Part 3 of 4:

Implementing the Schulze STV Method

(draft, 10 March 2017)

Markus Schulze
Markus.Schulze@Alumni.TU-Berlin.de

Summary. In this paper the Schulze STV method with proportional completion is applied to instance A53 of Tideman's database.

This paper is the third part of a series of papers that can be downloaded here:

http://m-schulze.9mail.de/schulze1.pdf
http://m-schulze.9mail.de/schulze2.pdf
http://m-schulze.9mail.de/schulze3.zip
http://m-schulze.9mail.de/schulze4.pdf

Instance A53 consists of N = 460 voters and C = 10 candidates running for M = 4 seats. This instance is very interesting because the Newland-Britton method, the Meek method, and the Warren method each chooses a different set of winners. The Newland-Britton (1997) method chooses {*a*,*b*,*g*,*j*}; the Meek (1969, 1970; Hill, 1987) method chooses {*a*,*d*,*g*,*j*}; and the Warren (1994) method chooses {*a*,*f*,*g*,*j*}.

Format 1:

The instances of Tideman's database have two different formats. 50 of the 66 instances of Tideman's database have format "1". For example, instance A53 shares this format. That means, the file *a53.dat* has to be read as follows:

353	3	99	99	99	4	99	2	99	99	1

The first column is the number of the voter. The second to the eleventh column are the preferences of voter v = 353 for candidate *a* to candidate *j*. Thus, voter v = 353 gives his first preference to candidate *j*, his second preference to candidate *g*, his third preference to candidate *a*, and his fourth preference to candidate *e*. And he keeps the candidates *b*, *c*, *d*, *f*, *h*, and *i* unranked.

Format 2:

16 of the 66 instances of Tideman's database have format "2". For example, instance A35 shares this format. That means, the file *a35.dat* has to be read as follows:

26	F	D	O	N	P	H	A

Voter v = 26 gives his first preference to candidate *f*, his second preference to candidate *d*, his third preference to candidate *o*, his fourth preference to candidate *n*, his fifth preference to candidate *p*, his sixth preference to candidate *h*, and his seventh preference to candidate *a*. And he keeps the candidates *b*, *c*, *e*, *g*, *i*, *j*, *k*, *l*, *m*, and *q* unranked.

Unfortunately, in 7 instances some voters give more than one preference to the same candidate. The following table lists all those voters who give more than one preference to the same candidate:

instance	voters who give more than one preference to the same candidate
A17	#72, #126, #152, #232, #275, #290, #370, #538, #793, #846
A19	#330, #816
A20	#98, #1783, #2193, #2221
A49	#16, #51, #133, #134, #315, #413, #463, #559
A83	#69, #85, #145, #205, #317, #757, #782, #802, #1001, #1046, #1088
A95	#267
A96	#415

When an individual voter ranks some candidates in a cyclic manner, then we presume that this voter is indifferent between all the candidates of this cycle.

File *a53_stv.dat* contains the strengths of the $(C!)/((M!)\cdot((C–M–1)!))$ = 1260 vote managements. File *a53_stv.dat* has to be read as follows:

104	A	D	F	G	J	88.714286	101.098901	101.351648	90.736264	78.098901

Then, row 104 says:

$N[\{d,f,g,j\},a] = 88.714286$

$N[\{a,f,g,j\},d] = 101.098901$

$N[\{a,d,g,j\},f] = 101.351648$

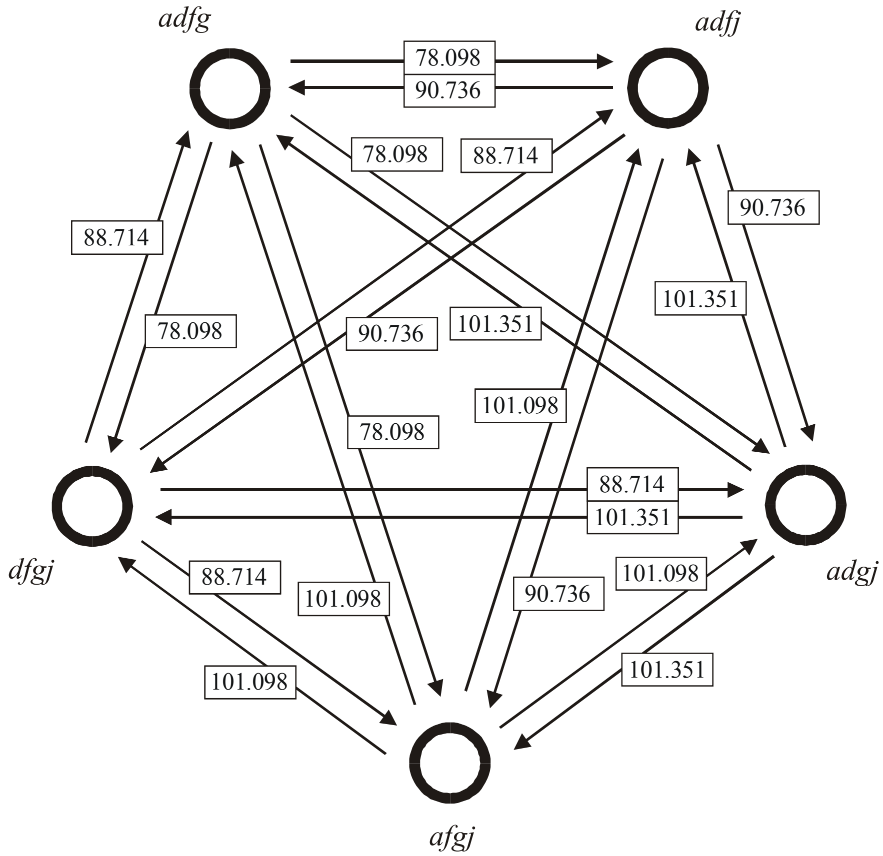
$N[\{a,d,f,j\},g] = 90.736264$

$N[\{a,d,f,g\},j] = 78.098901$

In the traditional head-to-head format, row 104 represents the following defeats:

	$\tilde{N}[*,\{adfg\}]$	$\tilde{N}[*,\{adfj\}]$	$\tilde{N}[*,\{adgj\}]$	$\tilde{N}[*,\{afgj\}]$	$\tilde{N}[*,\{dfgj\}]$
$\tilde{N}[\{adfg\},*]$	---	78.098901	78.098901	78.098901	78.098901
$\tilde{N}[\{adfj\},*]$	90.736264	---	90.736264	90.736264	90.736264
$\tilde{N}[\{adgj\},*]$	101.351648	101.351648	---	101.351648	101.351648
$\tilde{N}[\{afgj\},*]$	101.098901	101.098901	101.098901	---	101.098901
$\tilde{N}[\{dfgj\},*]$	88.714286	88.714286	88.714286	88.714286	---

In the graph theoretical format, row 104 represents the following links:



Candidates *a*, *g*, and *j* are Condorcet candidates because the strongest vote managements against these candidates are strictly below $N/(M+1) = 92$.

$$\max \{ N[\{z_1,\ldots,z_4\},a] \mid z_1,\ldots,z_4 \in A \setminus \{a\} \} = 89.332604$$

$$\max \{ N[\{z_1,\ldots,z_4\},g] \mid z_1,\ldots,z_4 \in A \setminus \{g\} \} = 91.345733$$

$$\max \{ N[\{z_1,\ldots,z_4\},j] \mid z_1,\ldots,z_4 \in A \setminus \{j\} \} = 78.524229$$

When the Schulze STV method with proportional completion is being used, then $\mathfrak{A} = \{a,f,g,j\}$ is the unique winning set, since it is the only set with $P[\mathfrak{A},\mathfrak{B}] \geq P[\mathfrak{B},\mathfrak{A}]$ for every other set $\mathfrak{B} \in A_4$.

The largest value in file *a53_stv.dat* is $N[\{a,e,g,j\},h] = 110.674779$. This is the strength of the vote management of the candidates $\{a,e,g,j\}$ against candidate *h*. This vote management is illustrated in table 1.

In table 1, the column “opinion” describes the opinion of the voters. A “1” means that this voter strictly prefers this candidate to candidate *h*. A “2” means that this voter is indifferent between this candidate and candidate *h*. A “3” means that this voter strictly prefers candidate *h* to this candidate. The column “voters #1” says how many voters share this opinion *before* proportional completion. The column “voters #2” says how many voters share this opinion *after* proportional completion. The column “distribution” describes how many voters of each group of voters vote for which candidate in the optimal vote management of the candidates in columns “1”, “2”, “3”, and “4” against the candidate in column “5”.

For example, row “4” says that *before* proportional completion there are 8 voters and *after* proportional completion there are 9.210514 voters who strictly prefer the candidates *a* and *e* to candidate *h* and who strictly prefer candidate *h* to the candidates g and *j*. In the optimal vote management of the candidates $\{a,e,g,j\}$ against candidate *h*, all 9.210514 voters vote for candidate *e*.

	opinion				voters #1	voters #2	distribution				
	1 *a*	2 *e*	3 *g*	4 *j*			1 *a*	2 *e*	3 *g*	4 *j*	5 *h*
1	1	1	1	1	76	174.783852	26.529814	42.206980	79.076900	26.970159	
2	1	1	1	3	7	11.478359		11.478359			
3	1	1	3	1	18	29.690120		29.690120			
4	1	1	3	3	8	9.210514		9.210514			
5	1	3	1	1	23	60.086335	60.086335				
6	1	3	1	3	7	10.278434	10.278434				
7	1	3	3	1	30	40.043541				40.043541	
8	1	3	3	3	13	13.780196	13.780196				
9	3	1	1	1	8	16.563163			16.563163		
10	3	1	1	3	3	4.586022		4.586022			
11	3	1	3	1	6	8.113086		8.113086			
12	3	1	3	3	5	5.389698		5.389698			
13	3	3	1	1	8	13.449067				13.449067	
14	3	3	1	3	14	15.034716			15.034716		
15	3	3	3	1	28	30.212012				30.212012	
16	3	3	3	3	17	17.300885					17.300885
17	1	1	1	2	3						
18	1	1	2	1	8						
19	1	1	2	2	5						
20	1	2	1	1	32						
21	1	2	1	2	17						
22	1	2	2	1	26						
23	1	2	2	2	8						
24	2	1	1	1	11						
25	2	1	1	2	11						
26	2	1	2	1	8						
27	2	1	2	2	6						
28	2	2	1	1	20						
29	2	2	1	2	11						
30	2	2	2	1	14						
31	2	2	2	2	8						
32	2	3	3	1	1						
sum					460	460	110.674779	110.674779	110.674779	110.674779	17.300885

Table 1: vote management of the candidates {*a*,*e*,*g*,*j*} against candidate *h*

In table 2, the Schulze STV method is applied to other instances of Tideman's (2000) database. The column "name 1" contains the name of the instance. If e.g. the name of the instance is A53, then the file *a53.dat* contains the raw data of this instance, the file *a53_stv.dat* contains the strengths of the vote managements to calculate the winning set of the Schulze STV method, and *a53_list.dat* contains the strengths of the vote managements to calculate the Schulze proportional ranking.

The column "name 2" contains the name of the same instance in Wichmann's (1994) database. *N* is the number of voters. *C* is the number of candidates. *M* is the number of seats.

The column "Schulze STV" contains the winning set of the Schulze STV method with proportional completion. The column "Schulze proportional ranking" contains the Schulze proportional ranking. Only in 6 of the 66 instances of Tideman's database (A10, A11, A13, A33, A34, A59), the winning set of the Schulze STV method differs from the first *M* candidates of the Schulze proportional ranking.

The programs *singl01.cpp* and *multi01.cpp* calculate the winning set of the Schulze STV method. The program *singl01.cpp* is single-threading; the program *multi01.cpp* is multi-threading. The column "runtime 1" contains the runtime for *singl01.cpp*. The column "runtime 2" contains the runtime for *multi01.cpp*. A 4-core "Intel Core i3-2100 CPU @ 3.10 GHz" is used for the calculations.

The programs *singl02.cpp* and *multi02.cpp* calculate the Schulze proportional ranking.

	name 1	name 2	*N*	*C*	*M*	Schulze STV	Schulze proportional ranking	runtime 1	runtime 2
1	A01	R006	380	10	3	*a h i*	*a i h d b c g j f e*	< 0.1 s	< 0.1 s
2	A02	R007	371	9	2	*c d*	*c d e b f a h i g*	< 0.1 s	< 0.1 s
3	A03	R008	989	15	7	*b d e f h k n*	*f h d k b e n g a l c i j o m*	31.0 s	14.7 s
4	A04	R009	43	14	2	*a i*	*i a k f e c b g d h m j l n*	< 0.1 s	< 0.1 s
5	A05	R010	762	16	7	*a c d e g l m*	*a c m e d g l k f o p h i j b n*	38.0 s	17.1 s
6	A06	R011	280	9	5	*b c e h i*	*i h e c b f g a d*	< 0.1 s	< 0.1 s
7	A07	R012	79	17	2	*d i*	*i d c o m p h a k g e j l n f b q*	< 0.1 s	< 0.1 s
8	A08	R013	78	7	2	*d g*	*d g c b f e a*	< 0.1 s	< 0.1 s
9	A10	R015	83	19	3	*m n p*	*n* ((*a p m q*) or (*m p q a*)) *g f s r l i b d j k e h o c*	0.2 s	< 0.1 s
10	A11	R016	963	10	6	*a c d e g h*	*a c e h j g d i b f*	< 0.1 s	< 0.1 s
11	A12	R017	76	20	2	*i r*	*r i l s g m a p b h t n e o k d* ((*f j*) or (*j f*)) *c q*	< 0.1 s	< 0.1 s
12	A13	R018	104	26	2	*k t*	*i t k m s j c f y z l u n a g e b p r d h v x o q w*	< 0.1 s	< 0.1 s
13	A14	R019	73	17	2	*b j*	*j b c n h q o i a l e d g k p m f*	< 0.1 s	< 0.1 s
14	A15	R020	77	21	2	*g l*	*l g t r m i c h p k j q s a b o d u n f e*	< 0.1 s	< 0.1 s
15	A17	R022	867	13	8	*a b d e f i j l*	*j b a e l f d i m h k c g*	1.2 s	0.6 s
16	A18	R023	976	6	4	*a b c f*	*b c f a d e*	< 0.1 s	< 0.1 s
17	A19	R024	860	7	3	*a e g*	*e a g c d b f*	< 0.1 s	< 0.1 s
18	A20	R025	2785	5	4	*a c d e*	*a d c e b*	< 0.1 s	< 0.1 s
19	A22	R027	44	11	2	*c k*	*k c a g b d i j h e f*	< 0.1 s	< 0.1 s
20	A23	R028	91	29	2	3 5	3 5 17 26 7 21 22 27 14 9 15 24 4 16 19 20 6 11 18 28 2 23 29 1 13 8 10 12 25	< 0.1 s	< 0.1 s

	name 1	name 2	*N*	*C*	*M*	Schulze STV	Schulze proportional ranking	runtime 1	runtime 2
21	A33	R038	9	18	3	*e o q*	*o a e i h c l n q f r d g* ((*b m p*) or (*b p m*) or (*m b p*)) *k j*	< 0.1 s	< 0.1 s
22	A34	R039	63	14	12	*a b c d e f g h j k m n*	*j b h e k n l g m c d a f i*	< 0.1 s	< 0.1 s
23	A35	R040	176	17	5	*a d e f q*	*f e a d q k b i m n c h j p o g l*	8.5 s	4.4 s
24	A48	R041	923	10	9	*a b c d e f g h j*	*d f b e c h j g a i*	< 0.1 s	< 0.1 s
25	A49	R042	575	13	3	*a c h*	*h c a j l d m g b i e f k*	< 0.1 s	< 0.1 s
26	A51	R044	42	6	3	*a d e*	*d a e f c b*	< 0.1 s	< 0.1 s
27	A52	R045	667	10	6	*a b c d e g*	*e d b g a c j f i h*	< 0.1 s	< 0.1 s
28	A53	R046	460	10	4	*a d g j*	*j a g d f b e c i h*	< 0.1 s	< 0.1 s
29	A54	R047	924	11	9	*a b d e f g h j k*	*e d f a k g h j b i c*	< 0.1 s	< 0.1 s
30	A55	R048	302	10	5	*a d f i j*	*i a j f d e h c g b*	< 0.1 s	< 0.1 s
31	A56	R049	685	13	2	*j k*	*j k f h m g d a e c b l i*	< 0.1 s	< 0.1 s
32	A57	R050	310	9	2	*d e*	*d e i b h c g f a*	< 0.1 s	< 0.1 s
33	A59	R052	694	7	4	*b d f g*	*f d e g b c a*	< 0.1 s	< 0.1 s
34	A63	R056	156	7	2	*c f*	*c f e d b a g*	< 0.1 s	< 0.1 s
35	A64	R057	196	3	2	*b c*	*b c a*	< 0.1 s	< 0.1 s
36	A65	R058	198	10	6	*a b e f g j*	*g b f e j a d c h i*	0.2 s	< 0.1 s
37	A66	R059	193	6	4	*b d e f*	*f d e b c a*	< 0.1 s	< 0.1 s
38	A67	R060	183	14	10	*b c e f g h i j k l*	((*f g*) or (*g f*)) *k b i e j l c h n m d a*	26.7 s	12.5 s
39	A68	R061	50	4	3	*a c d*	*a c d b*	< 0.1 s	< 0.1 s
40	A69	R062	86	9	3	*a c e*	*e c a f i d b h g*	< 0.1 s	< 0.1 s

	name 1	name 2	*N*	*C*	*M*	Schulze STV	Schulze proportional ranking	runtime 1	runtime 2
41	A70	R063	529	9	3	*e h i*	*e i h c d b a g f*	< 0.1 s	< 0.1 s
42	A71	R064	500	8	7	*a b c d e f g*	*d c g e a b f h*	< 0.1 s	< 0.1 s
43	A72	R065	272	3	2	*a c*	*a c b*	< 0.1 s	< 0.1 s
44	A73	R066	525	5	2	*c d*	*d c b a e*	< 0.1 s	< 0.1 s
45	A74	R067	253	3	2	*a c*	*a c b*	< 0.1 s	< 0.1 s
46	A76	R069	403	5	2	*a c*	*c a d b e*	< 0.1 s	< 0.1 s
47	A78	R071	486	4	3	*b c d*	*c d b a*	< 0.1 s	< 0.1 s
48	A79	R072	362	8	4	*a c e g*	*g a e c f d b h*	< 0.1 s	< 0.1 s
49	A80	R073	269	7	5	*a b c e g*	*a e c g b f d*	< 0.1 s	< 0.1 s
50	A81	R074	902	11	9	*a b c e g h i j k*	*h e c b j g a i k d f*	< 0.1 s	< 0.1 s
51	A83	R076	1123	4	3	*a b c*	*c a b d*	< 0.1 s	< 0.1 s
52	A84	R077	277	7	6	*a b c d e g*	*e b c d g a f*	< 0.1 s	< 0.1 s
53	A85	R078	158	4	3	*a b d*	*d a b c*	< 0.1 s	< 0.1 s
54	A86	R079	157	5	4	*a c d e*	*c a d e b*	< 0.1 s	< 0.1 s
55	A87	R080	120	4	3	*a b d*	*d b a c*	< 0.1 s	< 0.1 s
56	A88	R081	135	9	6	*a c e f g h*	*h e g c f a d b i*	< 0.1 s	< 0.1 s
57	A89	R082	256	5	3	*a d e*	*e d a b c*	< 0.1 s	< 0.1 s
58	A90	R083	366	20	12	*a b c d e f i l n o s t*	*a i t l e c s d f n o b p j m g k r h q*	218.4 s	130.0 s
59	A92	R085	540	13	3	*d f i*	*d f i e b h a m c j g k l*	< 0.1 s	< 0.1 s
60	A93	R086	561	4	2	*b d*	*b d c a*	< 0.1 s	< 0.1 s
61	A94	R087	579	4	2	*a d*	*a d b c*	< 0.1 s	< 0.1 s
62	A95	R088	587	7	2	*a b*	*a b f g d e c*	< 0.1 s	< 0.1 s
63	A96	R089	564	6	2	*a b*	*a b e f d c*	< 0.1 s	< 0.1 s
64	A97	R090	284	4	2	*a b*	*a b c d*	< 0.1 s	< 0.1 s
65	A98	R091	279	4	2	*a c*	*a c b d*	< 0.1 s	< 0.1 s
66	A99	R092	275	4	2	*a b*	*b a c d*	< 0.1 s	< 0.1 s

Table 2: Schulze STV method applied to instances of Tideman's database

In 18 of the 66 instances of Tideman's database, the winning set of the Schulze STV method differs from the winning set of traditional STV methods. These instances are listed in table 3. The column "Newland-Britton" contains the winning set of the Newland-Britton (1997) method. The column "Meek" contains the winning set of the Meek (1969, 1970; Hill, 1987) method. The column "Warren" contains the winning set of the Warren (1994) method.

	name 1	name 2	*N*	*C*	*M*	Newland-Britton	Meek	Warren	Schulze STV
1	A04	R009	43	14	2	*a i*	*i k*	*i k*	*a i*
2	A05	R010	762	16	7	*a c d e g k m*	*a c d e g k m*	*a c d e g k m*	*a c d e g l m*
3	A06	R011	280	9	5	*c e f h i*	*c e f h i*	*c e f h i*	*b c e h i*
4	A07	R012	79	17	2	*c i*	*c i*	*c i*	*d i*
5	A11	R016	963	10	6	*a c e g h i*	*a c e g h i*	*a c e g h i*	*a c d e g h*
6	A15	R020	77	21	2	*l r*	*i l*	*i l*	*g l*
7	A33	R038	9	18	3	[1]	[1]	[1]	*e o q*
8	A34	R039	63	14	12	*a b c d e f h j k l m n*	*a b c d e f h j k l m n*	*a b c d e f h j k l m n*	*a b c d e f g h j k m n*
9	A35	R040	176	17	5	*a e f n q*	*a e f k n*	*a e f k n*	*a d e f q*
10	A53	R046	460	10	4	*a b g j*	*a d g j*	*a f g j*	*a d g j*
11	A55	R048	302	10	5	*a d f i j*	*a d e f i*	*a d e f i*	*a d f i j*
12	A65	R058	198	10	6	*b d e f g j*	*b d e f g j*	*b d e f g j*	*a b e f g j*
13	A67	R060	183	14	10	*b c d e f g i j k l*	*b c d e f g h i j k*	*b c d e f g i j k l*	*b c e f g h i j k l*
14	A71	R064	500	8	7	*a b c d e g h*	*a b c d e g h*	*a b c d e g h*	*a b c d e f g*
15	A74	R067	253	3	2	*a b*	*a b*	*a b*	*a c*
16	A79	R072	362	8	4	*a e f g*	*a d e g*	*a d e g*	*a c e g*
17	A80	R073	269	7	5	*a b c e f*	*a b c e f*	*a b c e f*	*a b c e g*
18	A90	R083	366	20	12	*a b c d e f i k l n s t*	*a b c d e f i k l n s t*	*a b c d e f i l n o s t*	*a b c d e f i l n o s t*

Table 3: instances where the winning set of the Schulze STV method differs from the winning set of traditional STV methods

[1] In instance A33, 10 candidates received no first preferences, 7 candidates received one first preference each, and one candidate received two first preferences. The winning sets of the Newland-Britton method, the Meek method, and the Warren method depend on which candidates happen to be eliminated by random choice.

Acknowledgments

I want to thank Nic Tideman for giving me access to his database and for sending me a preprint of his paper “Better voting methods through technology”.

References

Hill ID, Wichmann BA, Woodall DR (1987), “Algorithm 123 — The Single Transferable Vote by Meek’s Method”, *Computer Journal* 30: 277–281

Meek BL (1969), “Une nouvelle approche du scrutin transférable”, *Mathématiques et Sciences Humaines* 25: 13–23 [Reprinted (1994) as: “A New Approach to the Single Transferable Vote — Part I”, *Voting Matters* 1: 1–7]

Meek BL (1970), “Une nouvelle approche du scrutin transférable (fin)”, *Mathématiques et Sciences Humaines* 29: 33–39 [Reprinted (1994) as: “A New Approach to the Single Transferable Vote — Part II”, *Voting Matters* 1: 7–11]

Newland RA, Britton FS (1997), “How to conduct an election by the Single Transferable Vote”, Electoral Reform Society of Great Britain and Ireland (ERS), 3rd Edition

Tideman TN, Richardson D (2000), “Better voting methods through technology: The refinement-manageability trade-off in the single transferable vote”, *Public Choice* 103: 13–34

Warren CHE (1994), “Counting in STV Elections”, *Voting Matters* 1: 12–13

Wichmann BA (1994), “An STV Database”, *Voting Matters* 2: 9