychology, and from the Easter Term 1901 also taught a course on "Diagrammatic Treatment of Pure Economic Theory"[2]. From 1893 to 1898 he was also University Teacher in the Theory of Education. In 1902, Johnson was finally appointed to the newly created Sidgwick Lectureship in Moral Science at the University and was elected a Fellow of King's College. He held these positions for the rest of his life. He continued to teach Logic, Economics, Psychology, and successively Probability until the academic year 1921-22. From then on, his lecturing was limited to Logic until his death on 14 January 1931.

Johnson did not publish much. Following his treatise on *Trigonometry* (1888), in the 1890s he published a paper on "The Logical Calculus" in *Mind* (1892), some reviews and notes on logical topics in the same Journal, and two other papers on economics appeared in the *Cambridge Economic Club* (Johnson 1891, Johnson and Sanger 1894). He also wrote fourteen entries in the first edition of *Palgrave's Dictionary of Political Economy* (1894-1899). In 1900 Bertrand Russell, at that time a colleague of Johnson's at Cambridge, presented a note by Johnson entitled "Sur la théorie des équations logiques" at the *Congrès International de Philosophie* held in Paris. The note subsequently appeared in the

proceedings of the Congress (Johnson 1901). For the next twelve years Johnson did not publish anything until the *EJ* article appeared in December 1913. Five years later, he published a paper on "The Analysis of Thinking" in *Mind* (1918). With the help of his pupil Miss Naomi Bentwich, in the 1920s Johnson finally succeeded in organizing and putting together his ideas on logic and published his three volume *Logic* (1921-24), which made him well known outside Cambridge. Because of Johnson's ill-health, a fourth volume devoted to the foundations of probability was planned but never completed. The first three chapters were published posthumously in 1932.

For a long time, the paucity of his publications was a source of anxiety for Johnson, since his King's Fellowship was only for a period of years and the question of prolonging it came up periodically. In the end, Johnson kept his Fellowship due to the high regard in which he was held by his colleagues. Indeed, Johnson's notable influence on the Cambridge moral scientists of the period was mainly exerted through personal exchanges or lectures (his pupils included J.M. Keynes, F. Ramsey, L. Wittgenstein, C.D. Broad). A student and later colleague of Johnson's compared him "primarily [to] a Socrates rather than [to] a Plato" (A.D. 1932, p. 136). All testimonies concur in considering him a highly autonomous mind, though a slightly eccentric person:

> He gave above all the impression of one steadily pursuing his own way, and of not caring very much whether his view became known or not, or whether anyone agreed with him (A.D. 1932, p. 137).

In the second part of his life, this attitude led Johnson to disregard the latest developments in his areas of research. His pupil C.D. Broad reports that:

> Though extremely well versed in the works of the great philosophers and in the classical physics, he read hardly any contemporary books on either physics or

> philosophy. I should doubt whether he had looked into any work by one of his colleagues since the first edition of Mr. Russell's *Principia Mathematica* [1910-1913] except Mr. Keynes's *Treatise on Probability* [1921] (Broad 1931, p. 509).

This, however, does not mean that Johnson was an isolated figure at Cambridge. J.M. Keynes recalls that Johnson's house was "one of the greatest centres of talk and social life in Cambridge" (quoted in Broad 1931, p. 510). University scholars as well as his current and former pupils were invited to Johnson's famous Sunday afternoon tea-parties. He was an extremely good conversationalist and talk was generally on serious subjects.

The last aspect of Johnson's life which is relevant for this paper is his knowledge of foreign languages, particularly those in which Pareto's main works appeared, namely Italian and French. Even though in his entries in *Palgrave's Dictionary*, Johnson refers to the writings of some Italian economists (L. Cossa, M. Pantaleoni, C. Supino) and some French ones (F. Bastiat, M. Block, A.E. Cherbuliez, A.A. Cournot, J.B. Say), he does not cite any passage from their works[3]. In a slightly more specific way, in his entry "Producer's Goods", Johnson refers to the distinction between the producer's good and the consumer's good "first given by J.B. Say (*Course de l'Economie Politique*, pt. i, ch. xii)" (cf. Johnson 1899a, p. 212). Checking Johnson's complete works on both economics and logic, I found very few quotations, and none from French or Italian authors. However, according to Johnson's pupil and biographer C.D. Broad, when Johnson was 19 he spent a winter at Hyères, Provence for health reasons and loved Switzerland as a holiday destination (cf. Broad 1931, pp. 497, 505). Moreover, it is worth pointing out that in those days proficiency in the French language was more common than it is today, particularly at Oxford and Cambridge, and that, as mentioned earlier, Johnson had published a note in French, with no indication of the translator. All these elements seem to indicate that Johnson may have at least been able to read French[4].

III. JOHNSON'S EARLY WORK IN ECONOMICS

Johnson's first 6-page economic paper is entitled "Exchange and Distribution" and was printed in the *Cambridge Economic Club*, Lent Term 1891. The paper presented a simple mathematical model of general equilibrium with a market for commodities and a market for production factors. The entrepreneurs produce commodities and maximize their profit by equaling the marginal revenue product of each factor to its given price. The agents of production (workers, land-owners and capital-owners) demand the goods supplied by the entrepreneurs and supply the production factors in their possession. In equilibrium, demand and supply for each commodity and for each production factor are equal. By using this model, Johnson attempted to show "how the exchange-prices of commodities and the different shares of remuneration amongst the agents of production mutually determine one another" (Johnson 1891, p. 67). However, he did not discuss any of the potential problems (e.g. solvability or existence problems) that this model presents.

Johnson wrote his second economic paper with C.P. Sanger, a young pupil of Marshall's whose role in the question of Johnson's knowledge of Pareto will be considered more closely in Section VIII. Their joint, 8-page article was entitled "On Certain Questions Connected with Demand" and published in the Easter 1894 term of the bulletin of the Cambridge Economic Club. Johnson and Sanger studied diverse issues of utility and demand theory, using a mathematichal approach. They considered the utility maximization problem under the budget constraint, adopting a general interdependent utility function instead of the additive utility function that was the standard one at the time. In working out the first-order conditions for the constrained optimum, they made use of what we currently call "indirect utility function" and spelled out different economic interpretations of the concept of marginal utility of money. In particular, they focused on the Bernoullian case in which the marginal utility of money is inversely proportional to the individual's amount of money.

Subsequently, Johnson and Sanger sough to identify a measure of the consumer's rent within a cardinal view of utility. Lastly, they examined variations in the elasticity of demand in connection with variations in price. Even though Johnson and Sanger dealt with all these topics in an innovative way, their paper did not have a great impact on the subsequent demand theory. This might be due to several reasons: the paper was extremely concise, the *Cambridge Economic Club* had just a local circulation, and, with reference to the Cambridge environment, the paper might have been too abstract for the standards of Marshallian economics.

Johnson's last contributions to economic theory before his 1913 article are the fourteen entries written for the first edition of *Palgrave's Dictionary of Political Economy* (1894-1899). They deal with demand and supply theory as well as with questions relating to economic methodology[5]. The most comprehensive entries on economic topics are "Supply" and "Supply and Demand", which appeared in the third volume of the *Dictionary* (Johnson 1899b and 1899c). These entries mainly follow the treatment given to these subjects by Marshall in the Books III and V of the *Principles*.

IV. TOWARDS THE *EJ* PAPER

Although between 1899 and 1913 Johnson did not publish anything on economics, in 1901 he began his course on mathematical economics which he continued until 1921. From 1901 to 1908 the title of the course remained "Diagrammatic Treatment of Pure Economic Theory", whereas from 1909 to 1921 the title became simply "Pure Economic Theory". Unfortunately the *Cambridge University Reporter*, the official bulletin of the University of Cambridge, does not report the syllabus of Johnson's course. In December 1913, Johnson's chief economic contribution appeared in the *EJ* with a title – "The Pure Theory of Utility Curves" – which strongly resembles the title of his Cambridge course. It therefore seems

plausible that the article was mainly an outgrowth of Johnson's lecture notes.

Johnson's *EJ* paper is divided into four Parts and the similarities with Pareto's *Manuel* are mainly found in the first two. In the section five, I will examine in detail Part I and II, stressing the passages where Johnson's paper resembles more closely Pareto's *Manuel*. When comparing the two, I shall refer to the French edition of Pareto's book since, at that time, it had a wider circulation outside Italy. In particular, it is noteworthy that the Cambridge Library has owned a copy of the *Manuel* since 1910: it was catalogued on 22 December 1910, that is, three year before the appearance of Johnson's article[6]. In section six, I shall present the content of the last two Parts of Johnson's paper.

V. PARTS I-II, AND THEIR SIMILARITIES TO PARETO'S *MANUEL*

In Part I of his paper, Johnson puts forward a modification of the indifference curves that F.Y. Edgeworth had drawn in the two-commodity case. In Edgeworth's graph, the abscissa represents the amount of a commodity acquired and the ordinate the amount of the other good which is sacrificed in return for the former (cf. Edgeworth 1881, pp. 28, 106). Johnson suggests representing in both axes the amounts of the goods that contribute positively to the utility, and proposes that the goods be purchased not by sacrificing another good but by means of an assigned amount of money, as in the 1894 article written with Sanger. This leads to the by now familiar indifference curves-budget line diagram (cf. Johnson 1913, p. 489, fig. 4).

Johnson's diagram is, however, very similar to some of the diagrams used by Pareto in the *Manuel* (cf. Pareto 1909, p. 184, fig. 12; p. 343, fig. 44; p. 345, fig. 46). The main difference between Johnson's and Pareto's diagrams lies in the interpretation of the constraint line. For Pareto, it represents a constant transformation function of a commodity into another through production or exchange, whereas for Johnson it simply represents the

monetary constraint, in line with the modern view.

Johnson subsequently shows how the new diagram makes it possible to analyze graphically the effects on demand of a variation in the individual's income as well as in the commodity prices. By connecting the bundles of maximum utility which are purchased when the income varies, Johnson obtains the "varying *expenditure* curve" (Johnson 1913, p. 490), now known as the income-consumption curve. By connecting the optimal bundles which are purchased when the price of one commodity varies, Johnson obtains the "varying *price* curve" (Johnson 1913, p. 490) now known as the price-consumption curve.

Finally, in the last section of Part I, Johnson points out the non-cardinal nature of his indifference curves-budget line diagram:

> There are no lines in the figure which measure the utility itself. The several utility-curves [*i.e.* the indifference curves] are arranged in a scale of increasing value as we pass to the right and above; and thus the 'distance' (measured arbitrarily) from one curve to another 'indicates' (without measuring) the increase in utility (Johnson 1913, p. 490).

This observation concurs with Pareto's emphasis on the ordinal nature of the indifference apparatus. Johnson also observes that:

> Economics [does not] need to know the marginal (rate of) utility of a commodity. What is needed is a representation of the *ratio* of one marginal utility to another (Johnson 1913, p. 490).

Despite these ordinal insights, many cardinal concepts remain in Johnson's demand theory: *i*) he talks about "*equal* additional increments of net utility" stemming from the continually increasing acquisition of a good (Johnson 1913, p. 485); *ii*) he attributes meaning to the distance between indifference curves (Johnson 1913, pp. 490-91, 499, 503); *iii*) most of all,

he makes assumptions regarding the sign and the magnitude of the second-order and cross-partial derivatives of the utility function (Johnson 1913, pp. 492-93)[7].

Regarding this last point, it is noteworthy that Pareto also refers to decreasing marginal utility and to the sign of the second-order cross-partial derivatives without seeming to realize that these properties are not invariant to an arbitrary, strictly increasing transformation of the utility function (cf. Pareto 1909, pp. 263 ff., 572 ff.). Therefore, an affinity emerges between Pareto's and Johnson's ordinal approaches as well as an affinity in their cardinal flaws.

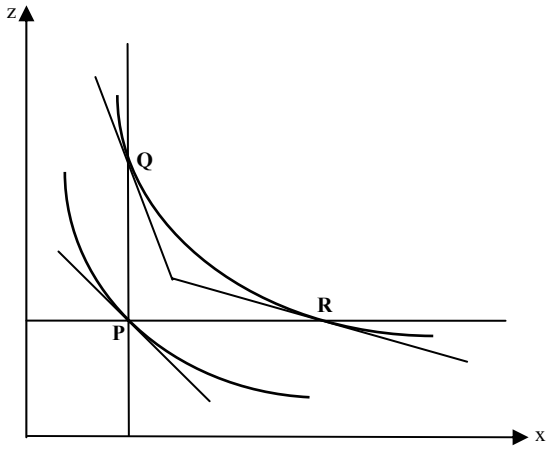
In Part II, Johnson discusses the analytical conditions that ensure the convexity of indifference curves in the two-commodity case. He considers the ratio between the marginal utility of x (u_x) and the marginal utility of z (u_z), and affirms that "in the standard case, a change in the amount of x would produce a greater relative change in the marginal utility of x than in that of z" (Johnson 1913, p. 493). In mathematical terms, Johnson assumes that:

$$\frac{\partial\left(\frac{u_x}{u_z}\right)}{\partial x} = \frac{u_{xx}u_z - u_x u_{xz}}{(u_z)^2} < 0 \tag{1}$$

where u_{ii} is the second-order derivative of the utility function with respect to the commodity i, and u_{ij} is the second-order cross-partial derivative with respect to the commodities i and j. Symmetrically, Johnson assumes that "[in the standard case] a change in z would produce a greater relative change in the marginal utility of z than in that of x" (Johnson 1913, p. 493), *i.e.*:

$$\frac{\partial\left(\frac{u_z}{u_x}\right)}{\partial z} = \frac{u_{zz}u_x - u_z u_{zx}}{(u_x)^2} < 0 \tag{2}$$

The economic meaning of these inequalities can be further illustrated in the following figure which reproduces with slight modifications Figure 8a of Johnson's paper:



The sign of $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ tells us how the slope of the indifference curves varies when the individual possesses a larger quantity of commodity *x*, and the quantity of commodity *z* at his disposal remains constant, as in the movement from point P to point R in the figure. This is a movement from one indifference curve to another, and not a movement on the same indifference curve, as when we consider the variations of the marginal rate of substitution. Since *x* becomes relatively more abundant with respect to *z* by going from P to R, Johnson deems that, in the standard cases, the individual will be willing to give up less quantity of *z* to consume one more unity of *x*, so that, in the standard cases, $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ is negative. Let us now consider the movement from P to Q. With an argument analogous to the previous one, since *z* becomes relatively more abundant with respect to *x*, Johnson estimates that the individual will generally be willing to give up more quantity of *z* to consume one more unity of *x*, so that $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial z$ is positive. Since the sign of $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial z$ is the opposite of the sign of $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$, in the standard cases $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ is negative, as requested by inequality (2).

Yet, Johnson's discussion on the standard signs of $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and of $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ is

extremely similar to that presented by Pareto in the Appendix of the *Manuel* (Pareto 1909, pp. 572-73). Furthermore, Johnson's Figure 8 seems to be an outgrowth of Pareto's Figure 59 (cf. Johnson 1913, p. 497, and Pareto 1909, p. 573).

Johnson admits that the standard relations $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x < 0$ and $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z < 0$ "do not hold universally" (Johnson 1913, p. 493). In such cases a change in the quantity of a commodity would produce a greater relative change in the marginal utility of the other commodity. According to Johnson, this phenomenon is due to the fact that one good "is more urgently needed" than the other (Johnson 1913, p. 494). These are, however, non-standard cases. In particular, Johnson assumes that if one ratio of marginal utilities varies in a non-standard way, the variation of the other ratio must be regular and greater in absolute value than the former. This means that the following inequality must hold:

(3) $\frac{u_{xx}}{(u_x)^2} - \frac{u_{xz}}{u_x u_z} + \frac{u_{zz}}{(u_z)^2} - \frac{u_{xz}}{u_x u_z} < 0$, that is, $2u_x u_z u_{xz} - u_{xx}(u_y)^2 - u_{yy}(u_x)^2 > 0$

Yet, this is exactly the condition for the convexity of indifference curves for the two-commodity case. Once more, Johnson's statement of the convexity condition starting from the analysis of the standard signs of $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and of $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ is almost identical to that expounded by Pareto in the Appendix of the *Manuel* (cf. Johnson 1913, p. 496, and Pareto 1909, pp. 572 ff.).

In the same pages of the *Manuel*, Pareto also relates the discussion about the sign of $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and of $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ to the distinction between complementary and competitive goods. He endorses the cardinal definition of complementary and competitive goods set forth by Edgeworth which relies on the sign of the second-order cross-partial derivatives of the utility function (cf. Edgeworth 1897a). According to this definition, two goods are complements if $u_{xz} > 0$ and substitutes if $u_{xz} < 0$. Pareto observes that, under the standard

assumptions of a positive but decreasing marginal utility, if the two goods are complements, $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ are negative (this can easily shown by a simple substitution). Therefore, with complementary goods the standard inequalities (1) and (2) hold, whereas non-standard cases may arise with competitive goods (cf. Pareto 1909, p. 573).

Even Johnson relates the discussion about the sign of $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ to the distinction between complementary and competitive goods, however going in the opposite direction with respect to Pareto. Instead of starting from a definition of complementarity and examining what happens to conditions (1) and (2) when the goods are complements, Johnson defines as "complements" the goods for which both the inequalities (1) and (2) are satisfied, that is, the pair of goods in which neither is more urgently needed. All residual goods are "substitutes" (cf. Johnson 1913, pp. 495-96). Johnson's distinction between complements and substitutes is therefore just a reformulation of his basic distinction between standard and non-standard cases. However, he neither motivates his definition of complementarity, nor uses it in the rest of his paper[8].

With reference to Johnson's and Pareto's definitions of complementary goods, three observations are in order. First, since Pareto's complements satisfy inequalities (1) and (2), they constitute a subset of Johnson's complementary goods. Second, unlike Pareto's definition, Johnson's is invariant to an increasing transformation of the utility function and could therefore be used in an ordinal utility framework. Although Johnson does not underline this aspect of his own definition, it would subsequently draw the attention of some economists (see below). Third, Johnson's and Pareto's definitions lead to the same graphical representation of the indifference maps for perfect substitutes and perfect complements. More precisely, Figure 7a (indifference maps for perfect substitutes) and 7b (indifference maps for perfect complement) of Johnson's paper are identical respectively to Figure 36 and

Figure 31 of the *Manuel* (cf. Johnson 1913, p. 495, and Pareto 1909, pp. 276, 280).

VI. THE LAST TWO PARTS OF JOHNSON'S PAPER

The first sections of Part III examine in a pioneering way how the demand for commodities varies in response to variations in the individual's income, assuming the commodity prices remain constant. Through a procedure that appears rather tricky to the modern reader, Johnson shows that, when $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ are negative, a rise in the income determines a rise in the demand of both commodities. According to current terminology, this means that in Johnson's standard cases both commodities are normal. Moreover, Johnson shows that, when $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ is negative but $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ is positive, *i.e.* when z is more urgently needed than x, an increase in the income produces a rise in the demand for z and a fall in the demand for x. In current terminology, z is a normal good and x is an inferior one. Symmetrically, when x is more urgently needed than z, x is normal and z is inferior (cf. Johnson 1913, pp. 499-502).

In the last sections of Part III, Johnson considers the variations in the demand for goods in response to variations in the price of one commodity, the other variables being constant. Although even in this case Johnson's analysis is rather peculiar, at least two notable, original results should be pointed out. First, he provides a mathematical treatment of "the case in which an increased price leads to an *increase* of the amount of the commodity bought (*i.e.* [the] Giffen's paradox)" (Johnson 1913, p. 484)[9]. Second, Johnson shows that the Giffen goods are a subset of the inferior goods. More precisely, he demonstrates that the Giffen case arises when a good is more urgently needed than the other over a certain value. Taking into account the above-mentioned results about the relationships between the relative urgency of a good and its normal/inferior nature, this means that the Giffen goods are

inferior goods for which the degree of relative urgency is particularly high (cf. Johnson 1913, p. 505).

In Part IV of his paper, Johnson examines the problem of the constrained maximization of a function of *n* variables. This problem has the mathematical form: *max* $f(x_1,...,x_n)$ subject to $\sum_{i=1}^{n} p_i x_i = m$. According to the interpretation given to f (as a utility function rather than as a production function), to $\sum_{i=1}^{n} p_i x_i$ (as the total cost of the goods purchased rather than the total cost of the factors employed), and to m (as the fixed income of the individual rather than as the fixed expenditure assigned by the entrepreneur to production), we obtain a pair of completely analogous optimization problems for the consumer and the firm. Johnson therefore discusses "the question of the maximum product (or utility) derivable from the expenditure of a given sum on *n* factors (or commodities)" as a single question (Johnson 1913, p. 484). He derives the first-order conditions for a maximum, and states the second-order conditions for the general *n*-variables case, observing that "these conditions are equivalent to the statement that the 'surfaces' [that is, the isoquants or the indifference curves] are in all directions *convex* to the co-ordinate axes" (Johnson 1913, p. 509)[10]. Lastly, Johnson considers the maximization problem for special types of production/utility function, namely those of the separable form $f(\psi(x_1,x_2),\chi(x_3,x_4))$ (Johnson 1913, pp. 512-13).

With reference to this last part of Johnson's paper, it is noteworthy that he never draws attention to the main difference between the utility maximization problem and the product maximization problem, namely, the ordinal nature of the former and the cardinal nature of the latter. This confirms the partially cardinal nature of Johnson's contribution.

VII. THE INFLUENCE OF JOHNSON'S PAPER

Like Johnson's previous papers, the 1913 article is extremely concise. However, this feature did not prevent it from drawing the attention of some other economists. In a paper published in the *EJ* in 1915, Edgeworth amply illustrated the content of Johnson's article which he judged "a unique contribution to the subject" (Edgeworth 1915, p. 49). Edgeworth was, however, not convinced of Johnson's definition of complementarity, claiming that the definition in terms of the sign of u_{xz} that he proposed and Pareto accepted was, in the end, preferable (cf. Edgeworth 1915, pp. 49-51). Notably, as will be shown below, in his paper Edgeworth explicitly linked Johnson's analysis to that of Pareto. Another *EJ* article, published by A.W. Zotoff in March 1923, was devoted to expounding "in a more simple, *but at the same time more general way*" (Zotoff 1923, p. 115) the question treated by Johnson in Part IV of his paper regarding the maximum product that can be derived from a given expenditure.

In 1924 A.L. Bowley published his influential book on *The Mathematical Groundwork of Economics* and placed Johnson's analytical contribution alongside those of the main mathematical economists of the time:

> I have attempted to reduce to a uniform notation, and to present as a properly related whole, the main part of the mathematical methods used by Cournot, Jevons, Pareto, Edgeworth, Marshall, Pigou, and Johnson (Bowley 1924, p. v).

In particular, Bowley referred to Johnson's analysis of demand functions and to his examination of the relationship between changes in the output and changes in the costs (cf. Bowley 1924, pp. 32, 57).

Johnson's article drew renewed attention for its non-cardinal insights in the years 1934-1939 which were characterized by the completion of the ordinal revolution in choice theory initiated by Pareto. For instance, in their ordinal reconsideration of demand theory, Hicks

and Allen deemed that Johnson's 1913 work was "much less dependent upon a 'cardinal' conception of utility than any of theirs [the works of Edgeworth and Pareto]" (Hicks and Allen 1934, p. 54). In that period, one of the notions that appeared to be abandoned because of its cardinal nature was the Edgeworth-Pareto (EP) definition of complementarity. The dismissal of the EP definition led to several attempts to replace it by a notion matching the new ordinal framework. Johnson's criterion for complementarity could serve the purpose thanks to its invariance to an increasing transformation of the utility function.

It was, in particular, R.G.D. Allen who recovered and developed Johnson's definition of complementarity as an ordinal alternative to the EP definition (cf. Allen 1934a, 1934b, 1934c). Allen observed that the sign of $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and of $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ can vary as x and z become relatively more abundant, so that, under Johnson's definition, two goods can result alternatively complements and substitutes. On the other hand, Allen considered only the variations of x and z in the region for which none of the commodities possessed has reached the point of satiety. This region is called "effective-region" (Allen 1934c, p. 171). By making these two specifications, Allen slightly modified Johnson's notion of complementary goods, defining "complements" those goods for which $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ and $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ remain negative at all positions within the effective region. All residual goods are labeled "substitutes" (cf. Allen 1934c, pp. 170 ff.). Allen, however, quickly abandoned this definition of complementarity. As a matter of fact, in his joint article with Hicks, he proposed a different definition of complementary and competitive goods which is equivalent to the current standard definition in terms of the effect on the demand for a good i of a compensated price change of good j (cf. Hicks and Allen 1934, pp. 69 ff. and 209-11).

In his *Theory and Measurement of Demand*, Henry Schultz thoroughly discussed Johnson's definition of complementarity and how Allen developed it (Schultz 1938, pp. 608

ff.). Schultz was interested in finding a criterion to use in statistical analysis to distinguish different type of interrelations among goods. However, Schultz's statistical findings on beef and pork, coffee and tea, and some other goods, always satisfy the Johnson-Allen inequalities so that all goods prove to be complementary. Therefore, concluded Schultz, "the Johnson-Allen test [...] gives us no clue as to the nature of the relationship existing between commodities" (Schultz 1938, p. 612). According to Schultz, only the Hicks-Allen definition of complementarity possesses empirical significance.

In 1939 Hicks published *Value and Capital* (Hicks 1939). The book contained a systematic presentation of the ordinal theory of demand as well as an improved exposition of the Hicks-Allen definition of complementarity which became the standard one from that time on. After the publication of *Value and Capital*, the ordinal insights and the notion of complementarity found in Johnson's paper lost all interest and the attention given to Johnson's paper vanished.

VIII. BACK TO CAMBRIDGE

In the previous sections, the question of Johnson's knowledge of Pareto was mainly addressed from an analytical viewpoint. I shall now consider the personal and intellectual relationships between Johnson and the other individuals who may have played some role in our story. Even though the results are not definitive, this particular research provides some new evidence regarding Johnson's knowledge of Pareto's work.

Sidgwick

Henry Sidgwick (1838-1900) was Johnson's mentor at Cambridge. He first recognized Johnson's merit and brought him forward, supporting him with «constant help and the advice and encouragement», as Johnson himself wrote in a letter of condolence to

Sidgwick's widow dated 29 October 1900[11]. After Sidgwick's death, it was decided to establish a Memorial to him, which took the form of a Sidgwick Lectureship. Alfred Marshall, who was a member of the executive committee for the Memorial, wrote to the economist and politician L.H. Courtney on 7 February 1901, that this Lectureship

> probably [would] go to W.E. Johnson of Kings, a man who has done first rate work for the University with no pay, living in poverty & supporting a meagre existence by taking pupils &c. Sidgwick subsidized him privately (quoted in Whitaker 1996, vol. 2, p. 298, fn. 2).

As mentioned earlier, Johnson was, in fact, appointed to the Lectureship.

Marshall

Although Johnson took Marshall's theories as a fundamental reference point, he was not a pupil of the founder of the Cambridge School. In fact, when Johnson was a student, Marshall was teaching at Bristol (1877-82) and later at Oxford (1883-84). Despite the fact that both taught economics (first for the Moral Sciences Tripos and later for the Economics Tripos created by Marshall) there is no evidence of a close personal relationship between the two. There are no letters to or from Johnson in Marshall's correspondence edited by Whitaker (1996) and, in addition to the above-quoted passage, there are just three other occasional instances in which Marshall refers to Johnson, each time in letters to John Neville Keynes, John Maynard's father[12].

John Neville Keynes

J. N. Keynes (1852-1949) was a pupil of Marshall's, who lectured on logic and economics in Cambridge for many years. He published two books, *Studies and Exercises in Formal Logic* (first edition 1884) and the well known *Scope and the Method of Political Economy* (1891), which became the reference text on the economic method for the Cambridge School.

Johnson was one of the closest friends of Keynes's family. He contributed to John Neville's *Studies and Exercises in Formal Logic*, writing some of the problems at the end of the chapters, and discussed in great detail the successive editions of the book (second edition 1887; third edition: 1894) with his friend, as shown by the acknowledgements in the prefaces. Johnson also reviewed the third edition of *Studies and Exercises* for *Mind* (Johnson 1895). Even John Maynard recalled the close relationship between Johnson and his father:

> He [Johnson] used, when I was a child, regularly to lunch at Harvey Road with my father; I should think almost once a week. My father was then writing his book on logic [this should be, probably, revising his book], which would frequently be a matter of conversation and discussion (quoted in Broad 1931, p. 513).

Four letters from Johnson to J.N. Keynes are preserved in the archives of the Cambridge Library. Unfortunately, they contain no useful information regarding the question of Johnson's knowledge of Pareto[13].

John Maynard Keynes

Johnson not only helped the father with his advice and criticism but also the son, especially when J.M. Keynes (1883-1946) was working on probability. Johnson was one of the two readers for the dissertation on "The Principles of Probability" Maynard submitted in 1908 for the fellowship at King's College, the other was the philosopher and mathematician Alfred North Whitehead. Even though Johnson's judgment was highly favorable, Whitehead expressed some doubts about the originality of J.M. Keynes' work. In the vote of 17 March 1908, J.M. Keynes was not elected a Fellow, but he was given the chance to improve his work and to apply for the fellowship the following year. On 23 March 1908, Maynard wrote to his friend Lytton Strachey:

> I was also damaged, I think, by Whitehead's report. […] Johnson's report is almost as favourable as it could possibly be. I spent most of Sunday talking to him, and he had made a great number of very important criticisms, which, with the exception of one fundamental point, are probably right (quoted in Harrod 1982, pp. 127-28).

When he was revising his dissertation, J.M. Keynes benefited from Johnson's comments and suggestions. Whitehead's report on the revised version of "The Principles of Probability" was positive and Johnson confirmed his very favorable assessment. As a matter of fact, in March 1909 J.M. Keynes was elected a fellow of King's College[14].

Although increasingly absorbed by other activities, Keynes still worked on his Fellowship dissertation during the following twelve years. It finally appeared in 1921 as *Treatise on Probability*. During this long period, Johnson, also a King's Fellow, continued to help him with criticisms and advice. This is, in fact, borne out by the numerous acknowledgements made in different parts of the *Treatise*, and by three letters from Johnson to Keynes preserved in the Keynes Papers at King's College in which Johnson discusses some passages or mathematical demonstrations in Keynes' manuscript[15].

When Johnson published his article in the *EJ*, J.M. Keynes was already the editor of the Journal, having replaced F.Y. Edgeworth at the beginning of 1912. Keynes' editorial notebook is preserved at King's College (JMK/EJ/4). It contains material concerning both the papers and finances of the *EJ*, and for the period 1911-1915 it records the names of contributors, subjects of articles and editorial responses. With reference to Johnson's article, the notebook only indicates that it was published in the issue of December 1913. There is no indication of the date it was submitted or of a referee report, thereby making one think that the article was informally submitted to the editor. We know that Keynes appreciated Johnson's article, not only because in Johnson's obituary Keynes referred to it as "an important contribution" (Keynes 1931, p. 349), but also because Roy Harrod, in his

biography of Keynes, recalls that:

> Maynard retained a great regard for Johnson. When I asked him in 1922 how much mathematics it was needful for an economist to know, he replied that Johnson in his article in the *Economic Journal* had carried the application of mathematical analysis to economic theory about as far as it was likely to be useful to carry it (Harrod 1982, p. 8).

Sanger

Charles Percy Sanger (1871-1930) studied mathematics at Cambridge and was second wrangler in the Mathematical Tripos of 1893. He then turned to economics, which until 1903 was included in the Moral Sciences Tripos, and had Marshall as a teacher. In the Moral Sciences Tripos of 1894, Sanger was placed in the first class. In the same year, the young Sanger collaborated with Johnson on the paper "On Certain Questions Concerned with Demand" and in 1895 he published an important survey of Pareto's early work on demand theory in the *EJ* (Sanger 1895). In 1896 Sanger was called to the Bar, moved to London and began a legal career. However, he continued to be interested in mathematical and statistical economics, and lectured on these subjects first at University College, London, and later at the London School of Economics. He continued to contribute copiously to the *EJ* until 1926 (51 entries between 1895 and 1926), mainly reviews often referring to Italian, German and French works.

Sanger's study of Pareto's early work on demand theory was published in the *EJ* in March 1895 under the title "Recent Contribution to Mathematical Economics". The paper presented English scholars with an exhaustive study of the main results found in Pareto's 1892-93 "Considerazioni" and in other articles of the Italian economist[16]. When Sanger illustrates Pareto's discussion of the probable form of the utility function, he examines more closely the Bernoulli hypothesis especially as regards the utility of money:

> The hypothesis that has attracted most attention is Bernoulli's, and Prof. Pareto is right in discussing it at considerable length. […] If U is the total utility derived from an amount of money m, then $U = U_0 \log \frac{m}{a}$; where a is the minimum quantity of money on which existence is possible […], and U_0 is a constant of utility depending on the individual (Sanger 1895, p. 119).

Subsequently, Sanger demonstrates that U_0 is the amount of utility a person obtains from an amount of money e times as large as that on which he can subsist, where e is the Napierian constant. Sanger acknowledges that this result was suggested to him by none other than Johnson:

> This was pointed out to me by Mr. W.E. Johnson (Sanger 1895, p. 119, note 3).

Johnson's point only deals with a specific mathematical question, but suggests that he may have been familiar with Sanger's paper and thus, at least indirectly, with Pareto's early work on demand[17]. Unfortunately, I found no trace of any correspondence between Johnson and Sanger to clarify this point. Although the Keynes Papers at King's College contain some letters between Sanger and Keynes, they do not provide any useful elements for this inquiry. Other letters written by Sanger are stored in the archives of the Cambridge Library and of Trinity College, but are irrelevant to this study.

Berry

Arthur Berry (1862-1929) studied mathematics at Cambridge and was senior wrangler in the Mathematical Tripos of 1885. Under Marshall's influence, he became interested in economics and was a co-founder of the Cambridge Economic Club. At Marshall's request, he began to teach a course on "Diagrammatic and Mathematical Treatment of Economic

Theory" for the Moral Sciences Tripos during Easter Term 1891. In Michaelmas Term 1892 the title of the course became "Diagrammatic and Mathematical Treatment of Pure Economic Theory" and in Easter Term 1896 it was simplified to "Diagrammatic Treatment of Pure Economic Theory". Even though Berry gradually drifted away from economics in the 1890s, he continued to teach his course on economic theory until Easter Term 1900, when Johnson took it over. Despite this relationship and the fact that both Berry and Johnson were fellows of King's College, there is no trace of any correspondence between the two.

Edgeworth

Francis Ysidro Edgeworth (1845-1926) was not from Cambridge. He studied at Dublin and Oxford and taught at King's College, London from 1880 to 1891, and for the rest of his life at Oxford. Edgeworth was the first editor of the *EJ* (1890-1911) and when Keynes succeeded him in 1912, he remained on the consultative board of the Journal. Even before 1913, he was quite familiar with Pareto's works, which he had either discussed or referred to in at least thirteen papers[18].

As mentioned earlier, in March 1915 Edgeworth published in the *EJ* a review of the latest contributions to mathematical economics and discussed at length Johnson's 1913 paper. Edgeworth explicitly linked Johnson's analysis to that of Pareto's *Manuel*:

> A good preparation for that requisite treatment [of Giffen goods] appears to be afforded by the exercises in abstract reasoning provided by Professor Pareto and Mr. Johnson. […] Mr. Johnson has thrown additional light on the peculiar case [in which either $\partial\left(\frac{u_x}{u_z}\right)\Big/\partial x$ or $\partial\left(\frac{u_z}{u_x}\right)\Big/\partial z$ is positive] to which Professor Pareto had already called attention (*Manuel* p. 573) (Edgeworth 1915, pp. 48, 49).

Edgeworth therefore seemed to be aware of the analogies between Johnson's analysis and

Pareto's *Manuel*. If this is the case and if he was consulted in the course of the publication of Johnson's article, Edgeworth could have pointed out these analogies. However, in checking the Edgeworth papers preserved in the archives of Nuffield College, Oxford, the London School of Economics, London, King's College, Trinity College and Cambridge Library, Cambridge, I found no reference to Johnson.

Pareto

The last figure in our story is Vilfredo Pareto himself. The basic question is to what extent the English economists were acquainted with Pareto's theories before Johnson's article appeared. Even thought Amoroso attributed Johnson's ignorance of Pareto's work to a more general ignorance on the part of English public, the Italian economist does not seem to have been unknown in England before December 1913. As mentioned earlier, Edgeworth was quite familiar with Pareto's economics and, in the *EJ* alone, his works were mentioned or discussed in at least 19 papers published between 1894 and 1913[19]. Furthermore, between 1892 and 1911 the *EJ* published six reviews of Pareto's works[20], and Pareto himself wrote a brief note on state expenditures in Italy (Pareto 1892a), Walras' obituary (Pareto 1910) and reviewed two books (Pareto 1911 and 1912).

On the other hand, almost all these references to Pareto concern his pre-1900 works, that is, not the ordinal demand theory developed in the *Manuel*. Moreover, it is true that Pareto was hardly ever mentioned in the key English works of the period, such as Marshall's *Principles* (successive editions), Wicksteed's *Common Sense of Political Economy* (1910) and Pigou's *Wealth and Welfare* (1912). However, this fact seems due more to theoretical (partial vs. general analysis), academic (Cambridge vs. Lausanne) and methodological ('empirical' vs. 'abstract' approach) divergences, than to lack of knowledge[21]. Especially with regard to the Cambridge context, it is important to note that the Cambridge Library

purchased both the *Cours* (Pareto 1896 and 1897) and, as mentioned earlier, the *Manuel* (1909). Lastly, neither in Pareto's works nor in his correspondence is there any reference to Johnson.

IX. CONCLUSIONS

The elements presented in the initial overview of Johnson's life, the analytical comparison between Johnson's 1913 article and Pareto's *Manuel*, and the previous analysis of Johnson's personal relationships, do not prove that Johnson's paper was influenced by knowledge of Pareto's work, and particularly by the *Manuel*. However, the available evidence gives some weight to the thesis that Johnson could not have been unaware of Pareto's *Manuel* when he wrote his controversial article. I will now sum up this evidence:

i) The strongest evidence remains the analytical one. Johnson's indifference curves-budget line diagram, his graph representing the variations in the slope of successive indifference curves, as well as the representation of the indifference maps for perfect substitutes and complements are very similar to some of the figures that can be found in the *Manuel*. Furthermore, Johnson's statement of the convexity condition for the indifference curves as well as his discussion of the complementary/competitive relationships between commodities resemble the analytical treatment of these topics presented in the *Manuel*'s mathematical Appendix. Lastly, Johnson's ordinal insights as well as the cardinal vestigial of his theory are similar to the Paretian ones.

ii) The *Manuel* has belonged to the Cambridge Library since 1910 and there are some indications that Johnson could read French. Moreover, the similarities between Pareto's *Manuel* and Johnson's article are mainly related to diagrams and mathematical expressions. These diagrams and expressions could be easily understood by an individual with a solid mathematical background like Johnson's, regardless of his knowledge of French.

iii) The fact that in 1895 Sanger acknowledges Johnson's observation regarding the Bernoulli hypothesis suggests that Johnson had heard of Pareto and might have been familiar with his early demand theory, if not directly at least through Sanger's survey. Pareto's early demand theory actually is quite different from the ordinal theory expounded later in the *Manuel*. However, since 1901 Johnson had held his course on mathematical economic theory and it therefore seems plausible that he kept himself informed about the subsequent developments in the discipline, and may have come across the Cambridge Library copy of the *Manuel*.

iv) More generally, in the period before the publication of Johnson's article, Pareto was not at all unknown to English scholars. Before December 1913, 25 papers or reviews referring to him appeared in the *EJ* alone and Pareto himself wrote in the Journal on four occasions. Moreover, Johnson's house was one of the most important centers of intellectual and social life in Cambridge so that Johnson certainly had the opportunity to become acquainted with what was going on in the debate on economics.

v) J.M. Keynes, the *EJ* editor, held Johnson in high regard: Johnson was an old friend of his, one who had given a positive assessment of his King's College dissertation and advised him regarding matters of logic. Keynes' *EJ* editorial notebook fails to indicate when Johnson's paper was submitted or the set of referees, thus suggesting that the work was published according to an informal procedure. In particular, Keynes may have been instrumental in getting Johnson's article printed promptly since it was Johnson's first publication after twelve years and it might have been important to having his Fellowship at King's College renewed.

vi) Lastly, Johnson's silence regarding his debt to Pareto can be partly explained by the relaxed rules of citation in the late Nineteenth century and early 1900s, and partly by the specific environmental circumstances that characterized Cambridge during the Marshallian

Age. Although Johnson was not a pupil of Marshall's, he frequently took Marshall's theories as a fundamental reference point. Furthermore, with all probability, Marshall had backed Johnson for the appointment of lecturer in mathematical economics when Berry left and, more importantly, for the Sidgwick Lecturership. It can therefore be argued that the Cantabrigian Johnson had no great incentive to acknowledge his theoretical debt to Pareto, one of the leaders of the continental school and a harsh critic of Marshall's partial-equilibrium approach.

All this evidence is, however, still not conclusive. Even if implausible, the possibility that the eccentric Johnson wrote his 1913 paper in a completely autonomous way cannot be excluded. Although the analysis of all the parts of the puzzle gives a certain weight to the thesis that Johnson could not have been unaware of Pareto's *Manuel*, the evidence gathered does not allow us to definitively solve this long-standing question in the history of economic analysis.

FOOTNOTES

[1] Johnson's dissertation, together with John Venn's report, is found in the archives of King's College, Cambridge, Coll. 42/1884/Johnson.

[2] Cf. *Cambridge University Reporter*, different years.

[3] In his entries for *Palgrave's Dictionary*, Johnson also cites or quotes many other English, American, German and Austrian economists, including J. Bentham, H.C. Carey, C.F. Dunbar, F.Y. Edgeworth, B. Hildebrand, W.S. Jevons, J.N. Keynes, K. Knies, F. List, T.R. Malthus, A. Marshall, C. Menger, J. Mill, J.S. Mill, D. Ricardo, W. Roscher, G. Schmoller, N.W. Senior, H. Sidgwick, A. Smith, P.H. Wicksteed. He cites neither Pareto nor L. Walras.

[4] This overview of Johnson's life is based on Tardieu 1900, Broad 1931, Keynes 1931, A.D. 1932, Venn 1947, Braithwaite 1949, Rhees 1981, Harrod 1982, Skidelsky 1983, Zabell 1987, McGuiness 1988, Monk 1990, Skidelsky 1992 and Whitaker 1996.

[5] Johnson's methodological entries include: "Economic Man" (in Palgrave 1894); "Hypothesis"; "Logic and Political Economy", "Method of Political Economy", "Motives, Measurable" (in

Palgrave 1896); "Political Economy: Applications of Logical Conceptions to", "Synthesis and Synthetic Method", "Theory" (in Palgrave 1899). The entries on demand and supply theory include: "Goods, Classification of" (in Palgrave 1896); "Producers' Goods", "Producers' Rent", "Services: Material and Personal", "Supply", "Supply and Demand" (in Palgrave 1899).

[6] Cf. Cambridge Library, Classmark *Econ.6.90.252*. Unfortunately, it is not possible to reconstruct the book's loan history for the years 1910-1913.

[7] In stressing the ordinal character of Johnson's contribution, Stigler 1950, Samuelson 1974 and, more recently, Lenfant 2003 seem to underestimate these cardinal residuals contained in his 1913 paper.

[8] Regarding Johnson's definition of complementarity see also Stigler 1950, pp. 385-386, Samuelson 1974, pp. 1281-1282, and Lenfant 2003, pp. 24-29.

[9] Although within a different analytical framework, Pareto had already provided the mathematical expression for the relationship between a price variation and the ensuing demand variation for a good, *i.e.* the formula for $\partial x_i / \partial p_i$, in the general case with *n* interdependent commodities (cf. Pareto 1892-93, part 5: 306, and Pareto 1909, pp. 581 ff.). Based on $\partial x_i / \partial p_i$, in the *Manuel* Pareto also briefly discussed the case in which "lorsque le prix de *Y* croît [...] la dépense pour achetez du *Y* croît" (Pareto 1909, pp. 583-584), that is, the Giffen case.

[10] The second-order conditions for the utility maximization problem in the general *n*-variables case can also be found in the Appendix of Pareto's *Manuel*, although in a different mathematical form (cf. Pareto 1909, pp. 578 ff.).

[11] Letter at Trinity College, Cambridge, Sidgwick Papers, Add. ms. c. 104/88. The Trinity College archives contain only two other letters from Johnson to Sidgwick, which deal with the method of *reductio ad absurdum* (the letter of 13 March 1894, Sidgwick Papers, Add. ms. c. 94/63, and the letter of 14 March 1894, Sidgwick Papers, Add. ms. c. 94/64). Another letter Johnson sent to Henry Jackson, dated 2 February 1903, is preserved at Trinity College (Add. ms. c. 38 [6,7]), but is of no importance to the present paper.

[12] In the first letter, dated 30 October 1897, Marshall wrote that he did not concur with J.N. Keynes' "endorsement of Johnson's classification of 'postulates'" (cf. Whitaker 1996, vol. 2, p. 205). In the letter dated 23 February 1899, Marshall regretted that the Moral Science Board did not increase J.N. Keynes' and Johnson's salary (cf. Whitaker 1996, vol. 2, p. 245). Finally, in the third letter dated 11 February 1902, Marshall claimed that the objection to the phrase 'Mental Science' referred to some subjects in the Moral Sciences Tripos "as implying a now exploded theory about logic & metaphysics" could be attributed to Johnson (cf. Whitaker 1996, vol. 2, pp. 355-356).

[13] In the letter of 5 October 1891, Johnson discussed some issues relating to the theory of value with J.N. Keynes (Cambridge Library, Add. 7562/64). The letters of 29 December 1894 (Add. 7562/74), of 28 October 1900 (Add. 7562/120) and of 2 April 1904 (Add. 7562/163) concern family matters. The archives of the Cambridge Library contain two other letters of Johnson's, addressed to G.E. Moore and dated 28 January 1921 and 29 April 1929 (Add. 8330/8J/6/1-2). The British Library, London also owns one of Johnson's letter, sent to G.K. Chesterton in 1906 (Add. 73238 f. 21). None of these letters is, however, relevant to the paper.

[14] The documents related to Keynes's election to the fellowship are kept at King's College, Cambridge, Keynes Papers, JMK/13/TP/4; the file also contains Johnson's report.

[15] The acknowledgements to Johnson in the *Treatise* are found in Keynes 1921, pp. xxv, 11, 74, 126, 132, 134, 166, 169, 171, 203. In the letter of 7 March 1914, Johnson asked for a clarification of some passages of Keynes' manuscript (King's College, Cambridge, Keynes Papers, JMK/L/14). In another undated letter of 1914, Johnson discussed some other passages of Keynes' theory (JMK/TP/13/C/2/36). Finally, in a third letter of 22 December 1920, Johnson expressed his opinion regarding a demonstration made by Keynes in the final version of the *Treatise* (JMK/TP/1/2/36). Johnson wrote to Keynes also on 24 March 1909 (JMK/L/09), 17 May 1916 (JMK/L/16), and 24 August 1925 (JMK/PP/45/349). These letters are, however, not relevant to this paper. The archives of King's College also contain an early letter of Johnson's, dated 9 May 1885 and addressed to a certain Mr. Grant (Coll. 7/3 "J").

[16] The other articles Sanger refers to, are: Pareto 1892b, 1892c, 1894a, 1894b, 1895.

[17] However, this supposition is attenuated by a handwritten note by Sanger which appears at the end of an issue of the *Cambridge Economic Club* where the Johnson-Sanger paper was printed (Easter 1894 term). In this note, which has no date and is published by Darnell in his edition of the paper, Sanger makes some minor corrections to the printed article and observes that: "If we integrate $\frac{\partial u}{\partial m} = \frac{U}{m}$ so that $u = \log \frac{m}{\mu}$, where μ is the smallest income for which life is worth living, we see that U denotes the utility derived from an income of $e\mu$" (Johnson and Sanger 1894, p. 77). Evidently, this observation is exactly the one that appears in Sanger's 1895 study and is ascribed to Johnson. If Sanger's handwritten note actually dates back to 1894, Johnson might have made his suggestion before Sanger wrote his study on Pareto. Therefore Johnson might not have been familiar with Sanger's 1895 paper.

[18] Cf. Edgeworth 1894, 1896, 1897a, 1897b, 1898a, 1898b, 1903, 1904, 1908, 1910, 1911a, 1911b, 1913.

[19] In addition to Edgeworth (1894, 1897b, 1898a, 1910, 1911a, 1911b, 1913), also Fisher (1896, 1897a, 1897b), Oncken (1896, 1897), Pantaleoni (1898), Gide (1898), Sanger (1899), Bowley (1904), Moore (1907) and Pigou (1908, 1910) referred to him in the *EJ*.

[20] Cf. Sanger 1895, Flux 1896 and 1897, Foville 1903, Edgeworth 1903, Wicksteed 1906.

[21] See Bruni 2002, pp. 104-123 for a more exhaustive discussion of the conflicting relationship between Pareto and the English School.

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