Dynamics and Equilibria

Sergiu Hart

Presidential Address, GAMES 2008 (July 2008)

Revised and Expanded (November 2009)

Revised (2010, 2011, 2012, 2013)

DYNAMICS AND EQUILIBRIA

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Papers

Papers

- Hart and Mas-Colell, *Econometrica* 2000
- Hart and Mas-Colell, *J Econ Theory* 2001
- Hart and Mas-Colell, *Amer Econ Rev* 2003
- Hart, *Econometrica* 2005
- Hart and Mas-Colell, *Games Econ Behav* 2006
- Hart and Mansour, *Games Econ Behav* 2010
- Hart, *Games Econ Behav* 2011

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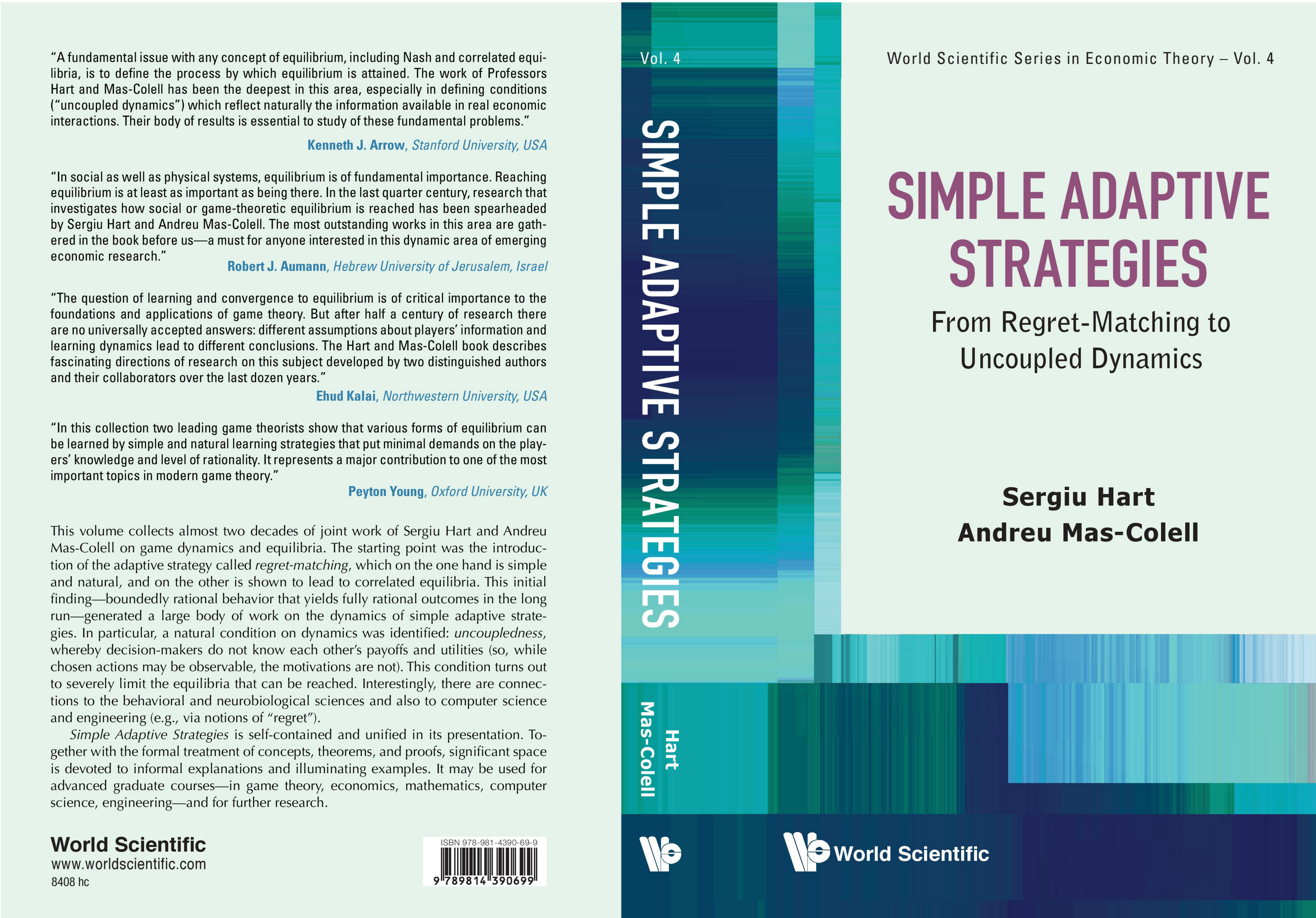
Book


"A fundamental issue with any concept of equilibrium, including Nash and correlated equilibria, is to define the process by which equilibrium is attained. The work of Professors Hart and Mas-Colell has been the deepest in this area, especially in defining conditions ("uncoupled dynamics") which reflect naturally the information available in real economic interactions. Their body of results is essential to study of these fundamental problems."
Kenneth J. Arrow, Stanford University, USA
"In social as well as physical systems, equilibrium is of fundamental importance. Reaching equilibrium is at least as important as being there. In the last quarter century, research that investigates how social or game-theoretic equilibrium is reached has been spearheaded by Sergiu Hart and Andreu Mas-Colell. The most outstanding works in this area are gathered in the book before us—a must for anyone interested in this dynamic area of emerging economic research."
Robert J. Aumann, Hebrew University of Jerusalem, Israel
"The question of learning and convergence to equilibrium is of critical importance to the foundations and applications of game theory. But after half a century of research there are no universally accepted answers: different assumptions about players' information and learning dynamics lead to different conclusions. The Hart and Mas-Colell book describes fascinating directions of research on this subject developed by two distinguished authors and their collaborators over the last dozen years."
Ehud Kalai, Northwestern University, USA
"In this collection two leading game theorists show that various forms of equilibrium can be learned by simple and natural learning strategies that put minimal demands on the players' knowledge and level of rationality. It represents a major contribution to one of the most important topics in modern game theory."
Peyton Young, Oxford University, UK
This volume collects almost two decades of joint work of Sergiu Hart and Andreu Mas-Colell on game dynamics and equilibria. The starting point was the introduction of the adaptive strategy called regret-matching, which on the one hand is simple and natural, and on the other is shown to lead to correlated equilibria. This initial finding—boundedly rational behavior that yields fully rational outcomes in the long run—generated a large body of work on the dynamics of simple adaptive strategies. In particular, a natural condition on dynamics was identified: uncoupledness, whereby decision-makers do not know each other's payoffs and utilities (so, while chosen actions may be observable, the motivations are not). This condition turns out to severely limit the equilibria that can be reached. Interestingly, there are connections to the behavioral and neurobiological sciences and also to computer science and engineering (e.g., via notions of "regret").
Simple Adaptive Strategies is self-contained and unified in its presentation. Together with the formal treatment of concepts, theorems, and proofs, significant space is devoted to informal explanations and illuminating examples. It may be used for advanced graduate courses—in game theory, economics, mathematics, computer science, engineering—and for further research.
World Scientific
www.worldscientific.com
8408 hc
ISBN 978-981-4390-69-9
Vol. 4
SIMPLE ADAPTIVE STRATEGIES
Hart
Mas-Colell
World Scientific Series in Economic Theory – Vol. 4
SIMPLE ADAPTIVE STRATEGIES
From Regret-Matching to Uncoupled Dynamics
Sergiu Hart
Andreu Mas-Colell
World Scientific

Nash Equilibrium

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John Nash, Ph.D. Dissertation, Princeton 1950

Nash Equilibrium

EQUILIBRIUM POINT:

John Nash, Ph.D. Dissertation, Princeton 1950

Nash Equilibrium

EQUILIBRIUM POINT:

"Each player's strategy is optimal against those of the others."

John Nash, Ph.D. Dissertation, Princeton 1950

Dynamics

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FACT

Dynamics

<u>FACT</u>

There are no general, natural dynamics leading to Nash equilibrium

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- ***"general"***

Dynamics

FACT

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- ***"general"*** : **in all games**

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  rather than: in specific classes of games

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- ***"general"*** : **in all games**
  rather than: in specific classes of games:
  - two-person zero-sum games
  - two-person potential games
  - supermodular games
  - . . .

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leading to Nash equilibrium

- ***"leading to Nash equilibrium"***

Dynamics

FACT

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- ***"leading to Nash equilibrium"*** :
  at a Nash equilibrium (or close to it) from some time on

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- ***"natural"***

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Dynamics

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There are no general, natural dynamics leading to Nash equilibrium

- *"natural"* :
  - **adaptive** (reacting, improving, ...)

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- *"natural"* :
  - **adaptive** (reacting, improving, ...)
  - **simple and efficient**

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  - **simple and efficient**:
    - **computation** (performed at each step)

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  bounded rationality

Dynamics

Dynamics that are **NOT** ***"natural"*** :

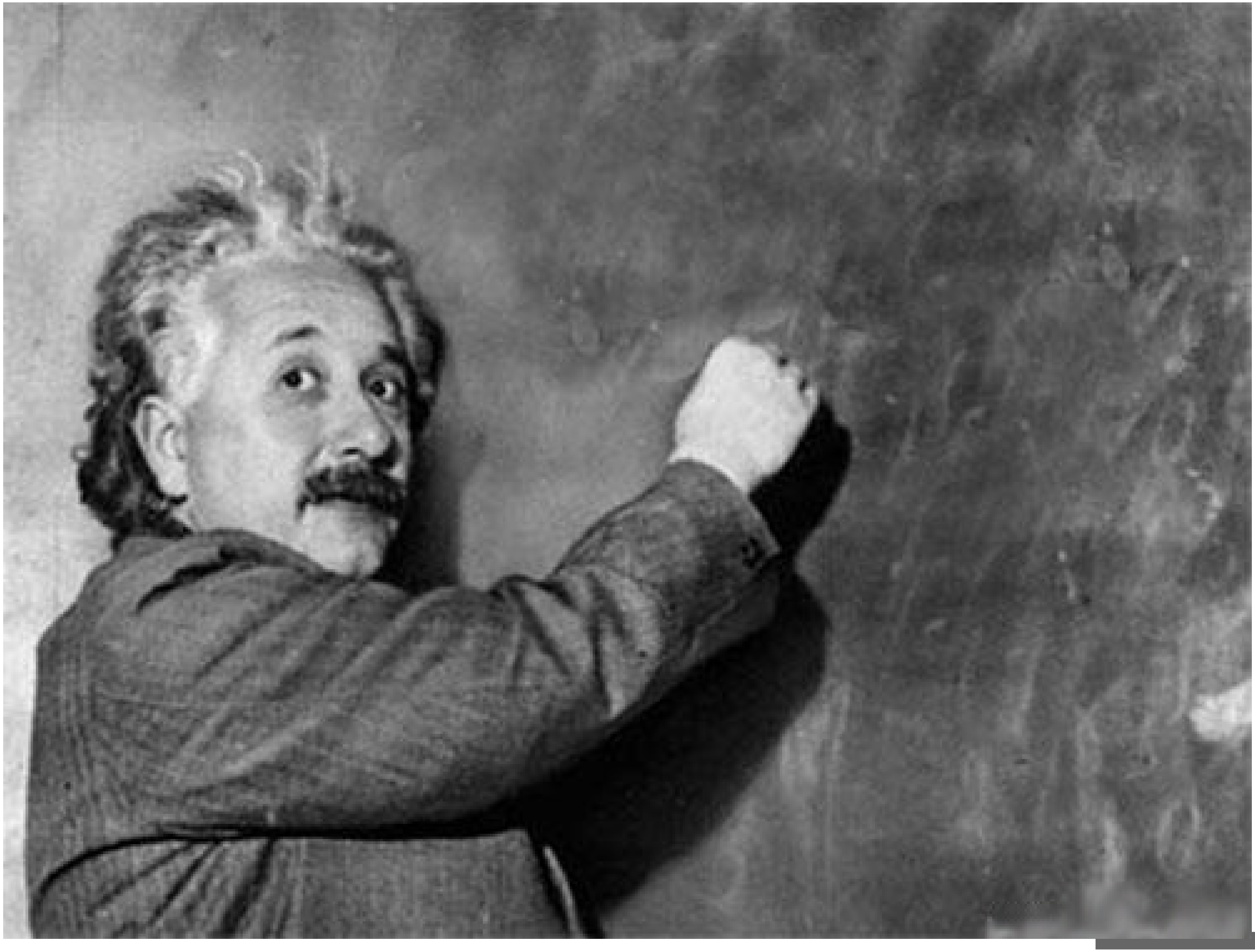
Dynamics

Dynamics that are **NOT** ***"natural"*** :

- **exhaustive search**
  (deterministic or stochastic)

Exhaustive Search

Exhaustive Search



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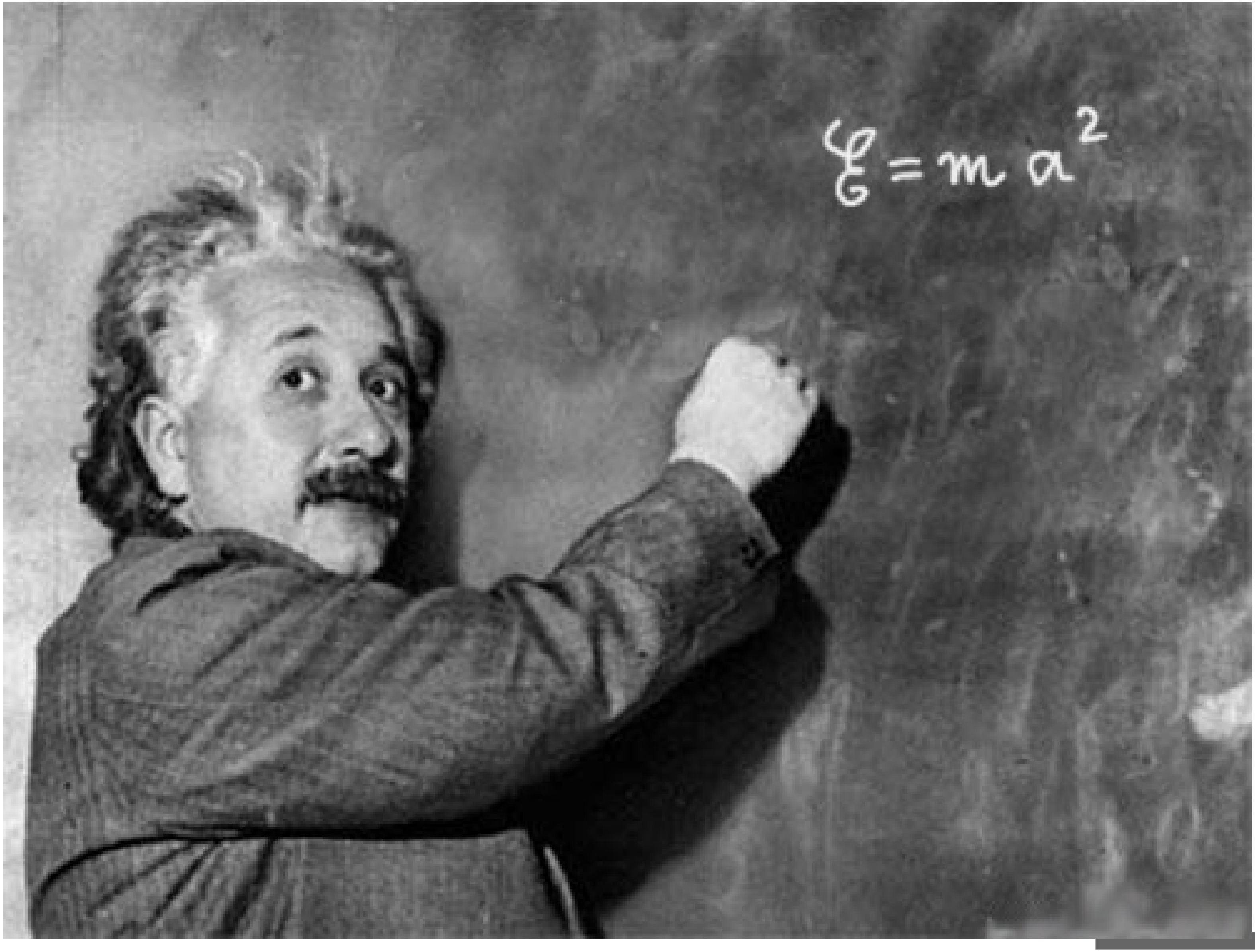

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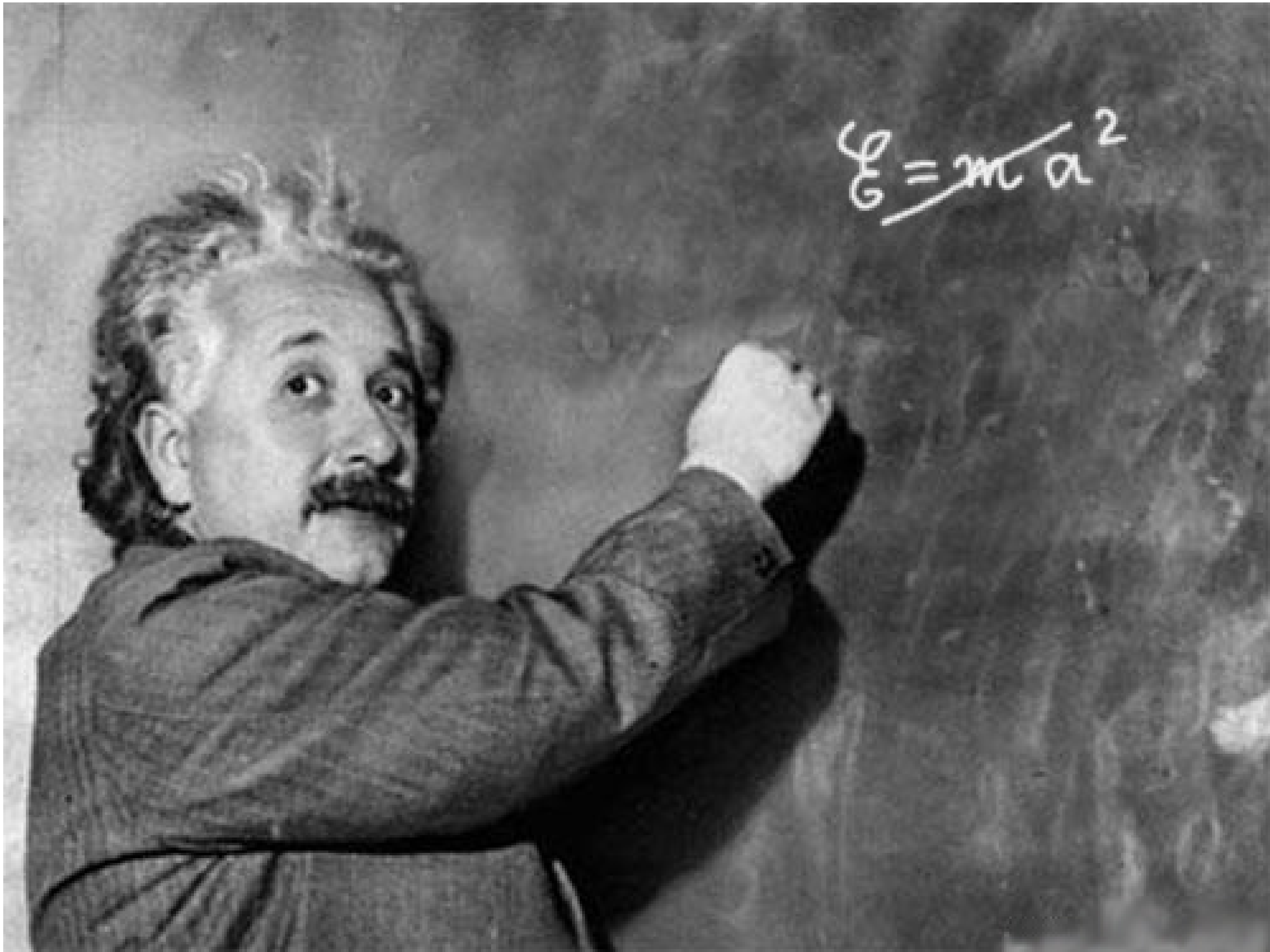

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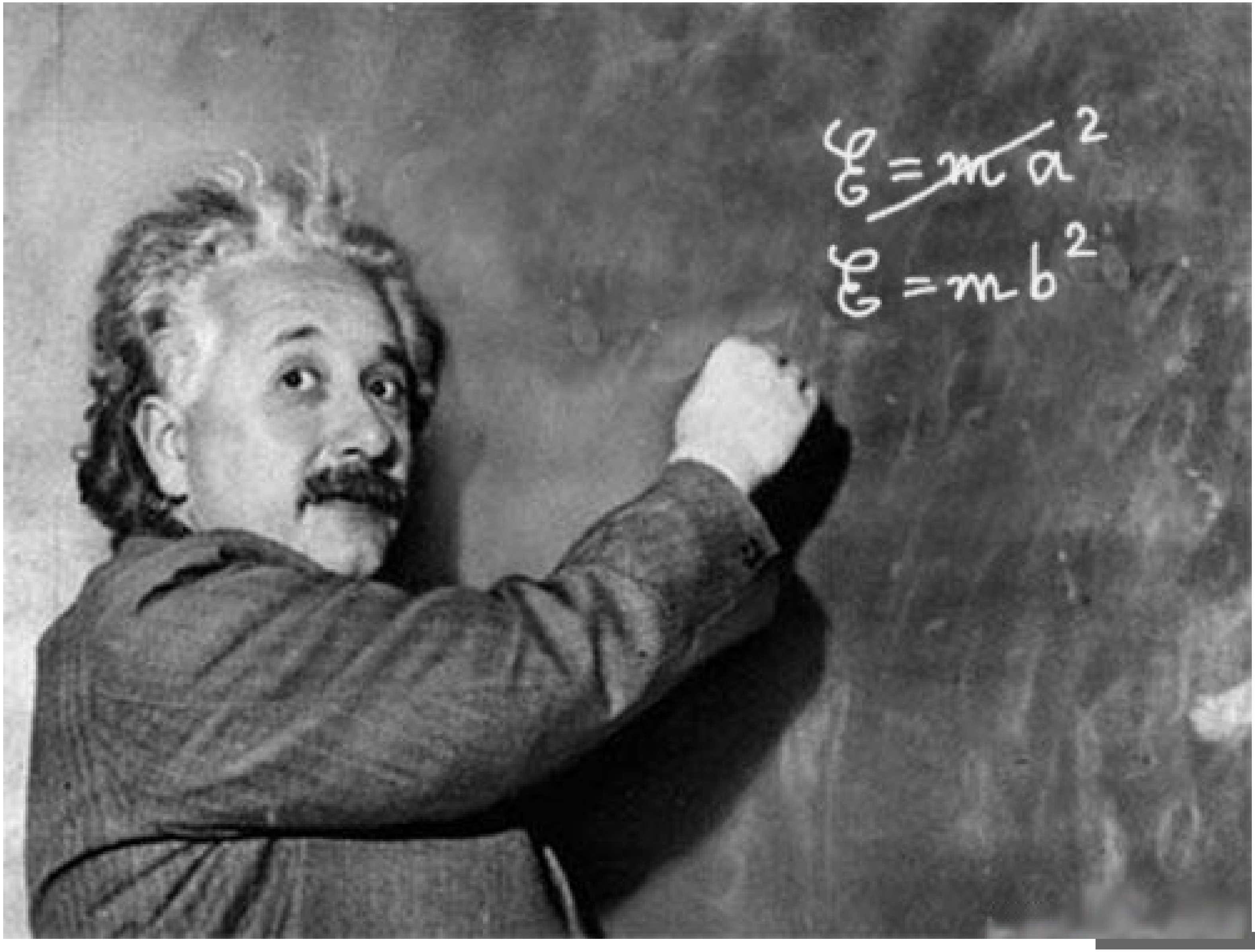

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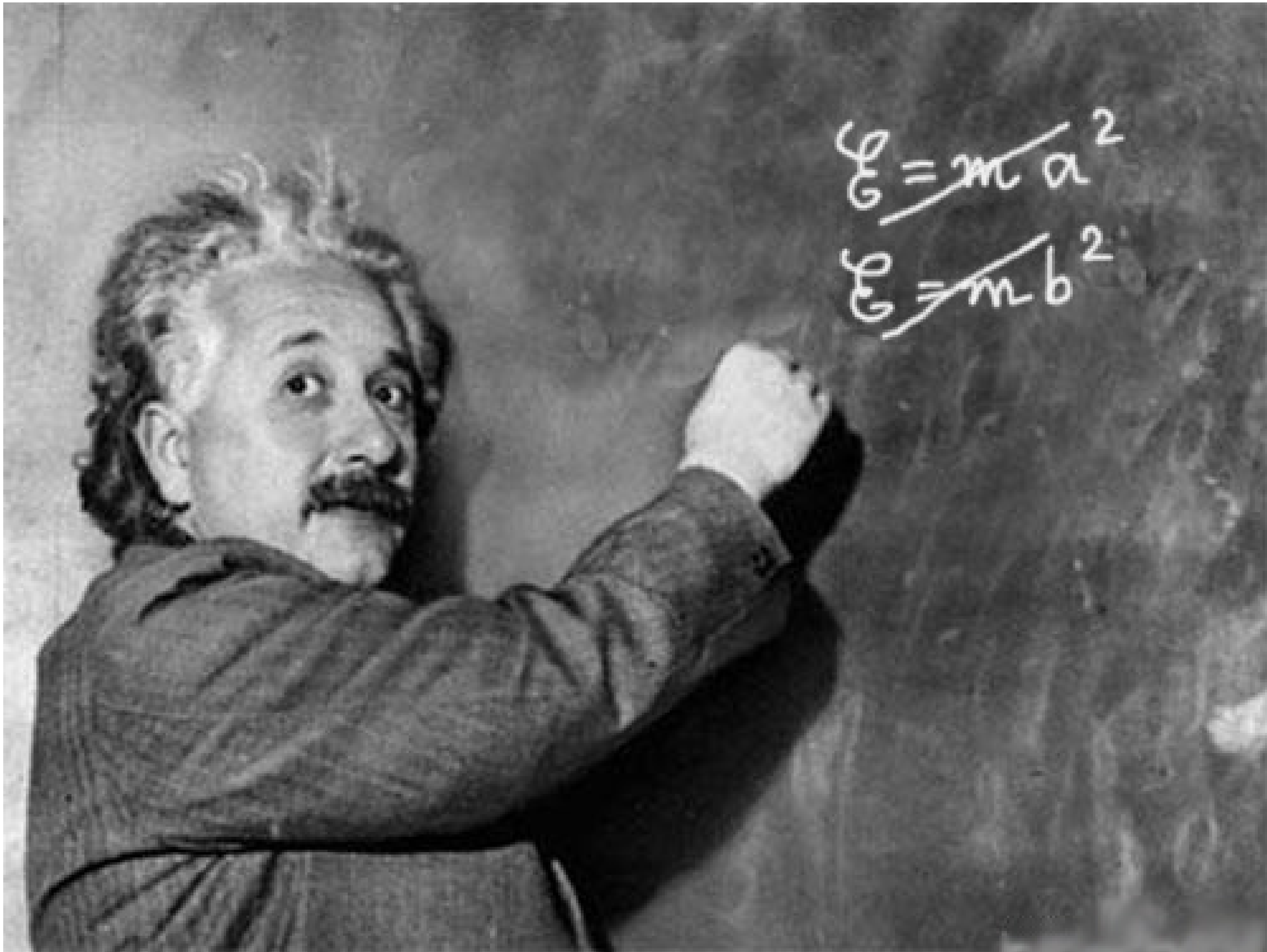

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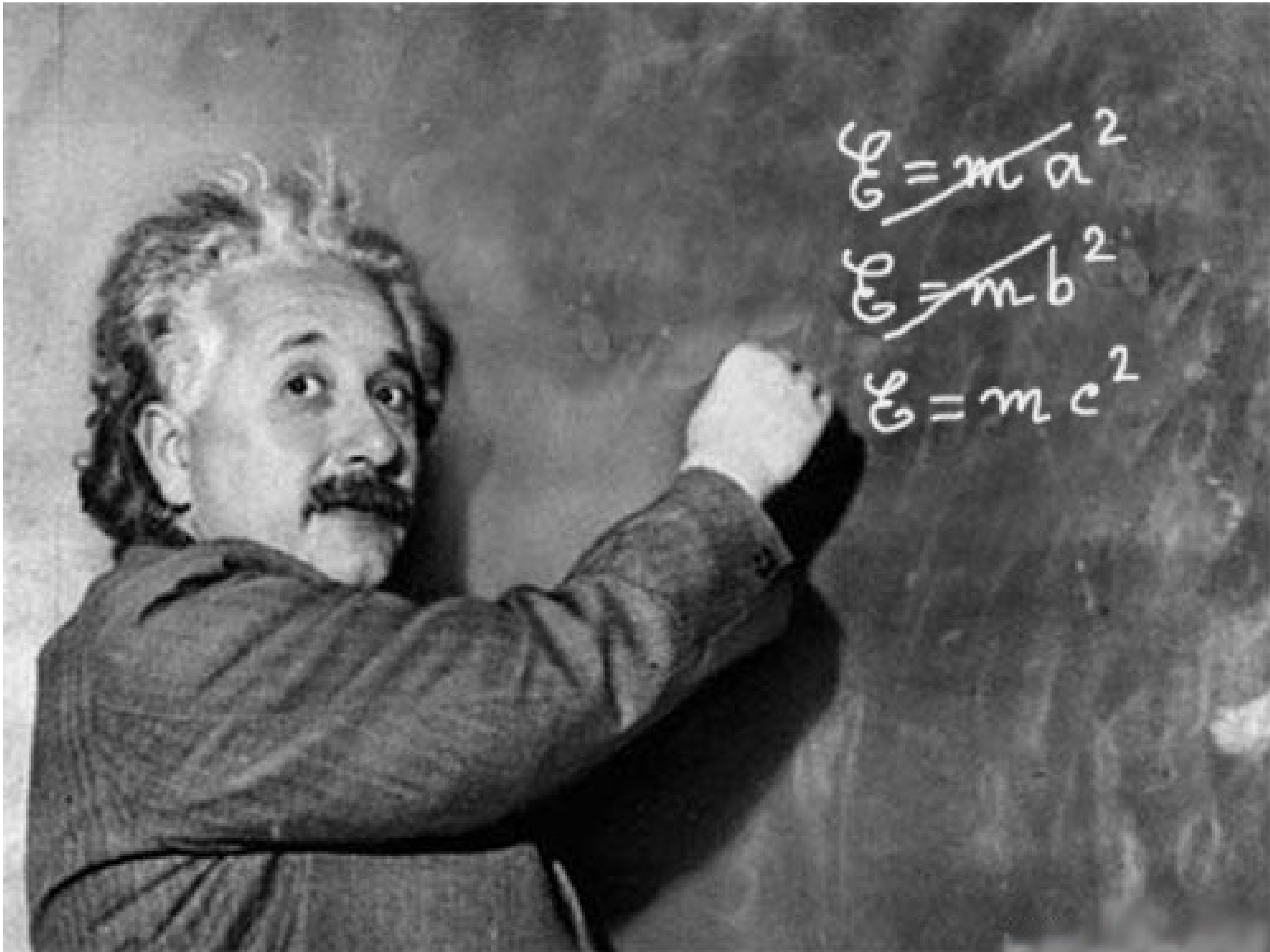

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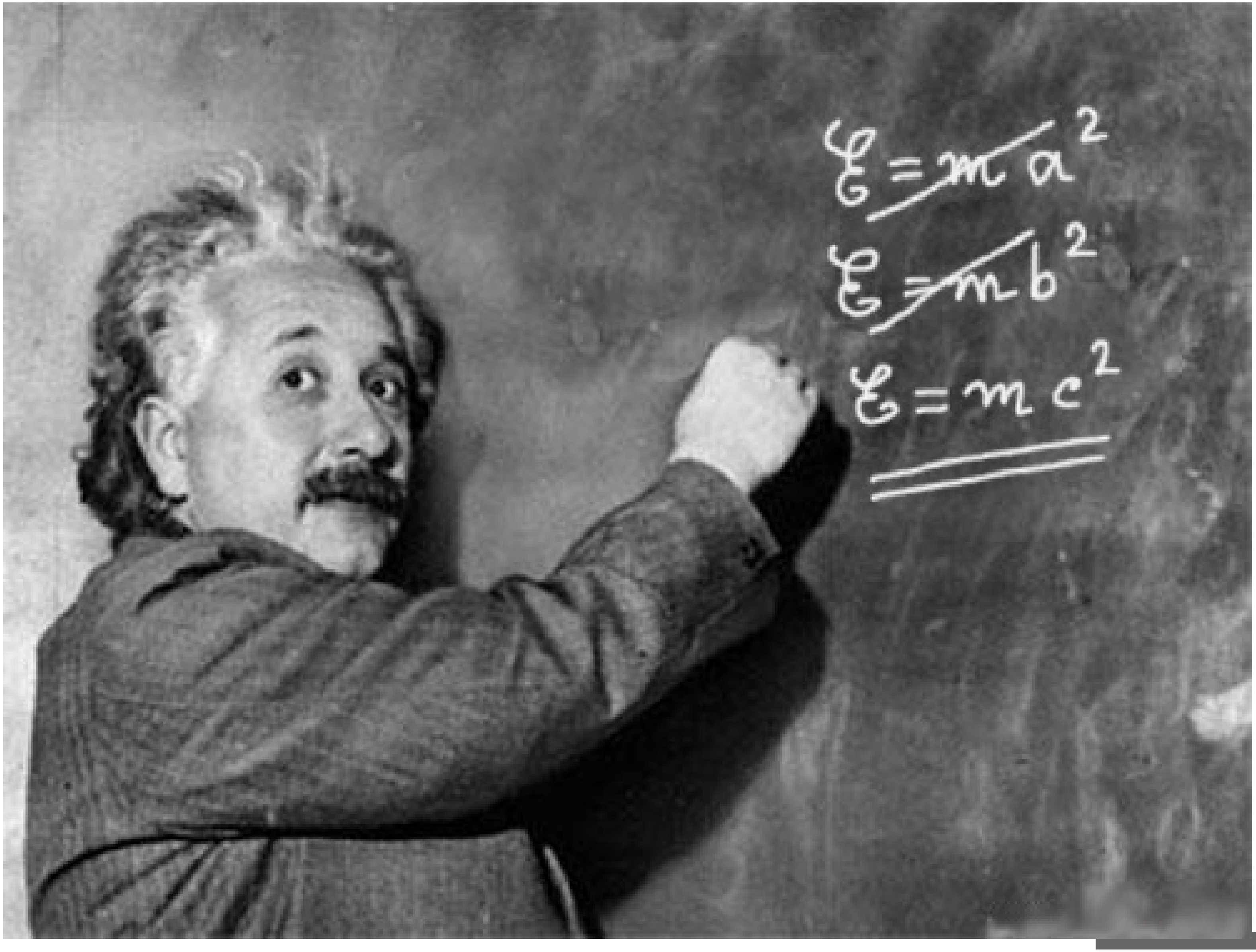

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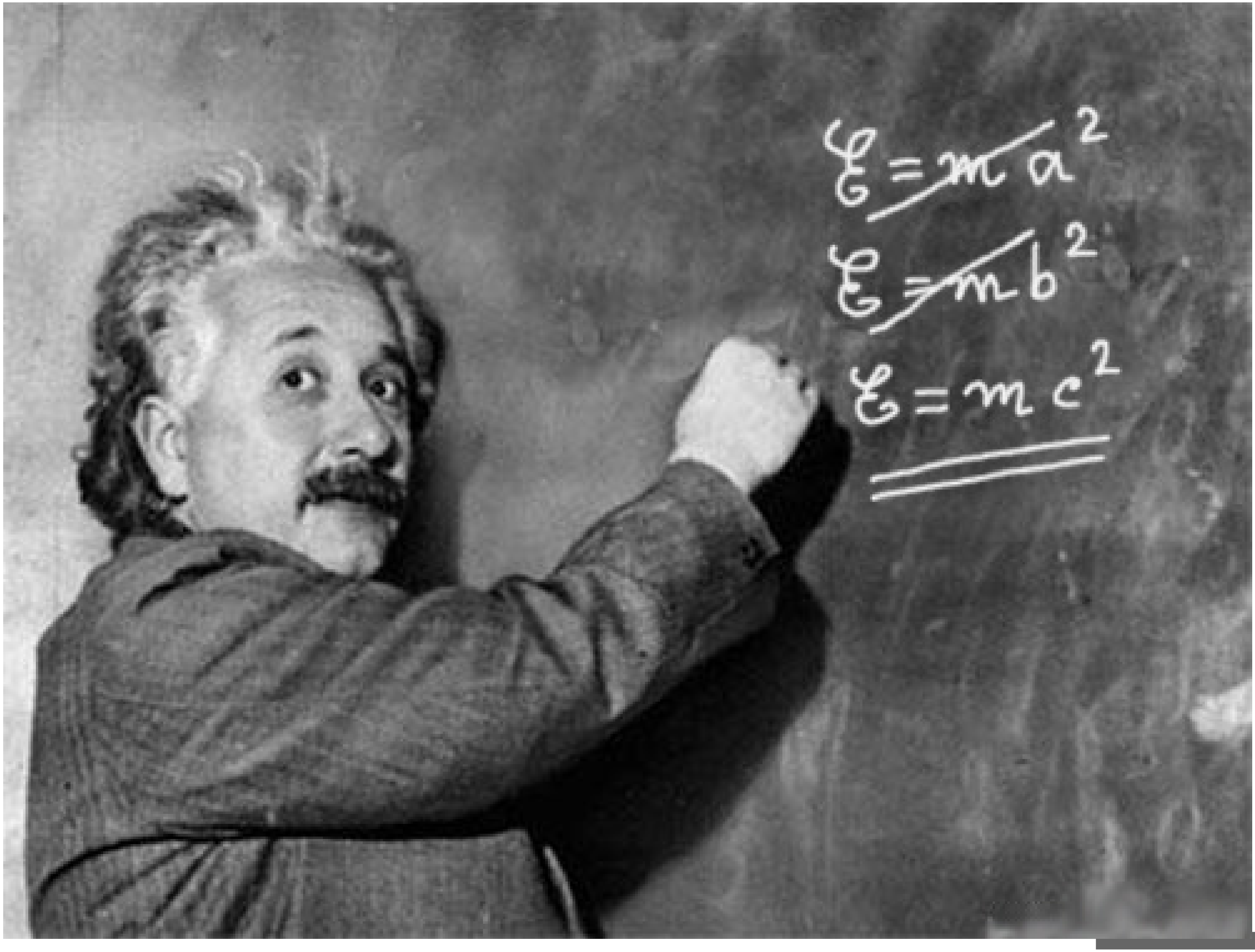
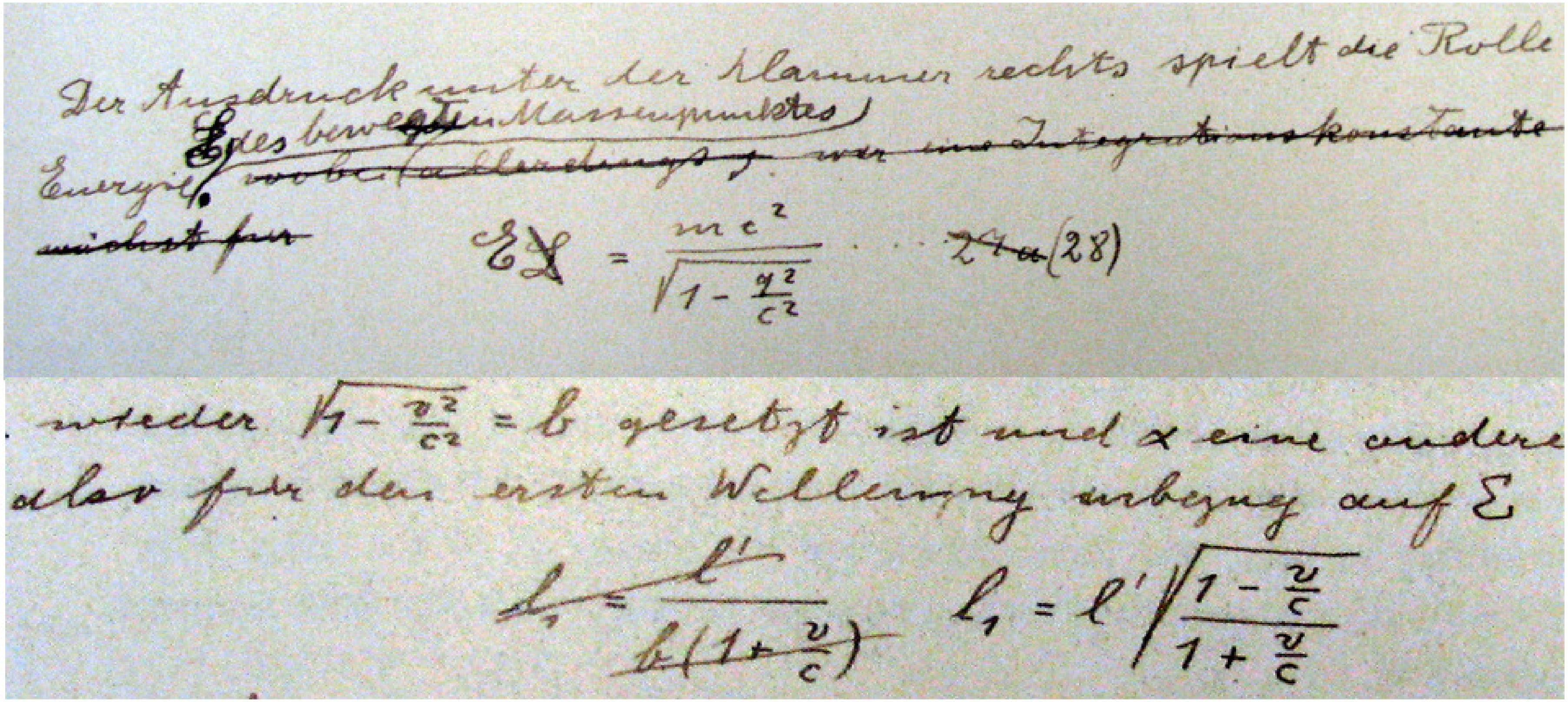


Einstein's Manuscript



Albert Einstein, 1912

On the Special Theory of Relativity (manuscript)

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  and then performing **joint** computation

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Dynamics that are **NOT** ***"natural"*** :

- **exhaustive search**
  (deterministic or stochastic)
- using a **mediator**
- **broadcasting** the private information and then performing **joint** computation
- **fully rational learning**
  (prior beliefs on the strategies of the opponents, Bayesian updating, optimization)

Dynamics

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There are no general, natural dynamics leading to Nash equilibrium

- *"natural"*:
  - adaptive
  - simple and efficient:
    - **computation** (performed at each step)
    - **time** (how long to reach equilibrium)
    - **information** (of each player)

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Natural Dynamics: Information

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Each player knows *only* his own payoff (utility) function

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Hart and Mas-Colell, AER 2003

Natural Dynamics: Information

UNCOUPLED DYNAMICS :

Each player knows *only* his own payoff (utility) function

(does *not* know the payoff functions of the other players)

(privacy-preserving, decentralized, distributed ...)

Hart and Mas-Colell, AER 2003

Games

N-person game in strategic (normal) form:

- Players

$$i = 1, 2, ..., N$$

Games

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- For each player i: **Actions**

$$a^i \text{ in } A^i$$

Games

N-person game **in strategic (normal) form**:

- **Players**

$$i = 1, 2, ..., N$$

- For each player i: **Actions**

$$a^i \text{ in } A^i$$

- For each player i: **Payoffs (utilities)**

$$u^i(a) \equiv u^i(a^1, a^2, ..., a^N)$$

Dynamics

- **Time**

$$t = 1, 2, \ldots$$

Dynamics

- **Time**

$$t = 1, 2, ...$$

- At period t each player i chooses an **action**

$$a_t^i \text{ in } A^i$$

Dynamics

- **Time**

$$t = 1, 2, \ldots$$

- At period t each player i chooses an **action**

$$a_t^i \text{ in } A^i$$

according to a probability distribution

$$\sigma_t^i \text{ in } \Delta(A^i)$$

Dynamics

Fix the set of players $1, 2, ..., N$ and their action spaces $A^1, A^2, ..., A^N$

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Uncoupled Dynamics

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$$\sigma_t^i \equiv f^i(a_{t-1}; u^i)$$

where $a_{t-1} = (a_{t-1}^1, a_{t-1}^2, ..., a_{t-1}^N) \in A$
are the actions of all the players
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  - Only last period matters (*“1-recall”*)

Uncoupled Dynamics

- Simplest **uncoupled dynamics**:

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where $a_{t-1} = (a_{t-1}^1, a_{t-1}^2, ..., a_{t-1}^N) \in A$ are the actions of all the players in the previous period

  - Only last period matters ("*1-recall*")
  - Time t does not matter ("*stationary*")

Impossibility

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Theorem. *There are* **NO** *uncoupled dynamics with 1-recall*

$$\boldsymbol{\sigma}_t^i \equiv \boldsymbol{f}^i(\boldsymbol{a}_{t-1}; \boldsymbol{u}^i)$$

that yield almost sure convergence of play to pure Nash equilibria of the stage game in all games where such equilibria exist.

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Hart and Mas-Colell, GEB 2006

Proof

Consider the following two-person game, which has a unique pure Nash equilibrium

	C1	C2	C3
R1	1,0	0,1	1,0
R2	0,1	1,0	1,0
R3	0,1	0,1	1,1

Proof

Consider the following two-person game, which has a unique pure Nash equilibrium **(R3,C3)**

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Assume ***by way of contradiction*** that we are given an uncoupled, 1-recall, stationary dynamic that yields almost sure convergence to pure Nash equilibria when these exist

Proof

- Suppose the play at time $t-1$ is **(R1,C1)**

	C1	C2	C3
R1	1,0	0,1	1,0
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Proof

- Suppose the play at time $t-1$ is **(R1,C1)**
- **ROWENA is best replying at (R1,C1)**

	C1	C2	C3
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R2	0,1	1,0	1,0
R3	0,1	0,1	1,1

Proof

- Suppose the play at time $t-1$ is **(R1,C1)**
- **ROWENA is best replying at (R1,C1)**
- $\Rightarrow$ **ROWENA will play R1 also at t**

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- Suppose the play at time $t-1$ is **(R1,C1)**
- **ROWENA is best replying at (R1,C1)**
- **$\Rightarrow$ ROWENA will play R1 also at t**

Proof:

- Change the payoff function of COLIN so that **(R1,C1)** is the unique pure Nash eq.

	C1	C2	C3
R1	1,0	0,1	1,0
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  - In the new game, ROWENA ***must*** play **R1** after **(R1,C1)** (by *1-recall*, *stationarity*, and *a.s. convergence* to the pure Nash eq.)

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  - By *uncoupledness*, the same holds in the original game

Proof

- Suppose the play at time $t-1$ is **(R1,C1)**
- **ROWENA is best replying at (R1,C1)**
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Proof

- ROWENA is best replying at $t-1$
- $\Rightarrow$ ROWENA will play the same action at t

Proof

Similarly for COLIN:

A player who is best replying cannot switch

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	C1	C2	C3
R1	1,0 $\longleftrightarrow$	0,1	1,0
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Similarly for COLIN:

A player who is best replying cannot switch

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R1	1,0 ↔	0,1 ↕	1,0 ↔
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$\Rightarrow$ (R3,C3) **cannot be reached**

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	C1	C2	C3
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$\Rightarrow$ (R3,C3) **cannot be reached**
(unless we start there)

2-Recall

2-Recall: Possibility

Theorem. THERE EXIST *uncoupled dynamics with* 2-RECALL

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THEN: At t player i *plays a^i again*: $a^i_t = a^i$

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THEN: At t player i *plays a^i again*: $a^i_t = a^i$

ELSE: At t player i *randomizes uniformly over A^i*

Possibility

"Good":

Possibility

"Good":

- simple

Possibility

"Good":

- simple

"Bad":

Possibility

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"Bad":

- exhaustive search

Possibility

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- *"natural"*:
  - adaptive
  - simple and efficient:
    - computation
    - time
    - information

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  - **simple and efficient**:
    - **computation**: *finite recall* ✓
    - **time** to reach equilibrium ?
    - **information**: *uncoupledness* ✓

Natural Dynamics: Time

HOW LONG TO EQUILIBRIUM?

Natural Dynamics: Time

HOW LONG TO EQUILIBRIUM?

Estimate the number of time periods it takes until a Nash equilibrium is reached

Natural Dynamics: Time

HOW LONG TO EQUILIBRIUM?

Estimate the number of time periods it takes until a Nash equilibrium is reached

- How?

Natural Dynamics: Time

HOW LONG TO EQUILIBRIUM?

Estimate the number of time periods it takes until a Nash equilibrium is reached

- How?
- An uncoupled dynamic

  $$\approx$$

  A distributed computational procedure

Natural Dynamics: Time

HOW LONG TO EQUILIBRIUM?

Estimate the number of time periods it takes until a Nash equilibrium is reached

- How?
- An uncoupled dynamic

  $\approx$

  A distributed computational procedure
- $\Rightarrow$ **COMMUNICATION COMPLEXITY**

Communication Complexity

Communication Complexity

- ***Distributed computational procedure***

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  - **START**: Each participant has some private information

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---

Yao 1979, Kushilevitz and Nisan 1997

How Long to Equilibrium

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- ***Uncoupled dynamic leading to Nash equilibria***

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---

Conitzer and Sandholm 2004

How Long to Equilibrium

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How Long to Equilibrium

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How Long to Equilibrium

- An uncoupled dynamic leading to Nash equilibria is **TIME-EFFICIENT** if the **TIME IT TAKES** is **POLYNOMIAL** in the number of players

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Hart and Mansour, GEB 2010

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Theorem. *There are* **NO TIME-EFFICIENT** *uncoupled dynamics that reach a pure Nash equilibrium in all games where such equilibria exist.*

In fact: exponential, like exhaustive search

Hart and Mansour, GEB 2010

How Long to Equilibrium

- **Intuition:**

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  - different games have different equilibria

How Long to Equilibrium

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  - the dynamic procedure must distinguish between them

How Long to Equilibrium

- **Intuition:**
  - different games have different equilibria
  - the dynamic procedure must distinguish between them
  - no single player can do so by himself

Dynamics and Nash Equilibrium

Dynamics and Nash Equilibrium

FACT

There are* NO *general, natural dynamics leading to Nash equilibrium

Dynamics and Nash Equilibrium

FACT

There are* NO *general, natural dynamics leading to Nash equilibrium

RESULT

There* CANNOT BE *general, natural dynamics leading to Nash equilibrium

Dynamics and Nash Equilibrium

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Dynamics and Nash Equilibrium

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- **Perhaps we are asking too much?**

Dynamics and Nash Equilibrium

RESULT

There CANNOT BE general, natural dynamics leading to Nash equilibrium

- **Perhaps we are asking too much?**
- For instance, the size of the data (the payoff functions) is *exponential* rather than polynomial in the number of players

Correlated Equilibrium

Correlated Equilibrium

CORRELATED EQUILIBRIUM

Aumann, JME 1974

Correlated Equilibrium

CORRELATED EQUILIBRIUM :

Nash equilibrium when players receive payoff-irrelevant information before playing the game

Aumann, JME 1974

Correlated Equilibrium

A **Correlated Equilibrium** is a Nash equilibrium when players receive payoff-irrelevant signals before playing the game

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- **Examples**:

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- **Examples**:
  - Independent signals

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  - Independent signals $\Leftrightarrow$ Nash equilibrium

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  - Public signals ("sunspots")

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Correlated Equilibria

"Chicken" game

	LEAVE	STAY
LEAVE	$5, 5$	$3, 6$
STAY	$6, 3$	$0, 0$

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another **Nash equilibrium**

Correlated Equilibria

"Chicken" game

	LEAVE	STAY
LEAVE	$5, 5$	$3, 6$
STAY	$6, 3$	$0, 0$

0	$1/2$
$1/2$	0

a **(publicly) correlated equilibrium**

Correlated Equilibria

"Chicken" game

	LEAVE	STAY
LEAVE	5, 5	3, 6
STAY	6, 3	0, 0

	L	S
L	$1/3$	$1/3$
S	$1/3$	0

another **correlated equilibrium**

- after signal L play LEAVE
- after signal S play STAY

Correlated Equilibrium

A **Correlated Equilibrium** is a Nash equilibrium when the players receive payoff-irrelevant signals before playing the game (Aumann 1974)

- **Examples**:
  - Independent signals $\Leftrightarrow$ Nash equilibrium
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  - Boston Celtics' front line

Correlated Equilibrium

- Signals (public, correlated) are **unavoidable**

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A joint distribution z is a **correlated equilibrium**

$$\Leftrightarrow$$

$$\sum_{s^{-i}} u(j, s^{-i}) z(j, s^{-i}) \geq \sum_{s^{-i}} u(k, s^{-i}) z(j, s^{-i})$$

for all $i \in N$ and all $j, k \in S^i$

Dynamics & Correlated Equilibria

Dynamics & Correlated Equilibria

RESULT

THERE EXIST ***general, natural dynamics***

leading to **CORRELATED EQUILIBRIA**

Dynamics & Correlated Equilibria

RESULT

THERE EXIST ***general, natural dynamics***
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- ***Regret Matching***

Hart and Mas-Colell, Ec'ca 2000

Dynamics & Correlated Equilibria

<u>RESULT</u>

THERE EXIST *general, natural dynamics*
***leading to* CORRELATED EQUILIBRIA**

- ***Regret Matching***
- General regret-based dynamics

Hart and Mas-Colell, Ec'ca 2000, JET 2001

Regret Matching

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- "REGRET": the increase in past payoff, if any, if a different action would have been used

Regret Matching

- "REGRET": the increase in past payoff, if any, if a different action would have been used

- "MATCHING": switching to a different action with a probability that is proportional to the regret for that action

Dynamics & Correlated Equilibria

THERE EXIST ***general, natural dynamics***
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Dynamics & Correlated Equilibria

THERE EXIST ***general*****, *natural dynamics***
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- ***"general"***: in all games

Dynamics & Correlated Equilibria

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Dynamics & Correlated Equilibria

THERE EXIST *general, natural dynamics*
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- ***"general"***: in all games
- ***"natural"***:
  - **adaptive** (also: close to "behavioral")

Dynamics & Correlated Equilibria

THERE EXIST ***general***, ***natural*** ***dynamics***
leading to CORRELATED EQUILIBRIA

- ***"general"***: in all games
- ***"natural"***:
  - **adaptive** (also: close to "behavioral")
  - **simple and efficient**:
    computation, time, information

Dynamics & Correlated Equilibria

THERE EXIST ***general, natural dynamics***
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- ***"general"***: in all games
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  - **adaptive** (also: close to "behavioral")
  - **simple and efficient**:
    computation, time, information
- ***"leading to correlated equilibria"***:
  statistics of play become close to
  CORRELATED EQUILIBRIA

Regret Matching and Beyond

Regret Matching and Beyond

www.sciencemag.org SCIENCE VOL 304 21 MAY 2004 1167

The Involvement of the Orbitofrontal Cortex in the Experience of Regret

Nathalie Camille,[1]* Giorgio Coricelli,[1,2]* Jerome Sallet,[1] Pascale Pradat-Diehl,[3] Jean-René Duhamel,[1] Angela Sirigu[1]†

Facing the consequence of a decision we made can trigger emotions like satisfaction, relief, or regret, which reflect our assessment of what was gained as compared to what would have been gained by making a different decision. These emotions are mediated by a cognitive process known as counterfactual thinking. By manipulating a simple gambling task, we characterized a subject's choices in terms of their anticipated and actual emotional impact. Normal subjects reported emotional responses consistent with counterfactual thinking; they chose to minimize future regret and learned from their emotional experience. Patients with orbitofrontal cortical lesions, however, did not report regret or anticipate negative consequences of their choices. The orbitofrontal cortex has a fundamental role in mediating the experience of regret.

Review TRENDS in Cognitive Sciences Vol.11 No.6

ELSEVIER

Full text provided by www.sciencedirect.com
ScienceDirect

Brain, emotion and decision making: the paradigmatic example of regret

Giorgio Coricelli[1], Raymond J. Dolan[2] and Angela Sirigu[1]

[1] Neuropsychology Group, Institut des Sciences Cognitives, CNRS, 67 Boulevard Pinel, 69675 Bron, France
[2] Wellcome Department of Imaging Neuroscience, 12 Queen Square, London, WC1N 3BG, UK

Human decisions cannot be explained solely by rational imperatives but are strongly influenced by emotion. Theoretical and behavioral studies provide a sound empirical basis to the impact of the emotion of regret in guiding choice behavior. Recent neuropsychological and neuroimaging data have stressed the fundamental role of the orbitofrontal cortex in mediating the experience of regret. Functional magnetic resonance imaging data indicate that reactivation of activity within the orbitofrontal cortex and amygdala occurring during the phase of choice, when the brain is anticipating possible future consequences of decisions, characterizes the anticipation of regret. In turn, these patterns reflect learning based on cumulative emotional experience. Moreover, affective consequences can induce specific mechanisms of cognitive control of the choice processes, involving reinforcement or avoidance of the experienced behavior.

change. People, including those with a deep knowledge of optimal strategies, such as Markowitz, often try to avoid the likelihood of future regret, even when this conflicts with the prescription of decisions based on rational choice; according to the latter, individuals faced with a decision between multiple alternatives under uncertainty will opt for the course of action with maximum expected utility, a function of both the probability and the magnitude of the expected payoff [4].

Here, we outline, for the first time, the neural basis of the emotion of regret, and its fundamental role in adaptive behavior. The following questions will be addressed: what are the neural underpinnings of 'powerful' cognitively generated emotions such as regret? What are the theoretical implications of incorporating regret into the process of choice, and into adaptive models of decision making? In line with recent work on emotion-based decision making [5,6], we attempt to characterize the brain areas underlying

Regret Matching and Beyond

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 57, NO. 2, FEBRUARY 2009 459

Decentralized Dynamic Spectrum Access for Cognitive Radios: Cooperative Design of a Non-Cooperative Game

Michael Maskery, Vikram Krishnamurthy, and Qing Zhao

***Abstract*—We consider dynamic spectrum access among cognitive radios from an adaptive, game theoretic learning perspective. Spectrum-agile cognitive radios compete for channels temporarily vacated by licensed primary users in order to satisfy their own demands while minimizing interference. For both slowly varying primary user activity and slowly varying statistics of "fast" primary user activity, we apply an adaptive regret based learning procedure which tracks the set of correlated equilibria of the game, treated as a distributed stochastic approximation. This procedure is shown to perform very well compared with other similar adaptive algorithms. We also estimate channel contention for a simple CSMA channel sharing scheme.**

***Index Terms*—Cognitive radio, dynamic spectrum access, game theory, stochastic approximation, correlated equilibrium.**

(channels) that are temporarily unoccupied by licensed users. Each radio dynamically selects several available channels so as to balance its own demand (competition) against system-imposed sharing incentives (cooperation). Selections are made independently by each radio, based only on its own performance history. We focus on applications where primary users' spectrum access activities either vary slowly with time (see [3], [4]), or where their spectrum access activities vary quickly, but average behaviour varies slowly. Example applications include the reuse of certain TC-bands that are not used for TC broadcast in a particular region.

Since optimal resource allocation in a decentralized, competitive environment is not straightforward, we propose to operate radios according to a game theoretic algorithm which

Regret Matching and Beyond

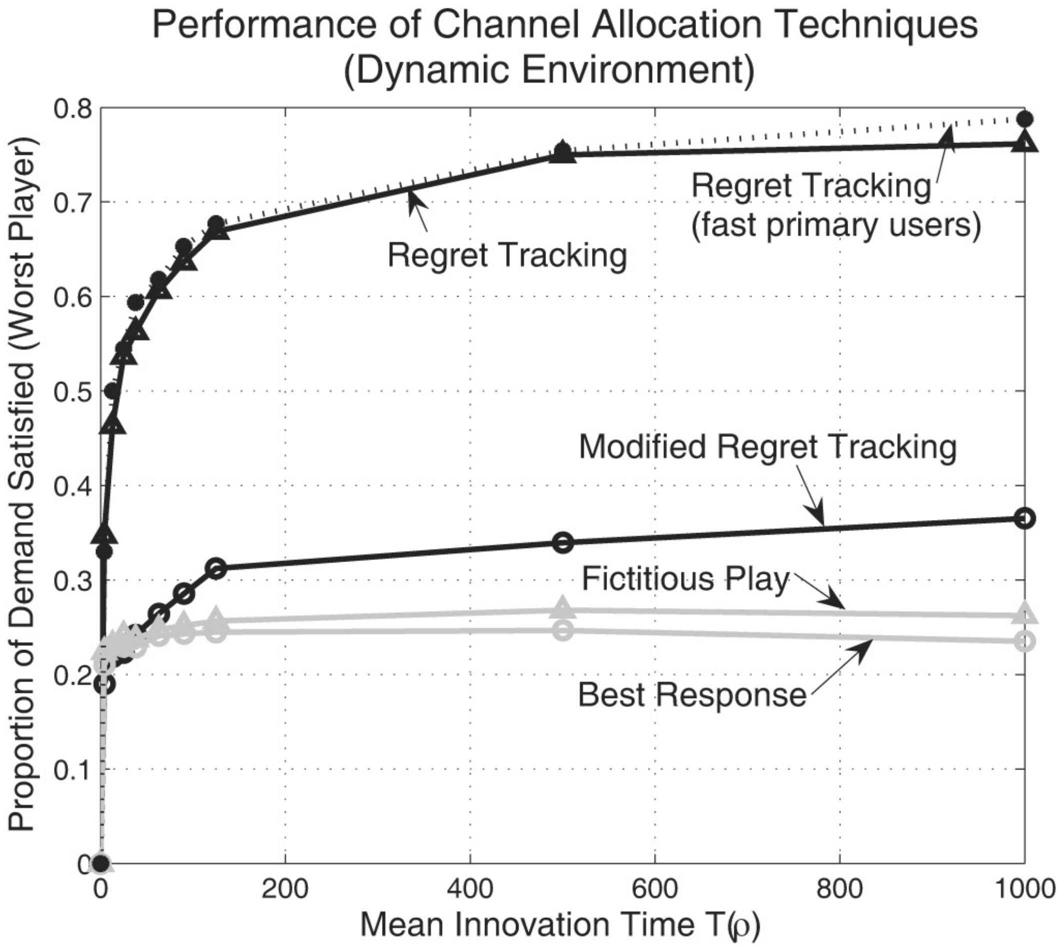

Fig. 6. Long-run average spectrum utilization in a dynamic environment for the channel allocation techniques of Section V. $T(\rho)$ (see Sec.VI-C) is the mean time between innovations in the system (changes in primary user activity, fast primary user statistics or cognitive radio demands).

Regret Matching and Beyond

17th IEEE International Conference on Control Applications
Part of 2008 IEEE Multi-conference on Systems and Control
San Antonio, Texas, USA, September 3-5, 2008

FrA04.3

Fully non-cooperative optimal placement of mobile vehicles

Shemin Kalam, Mahbub Gani and Lakmal Seneviratne

***Abstract*— In this paper, we consider optimal placement of autonomous mobile vehicles such that a cost function involving all the vehicles and possible locations of targets is minimized. This cost is proportional to the distance between the targets and vehicles. The optimal locations correspond to the vehicles being at the centroids of their own Voronoi cell which correspond to Centroidal Voronoi Tessellations (CVTs). We have adopted a game theoretical formulation to initially consider vehicle target assignment where a set of mobile vehicles choose their own targets. The movement of the vehicles towards the optimal locations is based on MacQueen's algorithm. But an important step of MacQueen's algorithm requires the knowledge of the nearest neighbour to be determined from a sample that is drawn from a fixed but unknown probability distribution. This calculation seems to be implicit in reported algorithms and brings in a hidden centralized process. We have used game theory as a framework to get around this problem and modelled the vehicles such that they are capable of making their own decisions and interested in optimizing their own utilities. Specifically, we have introduced an appropriate utility function and require the vehicles to negotiate their choice of targets via regret matching. We present simulations that illustrate that vehicles choose the targets optimally and converge to CVTs.**

Lloyd's descent algorithm [15] can be applied to solve the problem and it has been shown by Cortés et al [12] that the distribution of mobile sensors converges to Centroidal Voronoi Tessellations (CVTs). Considering a scenario where the spatial distribution is not known, MacQueen's algorithm [16] is a Monte-Carlo method of solving the problem [22]; in our problem we interpret MacQueen's algorithm as a real-time higher order control strategy. A drawback of the MacQueen's algorithm is that it requires the calculation of nearest neighbours. We have drawn upon game theory to get around the problem of this requirement. This reduces the communication burden on the system because the vehicles do not require the information about the distances between all the vehicles and targets which would have been needed for calculation of the nearest neighbour.

The game theoretic strategy that we adopt is inspired by the problem of autonomous vehicle-target assignment tackled by Arslan in [8]. To get to the problem of optimal placement of the mobile vehicles, initially we consider how a group of vehicle are to optimally assign themselves to a set of targets.

Regret Matching and Beyond

IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS VOL. 43, NO. 3 JULY 2007 843

Network-Enabled Missile Deflection: Games and Correlation Equilibrium

MICHAEL MASKERY
VIKRAM KRISHNAMURTHY
University of British Columbia

The problem of deploying countermeasures (CM) against antiship missiles is investigated from a network centric perspective in which multiple ships coordinate to defend against a known missile threat. Using the paradigm of network enabled operations (NEOPS), the problem is formulated as a transient stochastic game with communication where the appropriate strategy takes the form of an optimal stationary correlated equilibrium. Under this strategy, ships cooperate through real-time communication to satisfy both local and collective interests. The use of communication results in a performance improvement over the noncommunicating, Nash equilibrium scenario. This framework allows us to develop a theoretical foundation for NEOPS and captures the trade-off between information exchange and performance, while generalizing the standard Nash equilibrium solution for the missile deflection game given in [1]. The NEOPS equilibrium strategy is characterized as the solution to an optimization problem with linear objective and bilinear constraints, which can be solved calculating successive improvements starting from an initial noncooperative (Nash) solution. The communication overhead required to implement this strategy is associated with the mutual information between individual action probability distributions at equilibrium. Numerical results illustrate the trade-off between communication and performance.

Regret Matching and Beyond

Predicting Human Interactive Learning by Regret-Driven Neural Networks

Davide Marchiori[1] and Massimo Warglien[2]*

Much of human learning in a social context has an interactive nature: What an individual learns is affected by what other individuals are learning at the same time. Games represent a widely accepted paradigm for representing interactive decision-making. We explored the potential value of neural networks for modeling and predicting human interactive learning in repeated games. We found that even very simple learning networks, driven by regret-based feedback, accurately predict observed human behavior in different experiments on 21 games with unique equilibria in mixed strategies. Introducing regret in the feedback dramatically improved the performance of the neural network. We show that regret-based models provide better predictions of learning than established economic models.

www.sciencemag.org SCIENCE VOL 319 22 FEBRUARY 2008 1111

Regret Matching and Beyond

Computers in Biology and Medicine 39 (2009) 460 – 473

ELSEVIER

Contents lists available at ScienceDirect

Computers in Biology and Medicine

journal homepage: www.elsevier.com/locate/cbm

Classification of peptide mass fingerprint data by novel no-regret boosting method

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ARTICLE INFO

Article history:
Received 12 February 2007
Accepted 5 March 2009

Keywords:
Mass spectrometry
Peptidomics
FTICR
LC-MS
Boosting
Classifier
Cystic fibrosis
Ovarian cancer

ABSTRACT

We have developed an integrated tool for statistical analysis of large-scale LC-MS profiles of complex protein mixtures comprising a set of procedures for data processing, selection of biomarkers used in early diagnostic and classification of patients based on their peptide mass fingerprints.
Here, a novel boosting technique is proposed, which is embedded in our framework for MS data analysis. Our boosting scheme is based on Hannan-consistent game playing strategies. We analyze boosting from a game-theoretic perspective and define a new class of boosting algorithms called H-boosting methods.
In the experimental part of this work we apply the new classifier together with classical and state-of-the-art algorithms to classify ovarian cancer and cystic fibrosis patients based on peptide mass spectra.
The methods developed here provide automatic, general, and efficient means for processing of large scale LC-MS datasets. Good classification results suggest that our approach is able to uncover valuable information to support medical diagnosis.

Dynamics and Equilibrium

Dynamics and Equilibrium

- **NASH EQUILIBRIUM**:
  a ***fixed-point*** of a non-linear map

Dynamics and Equilibrium

- **NASH EQUILIBRIUM**:
  a ***fixed-point*** of a non-linear map
- **CORRELATED EQUILIBRIUM**:
  a solution of finitely many ***linear inequalities***

Dynamics and Equilibrium

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- **CORRELATED EQUILIBRIUM**:
  a solution of finitely many ***linear inequalities***
- ***set-valued*** fixed-point (curb sets)?

Dynamics and Equilibrium

Dynamics and Equilibrium

"LAW OF CONSERVATION OF COORDINATION":

Dynamics and Equilibrium

"<u>**LAW OF CONSERVATION OF COORDINATION**</u>":

***There must be some* COORDINATION —**

Dynamics and Equilibrium

"LAW OF CONSERVATION OF COORDINATION":

There must be some* COORDINATION *—

either in the* EQUILIBRIUM *notion,

Dynamics and Equilibrium

"<u>LAW OF CONSERVATION OF COORDINATION</u>":

There must be some **COORDINATION** ***—***

either in the **EQUILIBRIUM** ***notion,***

or in the **DYNAMIC**

The "Program"

The "Program"

A. **Demarcate** **the** BORDER **between**

The "Program"

A. **Demarcate the BORDER between**

- **classes of dynamics where convergence to equilibria CAN be obtained**

The "Program"

A. **Demarcate the BORDER between**

- **classes of dynamics where convergence to equilibria CAN be obtained, and**
- **classes of dynamics where convergence to equilibria CANNOT be obtained**

The "Program"

A. **Demarcate the BORDER between**

- **classes of dynamics where convergence to equilibria CAN be obtained, and**
- **classes of dynamics where convergence to equilibria CANNOT be obtained**

B. **Find NATURAL dynamics for the various equilibrium concepts**

Dynamics and Equilibrium

Nash Equilibrium

Dynamics and Equilibrium

Nash Equilibrium

"DYNAMICALLY DIFFICULT"

Dynamics and Equilibrium

Nash Equilibrium

"DYNAMICALLY DIFFICULT"

Correlated Equilibrium

Dynamics and Equilibrium

Nash Equilibrium

"DYNAMICALLY DIFFICULT"

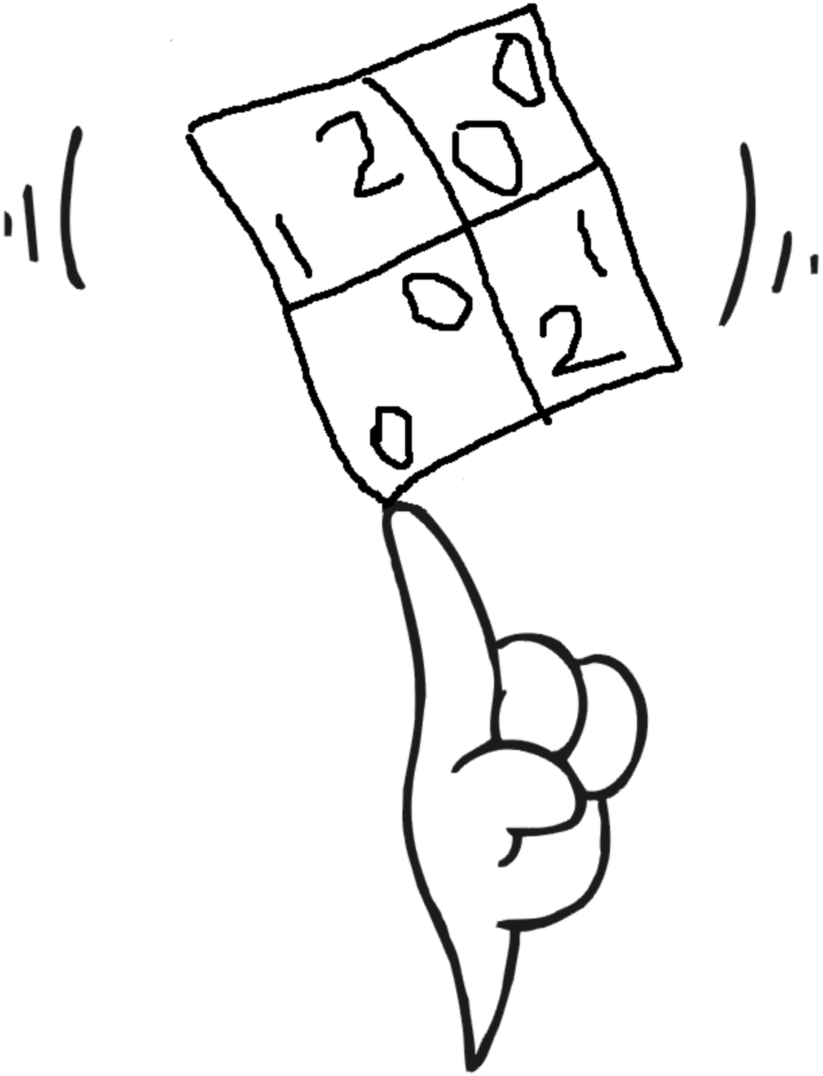
Correlated Equilibrium

"DYNAMICALLY EASY"

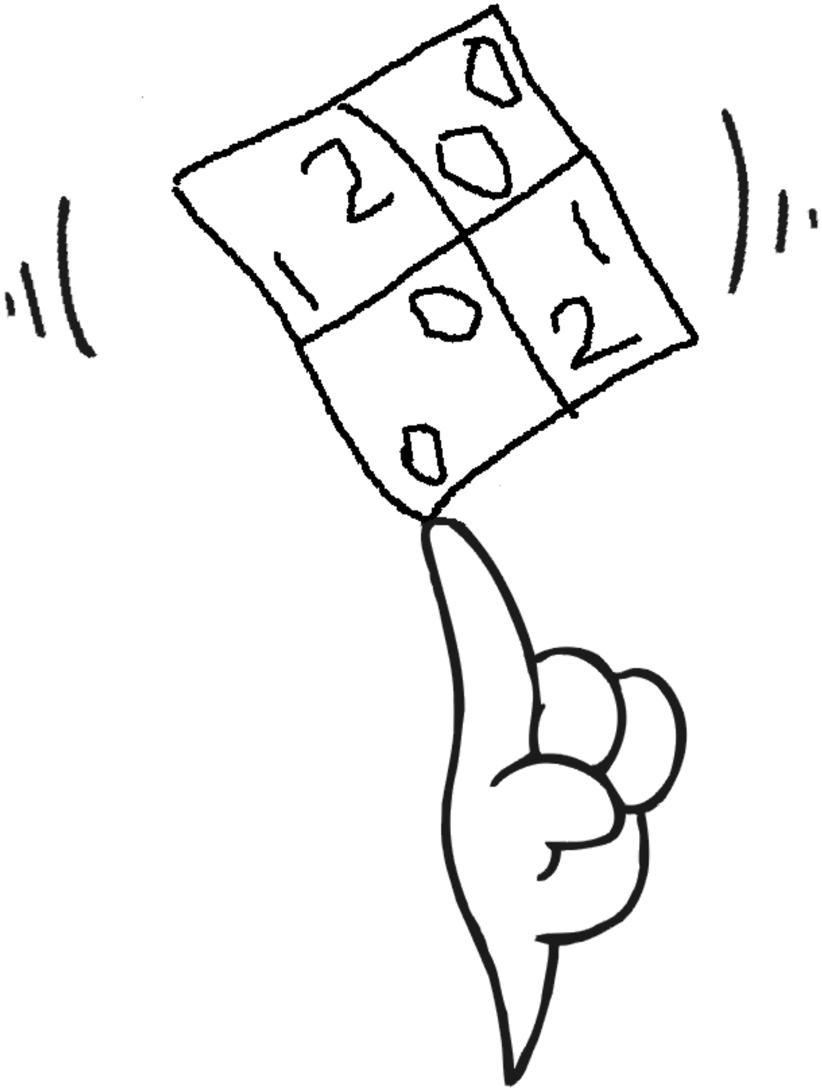
Dynamics and Equilibrium

Dynamics and Equilibrium

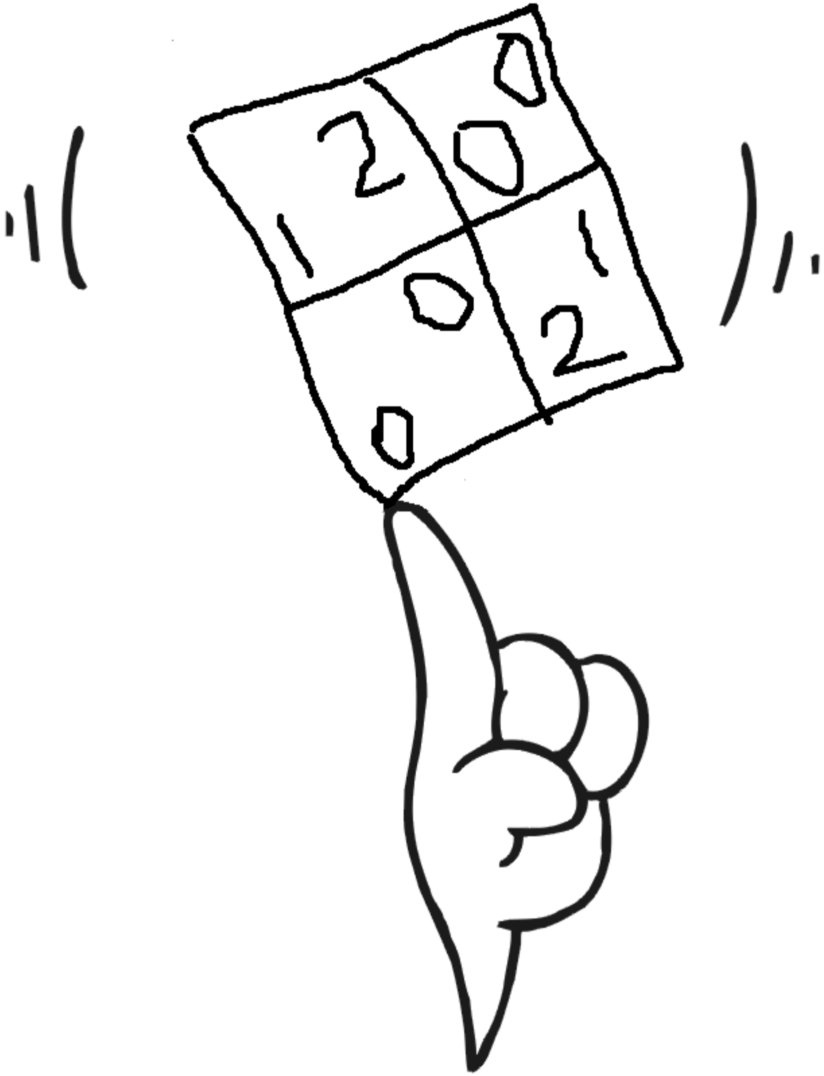
Dynamics and Equilibrium



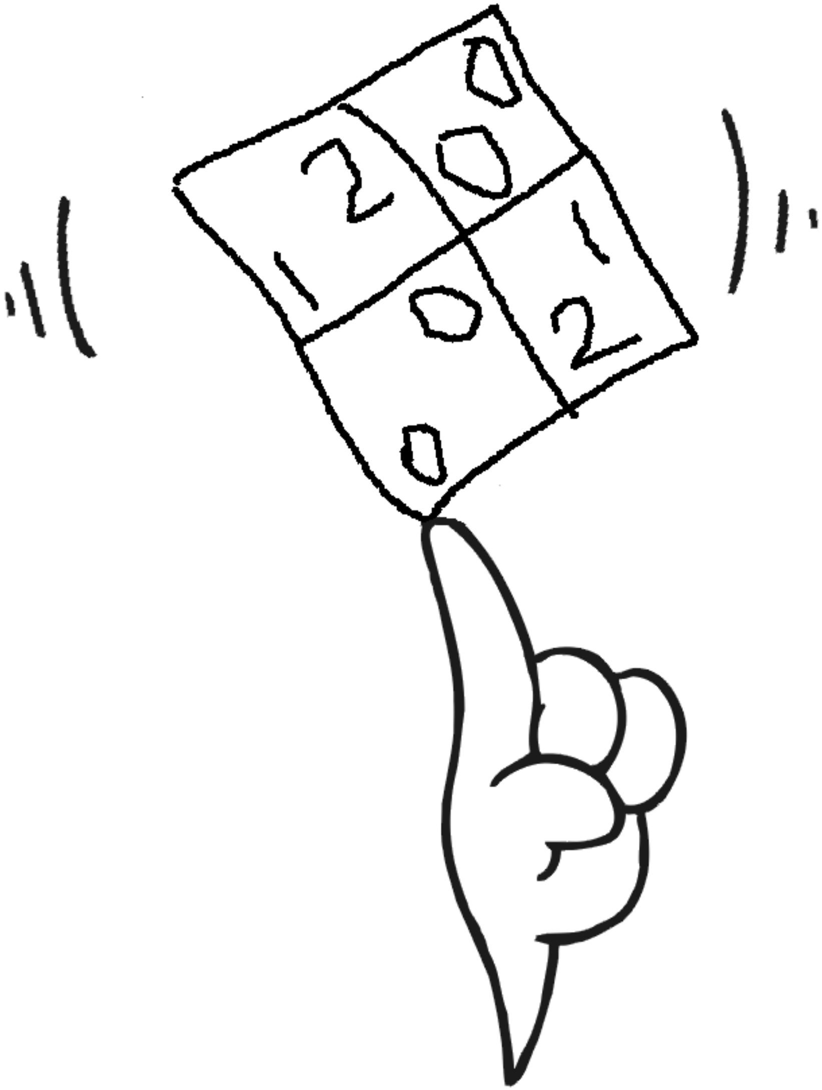
Dynamics and Equilibrium



Dynamics and Equilibrium



Dynamics and Equilibrium



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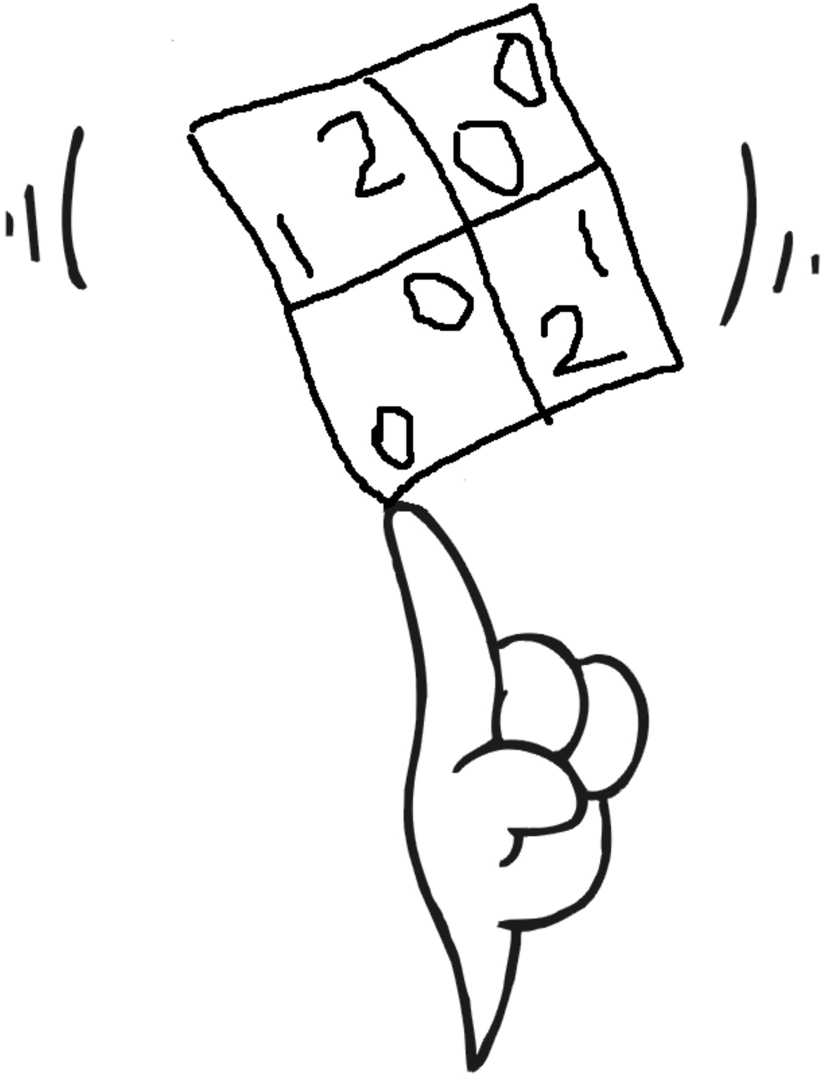

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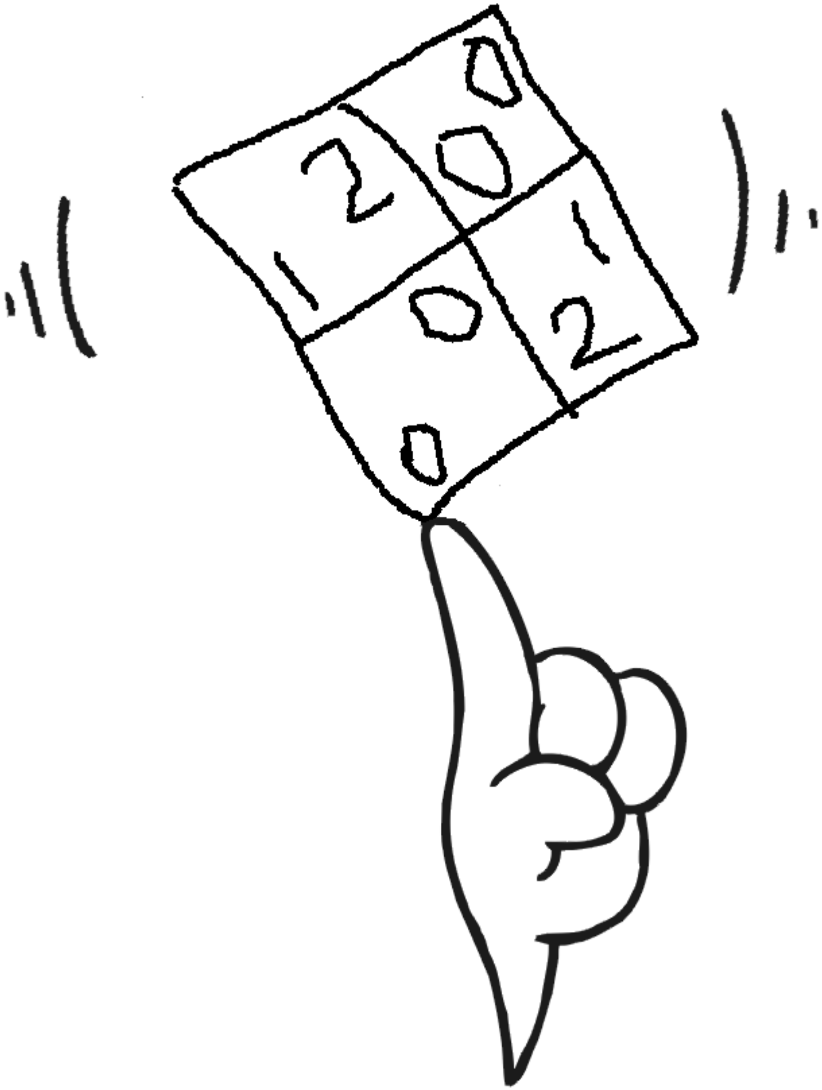

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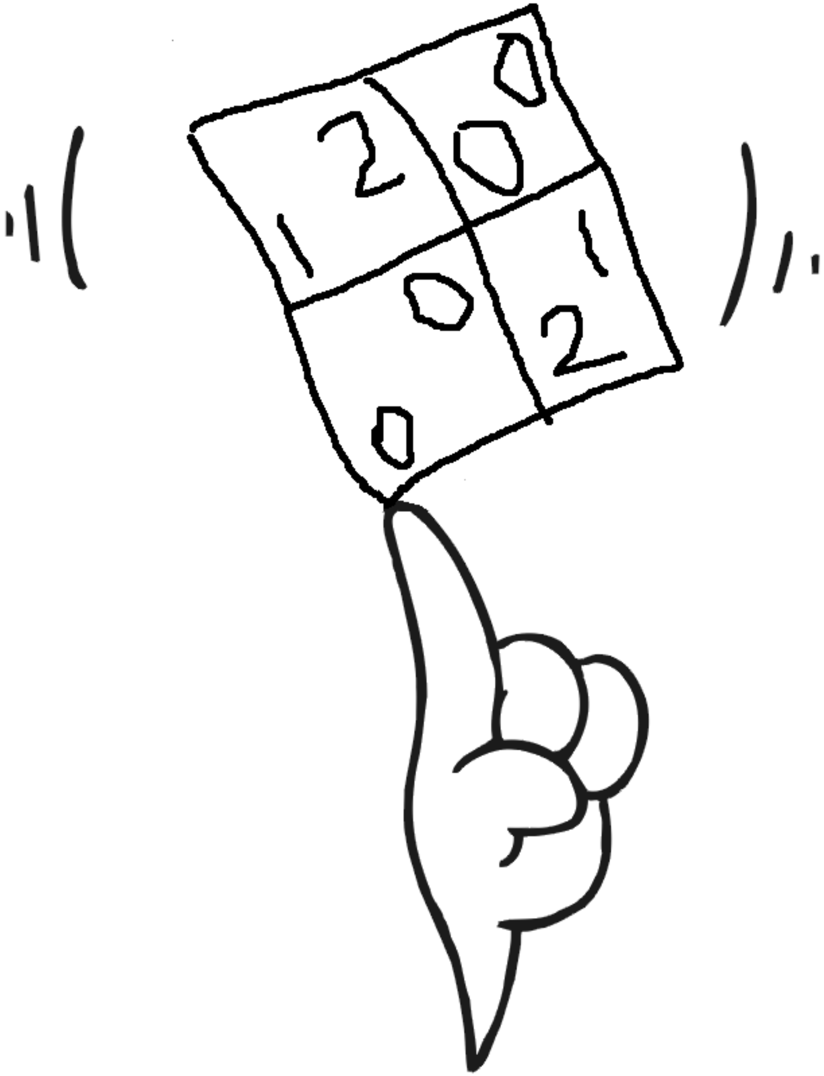

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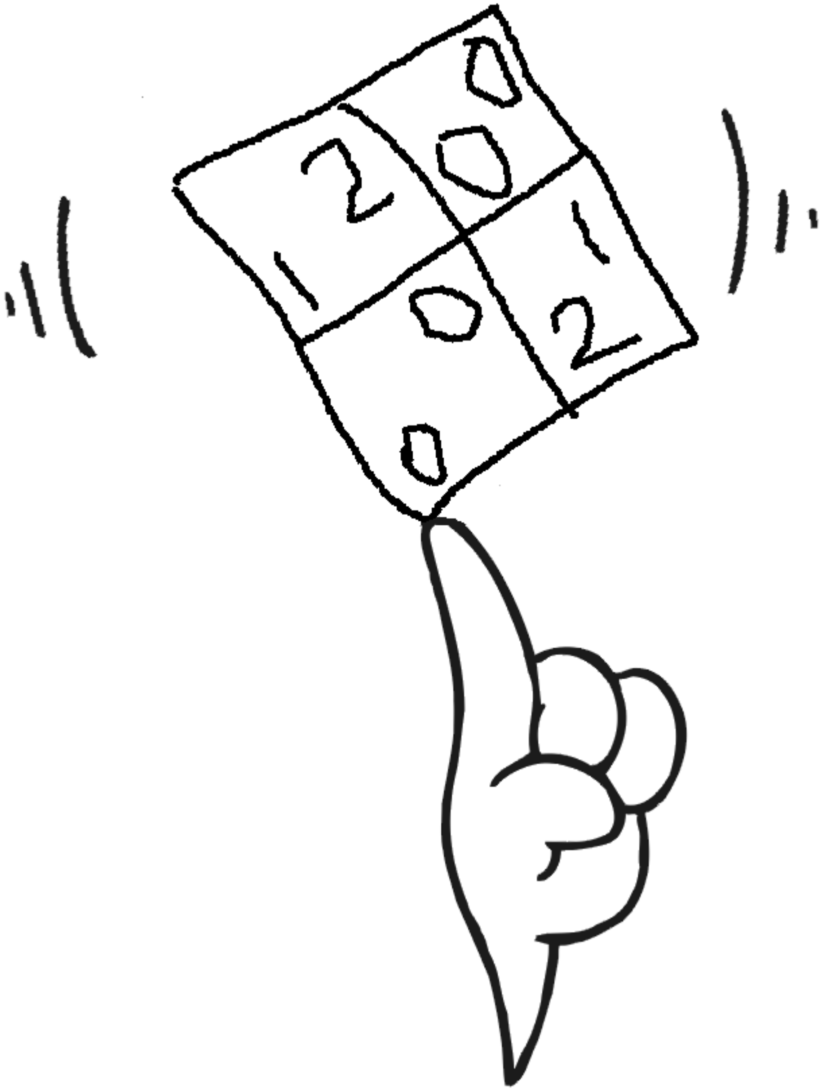

My Game Theory

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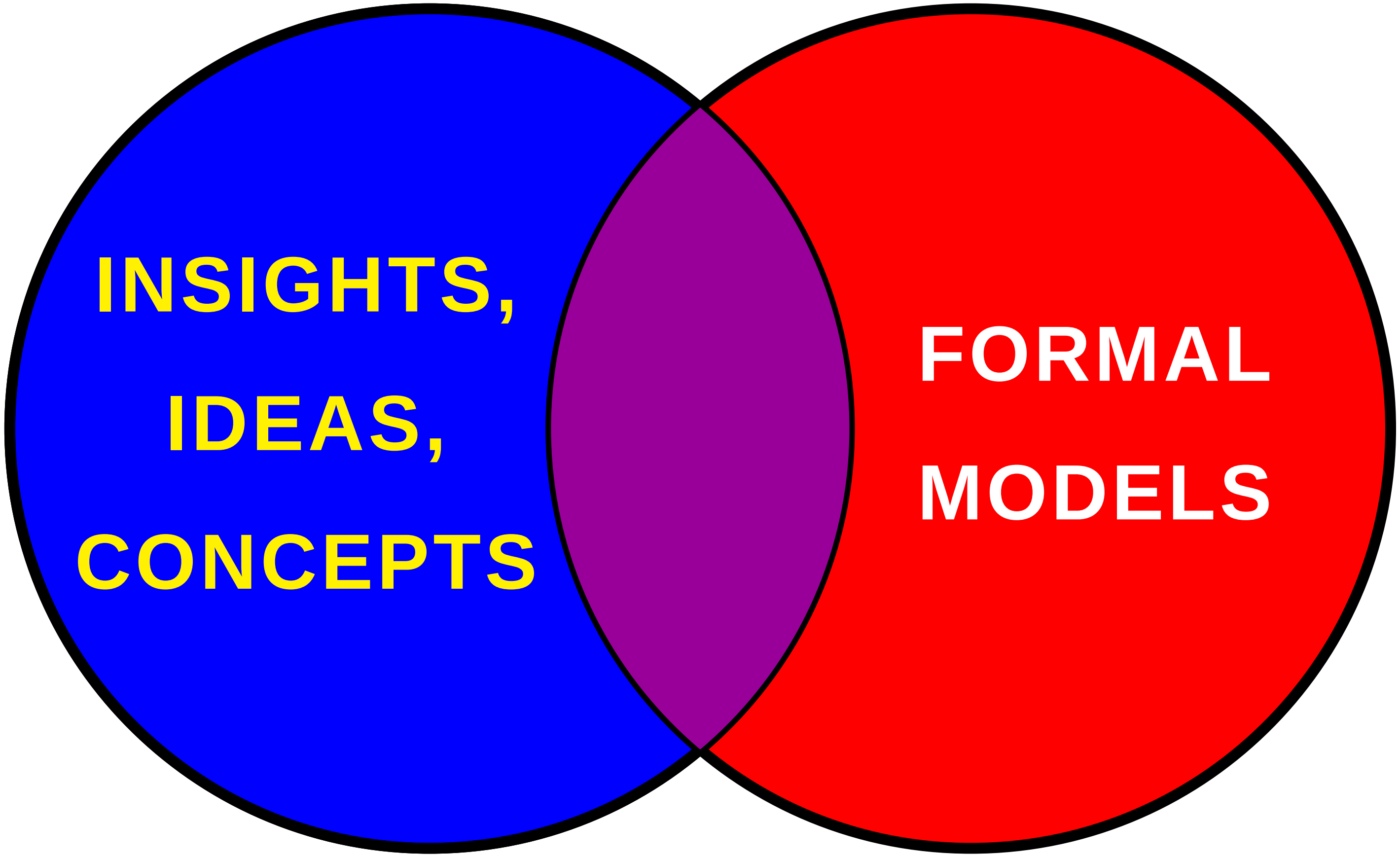

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INSIGHTS,

IDEAS,

CONCEPTS

FORMAL

MODELS

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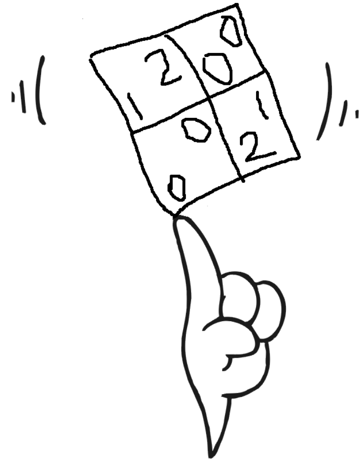

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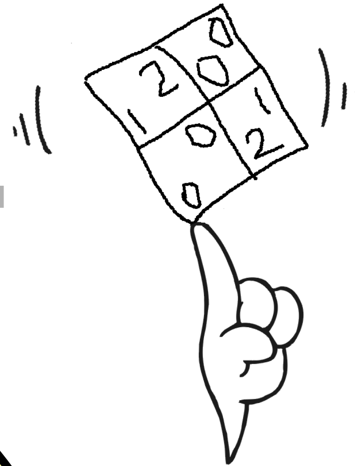

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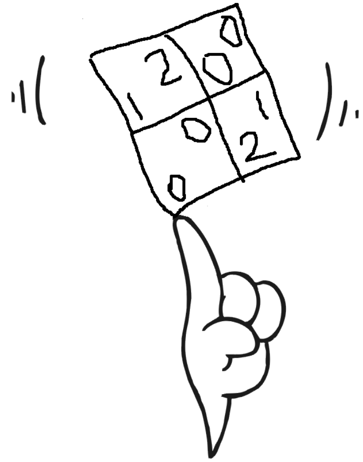

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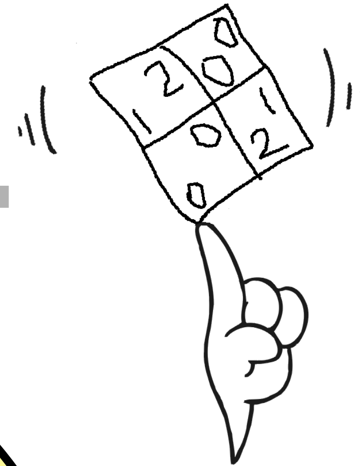

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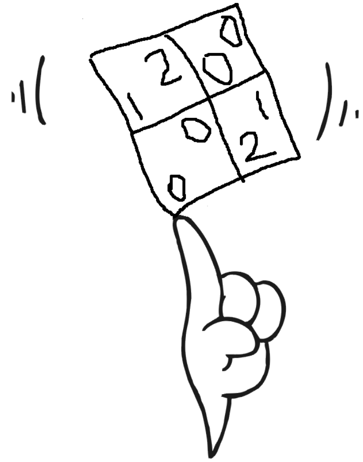

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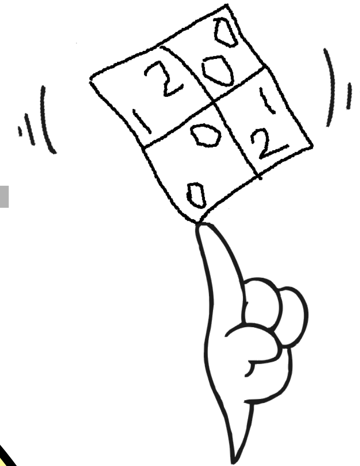

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