Efficient Subcarrier Allocation for Multiple Access in OFDM Systems

Stephan Pfletschinger, Gerhard Münz, Joachim Speidel
University of Stuttgart
Institute of Telecommunications, Pfaffenwaldring 47, 70569 Stuttgart
{pfletschinger|muenz|speidel}@inue.uni-stuttgart.de

Abstract – **In this paper we propose a computationally efficient subcarrier allocation algorithm for a multiuser OFDM system suited for downlink and uplink transmission. The algorithm considers user-individual bitrate and power constraints and allocates to each user the most appropriate subcarriers in a way that the total transmit power is minimized. The bit and power allocation for each user is done by a single-user bitloading algorithm on the base of this subcarrier allocation. The performance of the proposed algorithm is compared to a near-optimum algorithm which is based on Lagrange optimization. It is found that the proposed algorithm is computationally much more efficient while is has very little performance degradation, in some situations it even achieves better performance. The execution time for the proposed algorithm is compared for different numbers of users and OFDM sizes with the near-optimum algorithm and is found to offer advantages of various orders of magnitudes.**

I. INTRODUCTION

Recently, there has been a growing interest in wireless multiuser systems, such as WLAN and mobile communication systems. OFDM is considered a promising solution due to its elegant ability to combat multipath fading problems. For multiuser access, OFDMA is the straightforward extension: each user communicates with the base station over a set of dedicated subchannels. In a typical wireless transmission environment, the transfer function is different for each user. As a consequence, some subchannels might be in deep fade for one user while they might be fine for others. Thus, in case of a static (e.g. fixed wireless) or a slowly fading channel, the subchannel allocation should be adapted to the channel characteristics and adaptive modulation should be applied on each subchannel. If the channel is known to the transmitter and the receiver, it can be shown that OFDMA clearly outperforms other multiuser techniques [1].

For single-user OFDM, several algorithms for adaptive modulation based on the classical water-pouring theorem [2] have been developed (see [3] and the references therein). They are referred to as bitloading algorithms and determine the number of bits and the transmit power for each subchannel. Hence, in OFDMA, first a subcarrier allocation algorithm assigns the subchannels to the users, then a bitloading algorithm determines the constellation size and transmit power for each subchannel.

In this paper we present an algorithm which determines the subcarrier allocation such that the total transmit power is minimized for a given bitrate per user. This minimization is done with the constraint that the transmit power per user must not exceed an individual, predefined value. Hence, the algorithm is suited for the uplink of an OFDMA system. For the downlink the subcarrier allocation is simplified since there is only one power constraint to be considered whereas for the upstream there is one power constraint per user.

The subcarrier allocation problem has been studied under various premises. Wong et al. [4], [5] presented an algorithm which is based on Lagrange optimization and minimizes the total transmit power under bitrate constraints. This algorithm nearly reaches the optimal solution, but due to its complexity and its slow convergence it is computationally very expensive. Later, the same authors presented a strongly simplified faster algorithm [6]. Another step towards a fast implementation was made by Yin and Liu [7] who partitioned the task into two steps. Nevertheless, their algorithm still contains a highly complex assignment problem whose solution is shown for only two users. An algorithm based on CSMA (carrier sense multiple access) which maximizes the number of simultaneous users under bitrate and power constraints was introduced by [8], and [9] proposed a suboptimal algorithm to maximize the channel capacity of the user with smallest capacity.

Another approach for subcarrier allocation is multiuser waterfilling [10], [11], [12] which maximizes the total bitrate of all users under the constraint of a maximum transmit power per user. However, this solution does not guarantee a minimum bitrate for any user and is therefore not appropriate for most practical applications.

In the next section we introduce the system model and sketch an algorithm based on the water-filling theorem. In section 3 we present in detail a computationally efficient subcarrier allocation algorithm suited for *uplink* transmission. Section 4 presents simulation results and a comparison with a near-optimum solution.

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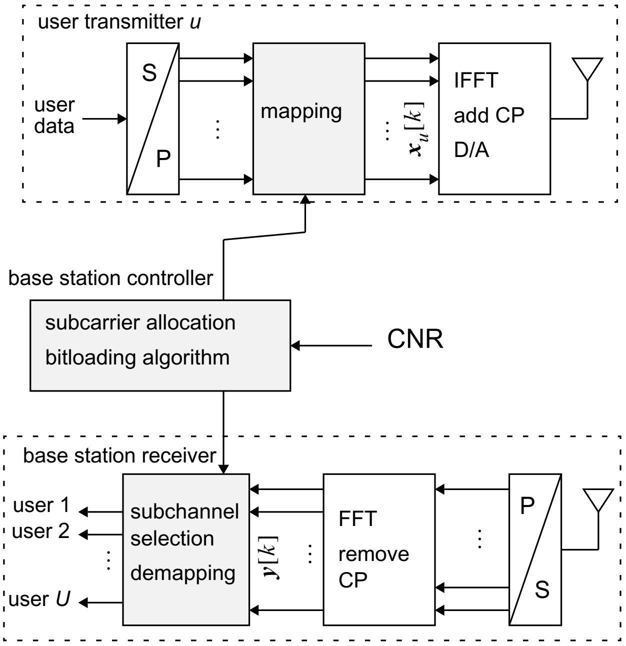

Fig. 1. User transmitter and multiuser receiver.

II. SYSTEM MODEL AND WATERFILLING

We consider the system model depicted in Fig. 1. The base station controller allocates for each user a set of subcarriers with corresponding power and constellation size. As input for the controller serves the channel gain to noise ratio (CNR) which is available after the channel estimation in the base station.

We assume the channel to be linear and (nearly) time-invariant. Provided the lengths of the impulse responses do not exceed the length of the cyclic prefix (CP), the channel can be decomposed into N independent flat fading subchannels with channel gain coefficients $H_{u,\nu}$ for user u and subchannel ν like illustrated in Fig. 2. Hence, a broadband channel with ISI is rendered into N independent flat channels. On the channel, additive Gaussian noise is assumed, thus the sequences $r_\nu[k]$ are independent Gaussian noise samples with zero mean. Since the noise is not necessarily white, the sequences $r_\nu[k]$ will generally have different noise powers $\sigma_\nu^2 = E[|r_\nu[k]|^2]$, where $E[\cdot]$ denotes the expectation operator. The input sequences $x_{u,\nu}[k]$ consist of $2^{b_{u,\nu}}$-QAM-modulated symbols with mean energy per symbol[1] $E_{u,\nu} = E[|x_{u,\nu}[k]|^2]$ and $b_{u,\nu} \in \{0, 1, \ldots, b_{max}\}$[2]. According to [13], the necessary symbol energy in order to transmit b bits with symbol error probability $P_{S,u}$ is given by

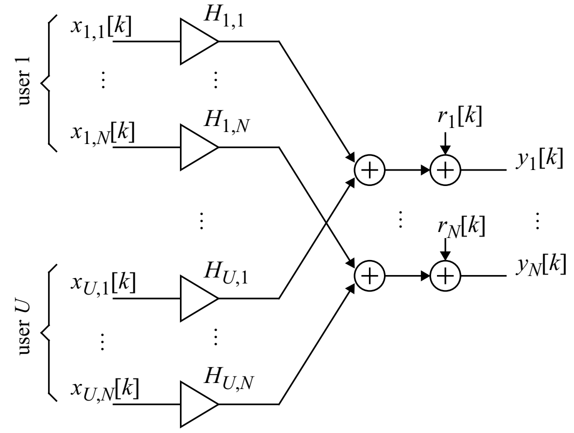

Fig. 2. Channel model for multiuser OFDM.

$$E_{u,\nu} = T_{u,\nu}^{-1} \cdot (2^b - 1) \text{ with } T_{u,\nu} = \frac{|H_{u,\nu}|^2}{\Gamma_u \sigma_\nu^2} \quad (1)$$

where the SNR gap Γ_u is defined as

$$\Gamma_u = \frac{1}{3} \cdot \left[Q^{-1}\left(\frac{P_{S,u}}{4}\right)\right]^2 \quad (2)$$

and $Q^{-1}(\cdot)$ is the inverse Q-function. By (2), different symbol error rates, possibly defining different QoS classes, can be defined for each user. A coding gain γ_c and a margin γ_m can be considered in Γ_u (see [13] for details), thus allowing different users to employ different coding schemes.

Before giving the details of our proposed low-complexity algorithm, we revisit the waterfilling theorem which provides the basis for the single-user bitloading algorithm that is applied after the subchannel allocation.

For a single-user system, i.e. for $U = 1$ in the model of Fig. 2, the channel gain to noise ratio (CNR), incorporating the SNR gap, is defined as

$$T_\nu = \frac{|H_\nu|^2}{\Gamma \cdot \sigma_\nu^2} . \quad (3)$$

This completely characterizes the channel. According to the waterfilling theorem, the symbol energy on subchannel ν is given by

$$E_\nu = [c_0 - T_\nu^{-1}]^+ \text{, where } [x]^+ = \begin{cases} x \ , x > 0 \\ 0 \ \text{else} \end{cases} \quad (4)$$

and the "water level" c_0 must be chosen such that

$$E_{tot} = \sum_{\nu=1}^{N} E_\nu . \quad (5)$$

The inverse CNR T_ν^{-1} can thus be imagined as the bottom of a bowl into which E_{tot} liters of water are poured, giving the water level c_0.

A bitloading algorithm based on this theorem gives the optimum solution for the single-user case. For the

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[1] Note that this is the same as the average transmit power of user u on subcarrier ν.

[2] $b_{\max}$ defines the maximum constellation size, which e.g. in ADSL is chosen as $b_{\max} = 15$.

multi-user channel, a generalization of the waterfilling theorem exists [10] and an algorithm has been derived [12]. The inconveniences of this nearly capacity achieving algorithm for practical use are its complexity and the lack of constraints: no minimum bitrate per user is considered and thus generally some users will be assigned higher rates than required while others will be left with no subcarrier at all.

III. TWO-STEP ALGORITHM

Yin and Liu described in [7] a two-step algorithm which divides the subcarrier allocation into two steps, based on the following reasoning:

- the resources for one user, i.e. the number of subcarriers and the transmit power mainly depend on its desired minimal bitrate $B_{min}(u)$ and on the *mean* CNR of its channel.
- *which* subchannel is assigned to a user depends on the CNR $T_{u,\nu}$ according to (1).

Thus, the subcarrier allocation can be realized in two steps. First, an estimation about how many subcarriers are conceded to each user is made, taking into account the users' mean CNRs, the desired minimum bitrates $B_{min}(u)$ and the users' maximum transmit powers $E_{max}(u)$. In the second step it is determined which subcarriers are given to which user.

As [7] is aimed at downlink transmission there is just one power constraint for the total transmit power. In the second step, the subcarriers are distributed in such a way that the total bitrate is maximized. This is a combinatorial problem with $N!/\prod_u k_u!$ possibilities, where k_u denotes the numbers of subcarriers assigned to user u. A solution for $U = 2$ users is given, but for a greater number of users, the complexity of the proposed algorithm will be enormous.

In the following, an algorithm is described which includes user-individual power and rate constraints and avoids the complicated combinatorial optimization.

Some typical CNR curves for 64 subcarriers and 4 users are shown in Fig. 3. The mean CNR of user u is defined as

$$\overline{T}_u = \frac{1}{N}\sum_{\nu=1}^{N} T_{u,\nu} \; . \qquad (6)$$

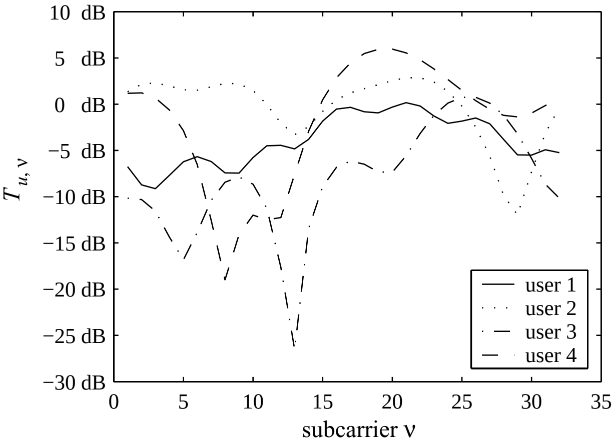

Fig. 3. Channel gain to noise ratio for four users.

A. Step 1: Estimation of the Number of Subcarriers for each User

Each user is assigned a number k_u of subcarriers such that the desired bitrate $B_{min}(u)$ can be reached with the given maximum energy $E_{max}(u)$:

$$E_{\max}(u) \ge E_{\text{tot}}(u) = k_u \overline{T}_u^{-1}\left(2^{B_{min}(u)/k_u} - 1\right) \; . \qquad (7)$$

For small $E_{max}(u)$ it might happen that the desired bitrate cannot be reached even if all subcarriers are conceded to user u. This is the case for

$$E_{min}(u) = N\overline{T}_u^{-1}\left(2^{B_{min}(u)/N} - 1\right) > E_{max}(u) \; . \qquad (8)$$

In this case the desired bitrate has to be reduced or the transmit power must be increased.

At the beginning, k_u is calculated as if the maximum number of bits per symbol b_{max} could be applied to all subcarriers:

$$k_u = \lceil B_{min}(u)/b_{max} \rceil \; .$$

Normally, in this first step much less subcarriers are assigned than available (otherwise the desired bitrates would already exceed the system's transmission capacities). Next, we assign for each user new subcarriers until the required energy does not exceed $E_{max}(u)$, in accordance with (7). If there are subcarriers left, i.e. $\sum_u k_u < N$ (which is the normal case), the maximum energies are lowered by a small step, and the procedure repeats until no subcarriers are left.

As this normally assigns some subcarriers more than allowed, we remove a subcarrier from the user which has to increase its transmit power by the smallest amount without this carrier. This is repeated until exactly N subcarriers are granted. The exact algorithm is detailed in Fig. 4.

B. Step 2: Distribution of the Subcarriers

The idea for the subcarrier distribution is that the users choose alternatingly the subcarrier with the best CNR. This is similar to the procedure that is used in physical education to form two sports teams: beginning with two team captains, the teams choose alternatingly one new player until nobody is left. For the subcarrier distribution there are more than two users which additionally have unequal numbers of subcar-

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$k_u := \lceil B_{\min}(u)/b_{\max} \rceil$
$E_{\text{tot}}(u) := k_u \overline{T}_u^{-1}(2^{B_{\min}(u)/k_u} - 1)$, $\forall u$
while $\sum_u k_u < N$
    for $u := 1, \ldots, U$
        while $E_{\text{tot}}(u) > E_{\max}(u)$
            $k_u := k_u + 1$
            $E_{\text{tot}}(u) := k_u \overline{T}_u^{-1}(2^{B_{\min}(u)/k_u} - 1)$
        end
    end
    if $\sum_u k_u < N$ then $E_{\max}(u) := (1 - \varepsilon)E_{\max}(u)$, $\forall u$
end
while $\sum_u k_u > N$
    $E_{\text{new}}(u) := (k_u - 1)\overline{T}_u^{-1}(2^{B_{\min}(u)/(k_u - 1)} - 1)$, $\forall u$
    $u' := \arg\min_u \{E_{\text{new}}(u) - E_{\text{tot}}(u)\}$
    $k_{u'} := k_{u'} - 1$ ; $E_{\text{tot}}(u') := E_{\text{new}}(u')$
end
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Fig. 4. First step of subcarrier allocation: estimate the number of subcarriers per user.

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$\mathbf{A} := \mathbf{0}$, $p_0(u) := k_u/N$, $\forall u$
$U := \{u | u = \arg\max_{u'=1\ldots U} \{k_{u'}\}\}$
for $u \in U$
    $\nu_1 := \arg\min_{\nu \in M} \{T_{u,\nu}^{-1}\}$, with $M := \left\{\nu | \sum_{u'=1}^{U} A_{u',\nu} = 0\right\}$
    $k_u := k_u - 1$
    $A_{u,\nu_1} := 1$
end
while $\sum_u k_u > 0$
    $p(u) := k_u / \sum_{u'} k_{u'}$ $\forall u$
    $U := \{u | u = \arg\max \{p(u') - p_0(u')\}\}$
    for $u \in U$
        $\nu_1 := \arg\min_{\nu \in M} \{T_{u,\nu}^{-1}\}$, $M := \left\{\nu | \sum_{u'=1}^{U} A_{u',\nu} = 0\right\}$
        $k_u := k_u - 1$
        $A_{u,\nu_1} := 1$
    end
end
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Fig. 5. Second step of subcarrier allocations: assign the subcarriers to the users.

riers. Therefore the order in which the users choose their subcarrier is important. A procedure based on priorities controls the order: the reference priority $p_0(u)$ is defined as the number of subcarriers of user u over the total number of subcarriers:

$$p_0(u) = k_u/N . \tag{9}$$

After user u has chosen one subcarrier, k_u is decremented by one; thus k_u here stands for the number of subcarriers that are still to assign. Hence we define the actual priority of user u as

$$p(u) = k_u / \sum_{u=1}^{U} k_u, \; u = 1, \ldots, U. \tag{10}$$

The user with the most subcarriers begins, then after each step the user with the greatest difference between reference and actual priority is picked out for the next turn. In the algorithm shown in Fig. 5, $\mathbf{A} = (A_{u,\nu})$ designates the subcarrier allocation matrix, with $A_{u,\nu} = 1$ if subcarrier ν is assigned to user u, and zero otherwise.

IV. SIMULATION RESULTS

The channel transfer functions for four users have been generated assuming a wireless channel as described in [14] and the noise was assumed to be white, giving the CNR curves in Fig. 3. The energy budget[1] has been chosen as 30 dB per user. With these parameters, the channel capacity was calculated with the multiuser waterfilling algorithm [12] as 320 bits per OFDM symbol. The actual bitrate achieved for an SNR gap of $\Gamma = 7$, which corresponds to a symbol error rate of $P_S = 10^{-5}$, was 229 bit/symbol. The discrepancy between the channel capacity and the bitrate depends heavily on the SNR gap. Note that the multiuser waterfilling algorithm acts on the assumption of a power budget per user and does not consider rate constraints. Although multiuser waterfilling is the dual of the considered optimization problem, it can serve in a way as a point of reference: it provides the maximum sum bitrate that can be achieved with the given CNRs.

While the multiuser waterfilling is the answer to a basic information theoretic problem, the power minimization with rate constraints is practically much more relevant. We compared the proposed two-step algorithm with a (nearly) optimum algorithm which is based on Lagrange optimization [4].

The two-step algorithm additionally considers user-individual power constraints. For the simulation, the power budgets where chosen large enough, so that the algorithm of Wong et al. did not violate these constraints. The minimum bitrates where selected as $B_{\min} = \{50, 45, 35, 55\}$, which was met by both algorithms. The allocated user energies are given in Table I, indicating that the two-step algorithm only consumes 0.8 dB more transmit power than the (nearly) optimum solution, but the two-step algorithm runs about 1000 times faster! The resulting bit-allocation is shown in Fig. 6. By comparing Fig. 3 with

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[1] All energies are normalized to the total noise power $P_N = \sum_{\nu=1}^{N} \sigma_\nu^2$.

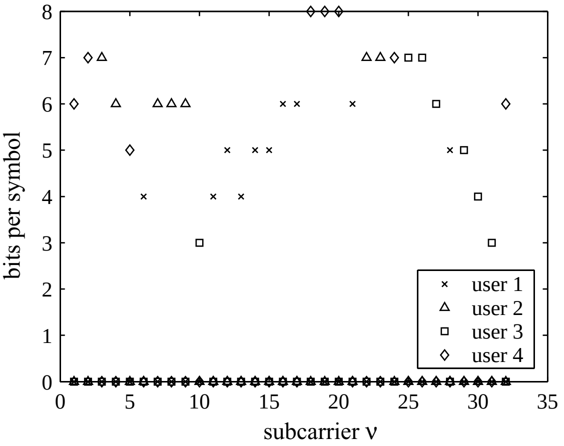

Fig. 6. Allocated bits per symbol.

Fig. 6 we recognize that usually a subcarrier is assigned to the user with the highest CNR. Of course, this is not always possible because of the rate constraints for each user.

TABLE I

ALLOCATED ENERGIES TO EACH USER AND TOTAL TRANSMIT ENERGY PER SYMBOL

user	1	2	3	4	total
Wong	27.2 dB	26.1 dB	25.8 dB	25.7 dB	32.3 dB
two-step	28.0 dB	25.7 dB	27.0 dB	27.2 dB	33.1 dB

In order to determine the computational efficiency of the proposed algorithm, it was tested with different numbers of users and subcarriers in comparison to Wong's reference algorithm. A WSSUS channel with exponential delay power spectrum [14] was used and for each user and simulation run, the stochastic channel coefficients $H_{u,\nu}$ were determined. Fig. 7 shows the execution times of both routines for different values of U and N. The two-step algorithm was performed with up to $N = 4096$ subcarriers and $U = 512$ users while the maximum values for Wong's algorithm where $N = 128$, $U = 32$. Especially for many users/subcarriers the execution times differ by some orders of magnitude while the achieved total transmit powers vary only slightly.

V. CONCLUSION

We have presented in detail a simple algorithm for adaptive subcarrier allocation in multiple access OFDM systems. The proposed algorithm minimizes the total transmit power while taking as constraints the users' desired bitrates and their power budgets. The comparison with a near-optimum algorithm reveals an enormous reduction of complexity while the performance is maintained.

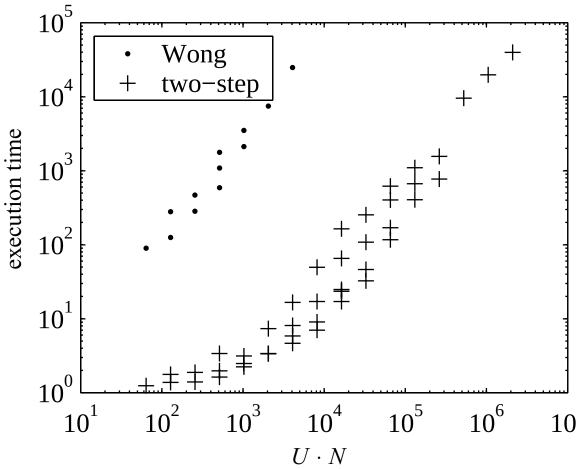

Fig. 7. Execution times of the proposed algorithm in comparison with Wong's near optimum algorithm for different numbers of users and subcarriers.

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