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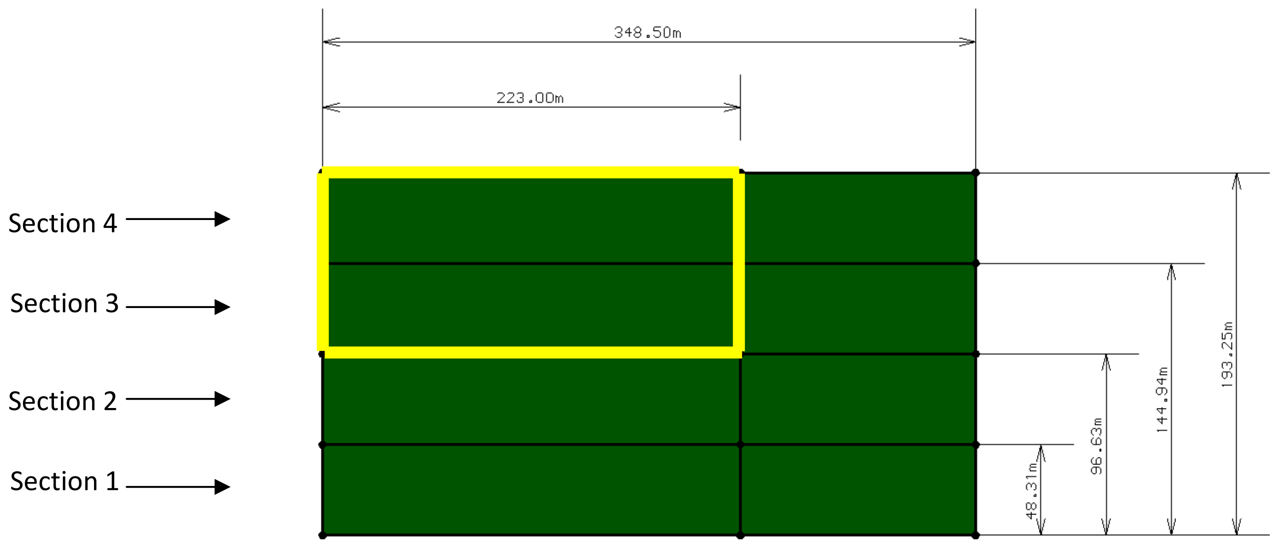
Assessment of a Pair of Reinforced Concrete Roof Slabs

Introduction

RMA have conducted analyses of a pair of reinforced concrete roof slabs for an American Precast company. RMA's finite element software EFE was used for this work. EFE conducts yield line, elastic and lower-bound limit analysis and the results from all three types of analysis applied to each slab are presented.

Geometry of Left Hand Slab



Dimensions in inches (not metres as suggested in figure)

Figure 1: Geometry and simple supports for roof slab

Properties for Left Hand Slab

The properties used for the elastic analysis were:

Elastic Modulus 3.63e6 lb/in^2 (25GPa)

Poisson's Ratio 0.2

We took account of the slab's thickness variation by breaking the slab into four sections as identified in Figure 1. The elastic analysis used the mean thickness for each section as shown in Table 1. Isotropic reinforcement was used and we assumed that the top steel had a constant cover so that the moment capacity varied linearly across the slab. In each section we used the moment capacities shown in Table 1.

Section	Thickness (in)	Moment Capacity (lb/in^2)
1	8.5	10,000
2	9.5	11,176
3	10.5	12,353
4	11.5	13,529

Table 1: Slab thicknesses and moment capacities

Loads/Boundary Conditions for Left Hand Slab

The slab was assumed to be simply supported on the four lines shown in Figure 1. The slab was loaded with uniform distributed loads corresponding to the sum of a dead load and a live load. The dead load was based on an assumed density of concrete/steel composite over the given section thickness whilst the live load was taken as a constant for the entire slab:

Dead load – density of concrete/steel composite 0.09lb/in^3 (2500kg/m^3)

Live load - 0.11lb/in^2 (0.75kN/m^2)

Table 2 lists the total loads (dead plus live) for each section.

Section	Total Load (lb/in^2)
1	0.874
2	0.964
3	1.054
4	1.144

Table 2: Slab uniformly distributed loads

Elastic Analysis Results for Left Hand Slab

A mesh of 720 square elements was used for the analyses. The elastic analysis used quadratic moment fields and the displaced shape is shown in Figure 2.

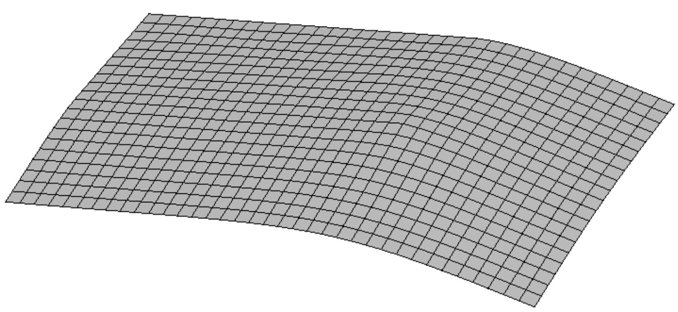


Figure 2: Displaced shape (elastic)

For the assumed elastic properties maximum displacement was -0.38in and occurred at the bottom right hand corner – as expected. A contour plot of displacement is shown in Figure 3. The contours are displaced away from the base plane mesh to aid understanding. It is seen that the slab deflects upwards inside the constrained region and the maximum positive displacement is 0.01in.

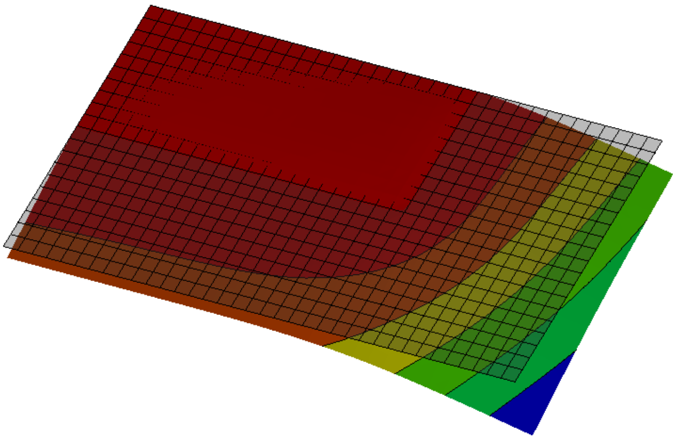


Figure 3: Displacement contours (elastic)

Whilst there are many results that we could contour, we have chosen to show only the maximum principal bending moment.

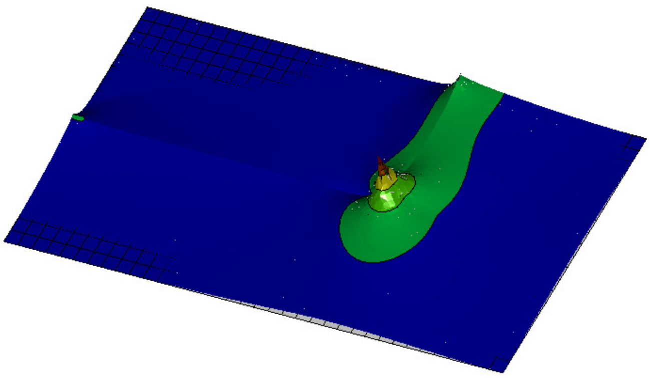


Figure 4: Maximum principal bending moment contours (elastic)

The contours of maximum principal bending moment of Figure 4 show, as expected, ridges over the internal supports and a peak value at the corner of the internal supports.

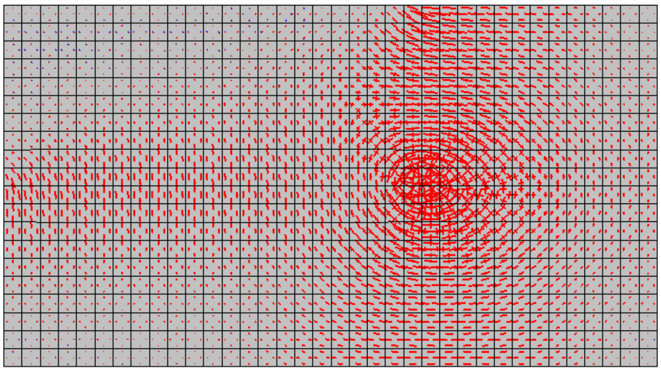


Figure 5: Principal bending vectors (elastic)

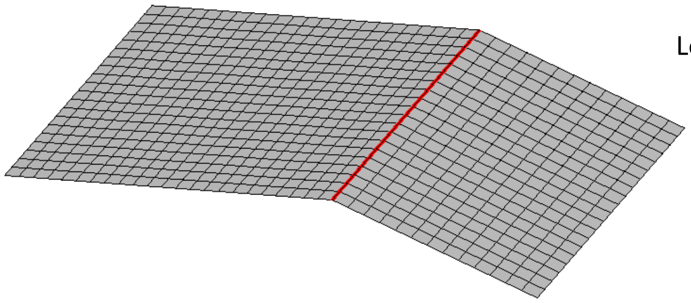
The principal moment vectors shown in Figure 5 provide more information on the distribution of moments. We use the convention of Red for Hogging and Blue for Sagging moments. The figure thus shows that the predominant moments are, as expected, hogging.

Limit Analysis Results for Left Hand Slab

Limit analysis attempts to identify the load at collapse. Collapse here is understood as collapse due to flexural (bending) failure. We use conventional yield line techniques which in terms of plasticity theory provide upper-bound or unsafe approximations to the collapse load. We also use equilibrium techniques for lower-bound or safe approximations to the collapse load. In this manner we are able to place bounds for the true collapse load for the slab.

The collapse load coming from limit analyses is expressed in terms of a load factor. The load factor is the factor that needs to be applied to the loading to cause collapse of the slab. The load factor is, obviously, dependent on both the assumed reinforcement and the applied load. We use the values previously defined for these quantities and present the resulting load factors. If other reinforcement or load is considered then the results may of course be scaled (providing the same patterns of reinforcement and load are used) such that the load factor is inversely proportional to the load and proportional to the assumed moment capacities.

A yield line analysis of the slab produced the result shown in Figure 6. The yield line pattern comprises a hogging line (red) across the slab coinciding with one of the lines of internal support. The load factor from this analysis is 1.48 implying that the slab can carry 1.48 times the applied load before collapse. Note, however, that the yield line technique is an upper bound approach and the true value may be less than this value.

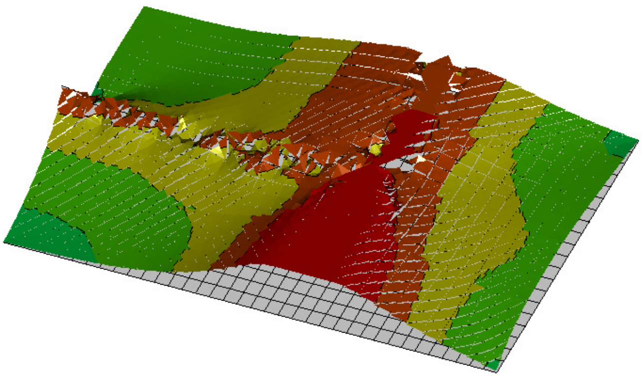


Load Factor = 1.48

Figure 6: Yield line pattern and deflected shape (Top and bottom steel identical)

An equilibrium limit analysis of the slab using equal top and bottom steel provided a load factor of 1.45. As the yield line analysis and the equilibrium limit analysis provide results that bound the true solution then we can say that the true load factor lies between 1.45 and 1.48. For this particular example a tight bound has been found but in general we would advocate taking the lower of the two values for reasons of safety.

We present contours of maximum principal moment and principal moment vectors for the equilibrium limit solution in Figures 7 and 8.



Load Factor = 1.45

Figure 7: Maximum principal bending moment contours (equilibrium limit - Bottom = Top Steel)

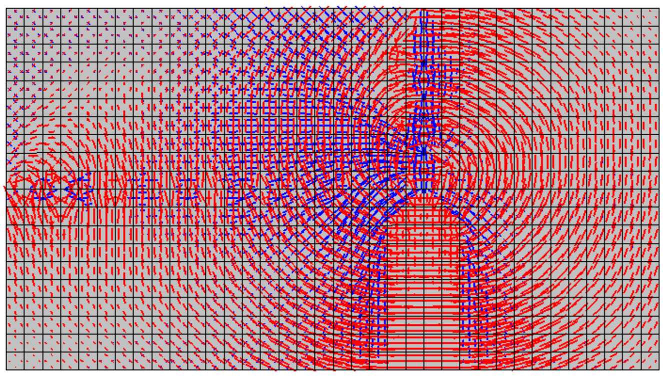


Figure 8: Principal moment vectors (equilibrium limit - Bottom = Top Steel)

Equilibrium limit analysis may be considered as a process of moment redistribution - a process of optimising a moment field so as to maximise load carrying capacity whilst not violating a given yield criterion – for the case of reinforced concrete the yield criterion is the Nielsen Biconic Yield Criterion. By comparing Figure 7 with Figure 4 one can see how, in this case the maximum principal moments are redistributed with the peak around the internal corner and ridges along the internal supports turning into flatter but wider distributions. Comparing Figure 8 with Figure 5 shows in further detail how the moment distribution changes with the limit solution invoking sagging moments (blue vectors) not present in the elastic solution.

Noting that for this problem the moment field is dominated by hogging moments, a further limit analysis was conducted with reduced bottom steel. For this analysis we reduced the bottom steel to 10% of the top steel. The yield line solution was exactly the same as for equal top and bottom steel. But the equilibrium limit analysis changed as shown in Figures 9 and 10.

Whilst there are changes evident in the maximum principal bending moment contours the biggest change is noted in the principal moment vectors where we see that the sagging vectors evident in Figure 8 have been eliminated. Contrary to intuition the load factor is slightly increased by reducing the bottom steel – we think we understand the reason for this but it does need further investigation.

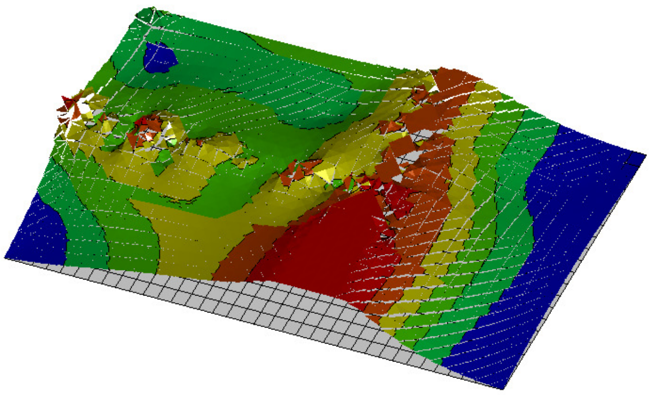


Figure 9: Maximum principal bending moment contours (equilibrium limit - Bottom = 0.1xTop Steel)

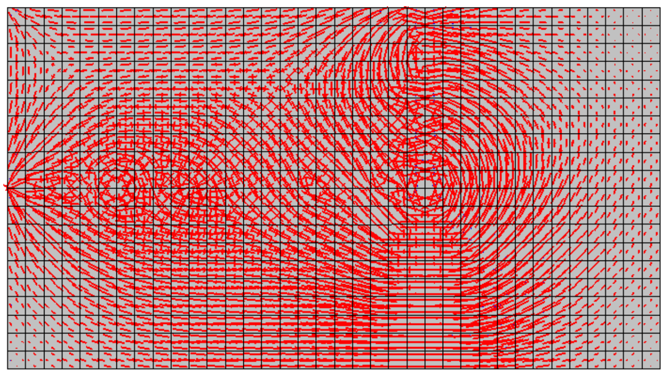
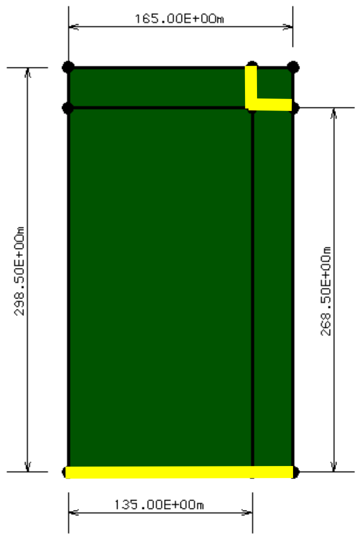


Figure 10: Principal moment vectors (equilibrium limit - Bottom = 0.1xTop Steel)

Discussion for Left Hand Slab

The analysis of this slab has produced a closely bounded solution giving load factor between 1.45 and 1.48. Noting that the moment field is dominated by hogging moments and analysis was conducted with a 90% reduction in bottom steel and with no significant change in the load carrying capacity of the slab this highlighted a way of economising on the steel used for the slab.

Geometry for Right Hand Slab



Dimensions in inches (not metres as suggested in figure)

Figure 11: Geometry and simple supports for roof slab

Properties for Right Hand Slab

The properties used for the elastic analysis were the same as those used for the left hand slab.

Although the slab has variable thickness, a uniform thickness of 10in was assumed for this work and a moment capacity of 10,000lb/in^2 – equal top and bottom steel.

Loads/Boundary Conditions for Right Hand Slab

The slab was assumed to be simply supported on the three lines shown in Figure 11. The slab was loaded with a uniform distributed load corresponding to the sum of a dead load and a live load. The dead load was based on an assumed density of concrete/steel composite over the given thickness whilst the live load was taken as a constant for the entire slab – the values used were the same as those for the left hand slab.

Elastic Analysis Results for Right Hand Slab

A mesh of 637 quadrilateral elements was used for the analyses. The elastic analysis used quadratic moment fields and the displaced shape is shown in Figure 12.

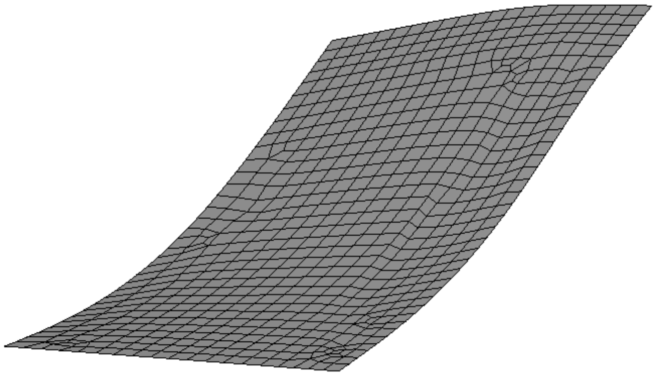


Figure 12: Displaced shape (elastic)

For the assumed elastic properties the maximum displacement was -0.31in towards the centre of the longest unsupported edge. A contour plot of displacement is shown in Figure 13. It is seen that the slab deflects upwards inside the column region and the maximum positive displacement is 0.02in at the free corner.

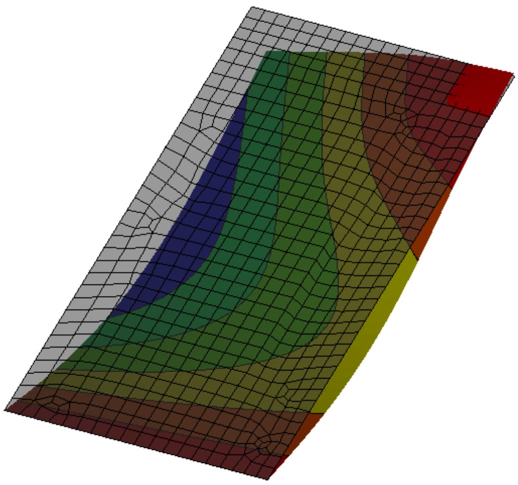


Figure 13: Displacement contours (elastic)

The principal bending moments are shown in Figure 14.

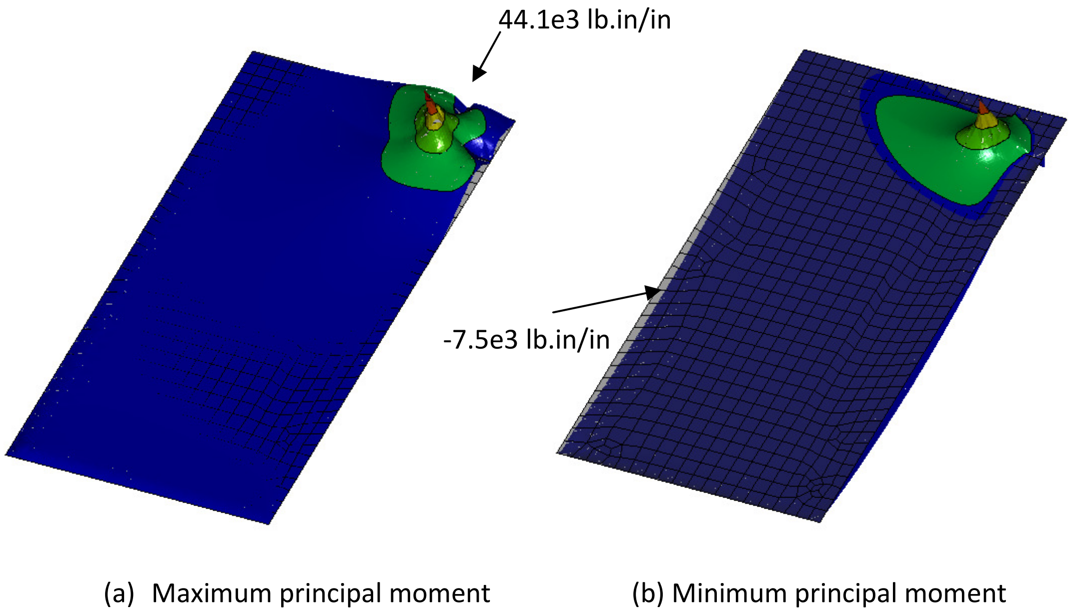


Figure 14: Principal bending moment contours (elastic)

Both principal bending moments (maximum and minimum) peak, as expected, over the corner of the column. The maximum sagging moment appears towards the middle of the longest unsupported edge.

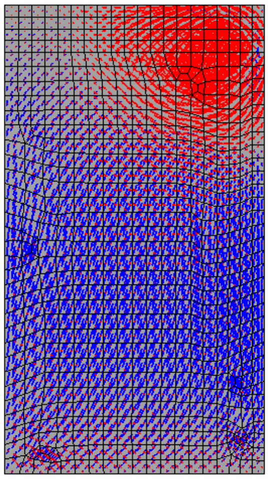
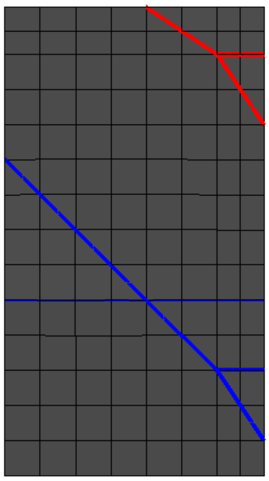


Figure 15: Principal bending vectors (elastic)

The principal moment vectors shown in Figure 15 provide more information on the distribution of moments. The figure shows a region of hogging around the column and a large sagging region between the column and wall support.

Limit Analysis Results for Right Hand Slab

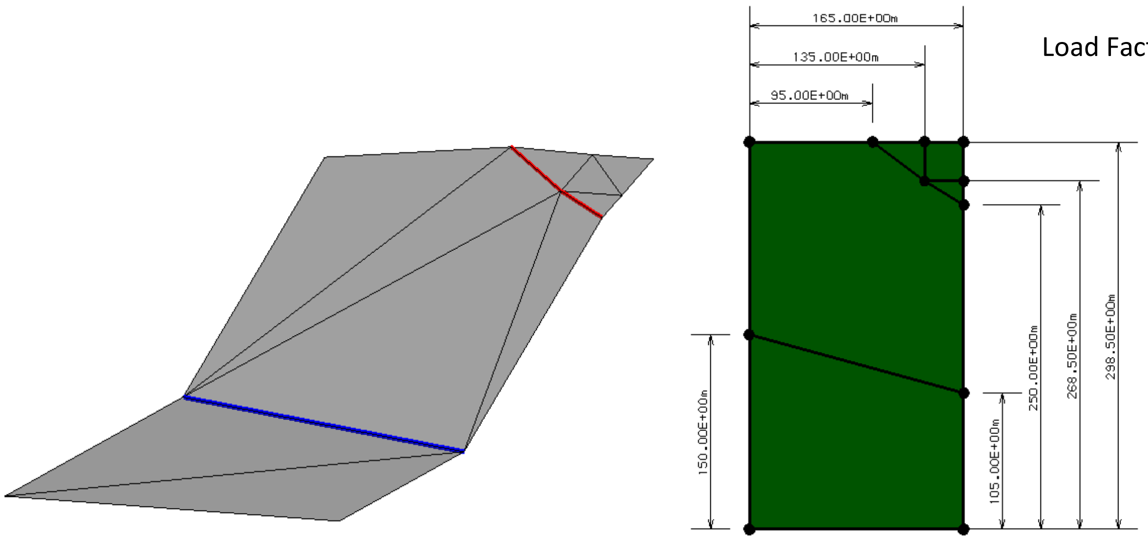
We started this analysis with a regular mesh of 112 rectangular elements. The yield line analysis on this mesh produced the result shown in Figure 16. An equilibrium limit analysis on the same mesh produced a load factor of 1.15. The difference between the upper and lower bound load factors is quite large indicating that there is room for optimisation of the yield line pattern.



Load Factor = 1.47

Figure 16: Yield line pattern

A simplified mesh was constructed incorporating the collapse mechanism anticipated from the analysis of the regular mesh. By manually moving the four points defining the collapse mechanism the yield line load factor was brought down to 1.22 - almost 20% reduction in the predicted collapse load.



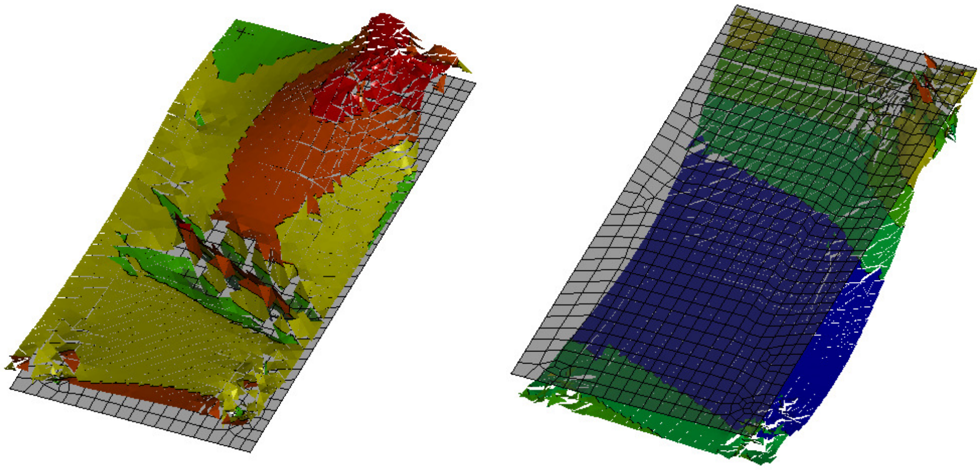
Load Factor = 1.22

Figure 17: Yield line pattern on deflected shape and optimal positions of points

An equilibrium limit analysis of the slab using equal top and bottom steel provided a load factor of 1.16. As the yield line analysis and the equilibrium limit analysis provide results that bound the true solution then we can say that the true load factor lies between 1.16 and 1.22. For this particular example a tight bound has been found but in general we would advocate taking the lower of the two values for reasons of safety.

We present contours of principal moment and principal moment vectors for the equilibrium limit solution in Figures 18 and 19.

Load Factor = 1.16



(a) Maximum principal moment (b) Minimum principal moment

Figure 18: Principal bending moment contours (equilibrium limit)

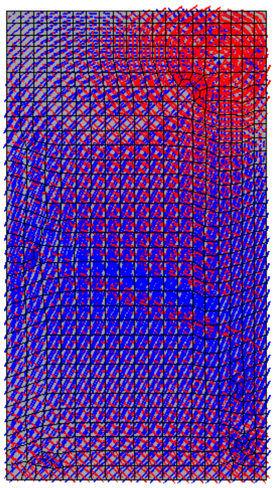


Figure 19: Principal moment vectors (equilibrium limit)

Comparing the elastic and equilibrium limit principal moments (Figures 14 & 18) shows that the effect of moment redistribution is to crop the elastic peak occurring around the corner of the column and concentrate the sagging moments in a diagonal band corresponding closely to the sagging yield line shown in Figure 17.

Discussion for Right Hand Slab

The analysis of this slab has produced a closely bounded solution giving load factor between 1.16 and 1.22. Unlike the left hand slab where the moment field was dominated by hogging moments, the right hand slab considered here has significant regions of both hogging and sagging moments. As such if single regular reinforcement mats are to be used for top and bottom steel there seems little sense in attempting to optimise the moment capacities. If, on the other hand, one were prepared to consider subdividing the slab into regions of differing reinforcement, the moment vector plot of Figure 19 provides a useful indication of how one might perform this subdivision and what sort of reinforcement would be required in each region.

Closure

This document presents the results from elastic and limit analysis (both yield line and equilibrium limit) for a pair of reinforced concrete roof slabs. The two forms of limit analysis provide upper and lower bounds, respectively, for the collapse load of the slabs. In both cases tight bounds on the collapse load were obtained. For the right hand slab the initial mesh produced a significant (20%) overprediction of the true collapse load highlighting a potential defficiency in using the yield line technique for slab assessment – namely that the predicted collapse load is strongly dependent on the assumed collapse mechanism. Mesh refinement alone is not sufficient in yield line analysis as unless the mesh happens to place edges such that the true collapse mechanism can be formed, it produces an unsafe prediction of the collapse load. The lower bound technique (equilibrium limit analysis), on the other hand always provides a safe estimate of the collapse load irrespective of the mesh and quickly converges towards the true solution with mesh refinement.

EFE Equilibrium Finite Elements

for safe structural analysis

and design optimisation