BCN
Stability and Fairness

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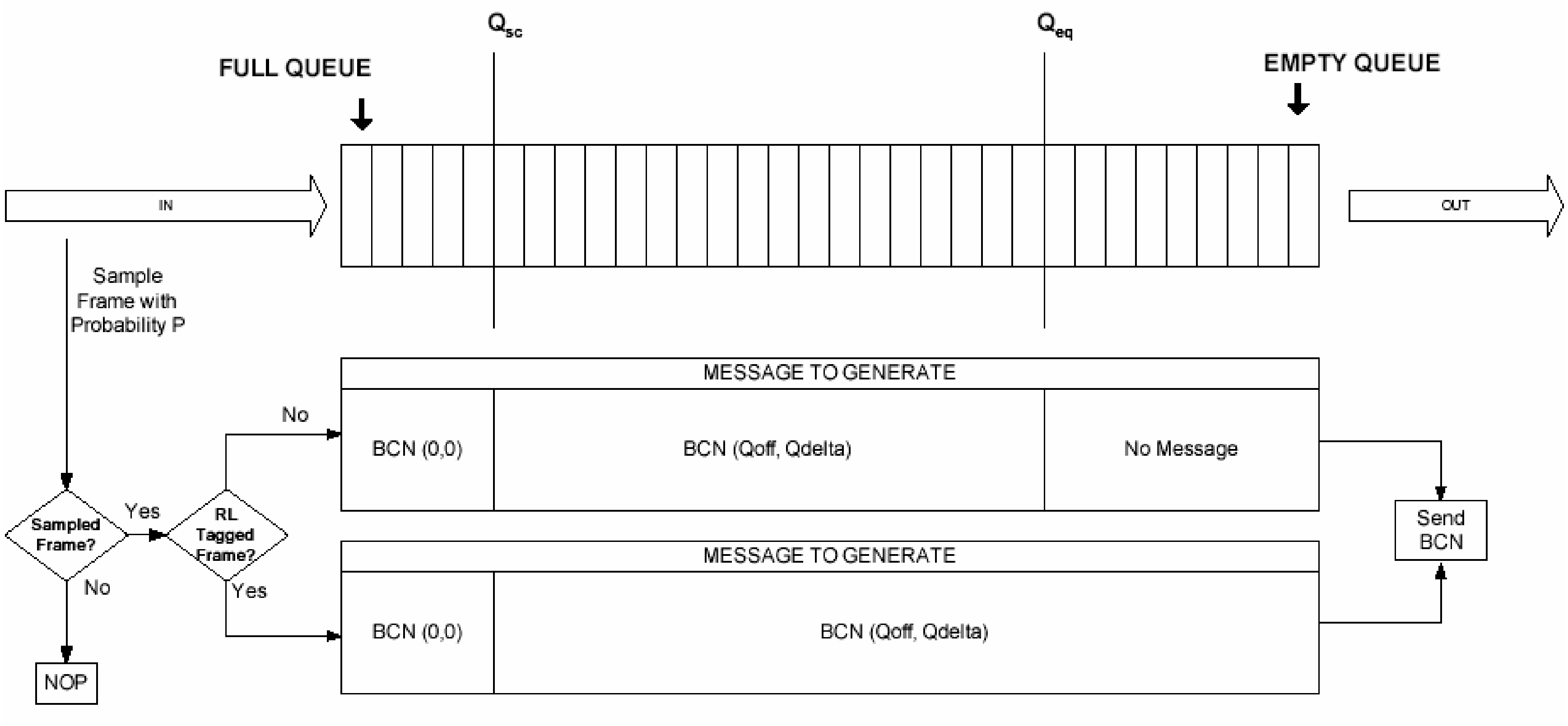
Outline

1. Stability analysis
   - Explicit parameterization of stability region
   - Sufficient condition for overall stability

2. Self-increase
   - Stability
   - Fairness (?)
   - Flow completion time

BCN Signals



Q_{sc}
Q_{eq}
FULL QUEUE
EMPTY QUEUE
IN
OUT
Sample Frame with Probability P
MESSAGE TO GENERATE
No
BCN (0,0)
BCN (Qoff, Qdelta)
No Message
Sampled Frame?
Yes
RL Tagged Frame?
No
Yes
Send BCN
MESSAGE TO GENERATE
BCN (0,0)
BCN (Qoff, Qdelta)
NOP

Fluid-Model Equations

- The CP equations (not linearized)

$$\frac{dq(t)}{dt} = N \times R(t) - C.$$

- The RP equations

$$F_b(t) \quad = -\left[(q(t) - q_{eq}) + \frac{wS}{CP} \times \frac{dq(t)}{dt}\right] /S.$$

If $F_b(t-\tau) > 0$,

$$\frac{dR(t)}{dt} \quad = \left[G_i R_u \times F_b(t-\tau) \times R(t-\tau) \times P\right] /S.$$

If $F_b(t-\tau) < 0$,

$$\frac{dR(t)}{dt} \quad = \left[G_d \times R(t) \times F_b(t-\tau) \times R(t-\tau) \times P\right] /S.$$

Fluid-Model Equations

- Continuous time
- No stochastic processes
- No discrete packet sizes
- Assume infinite buffer size

- Control analysis stability
  - Help us set parameters
  - Prerequisite for stochastic stability

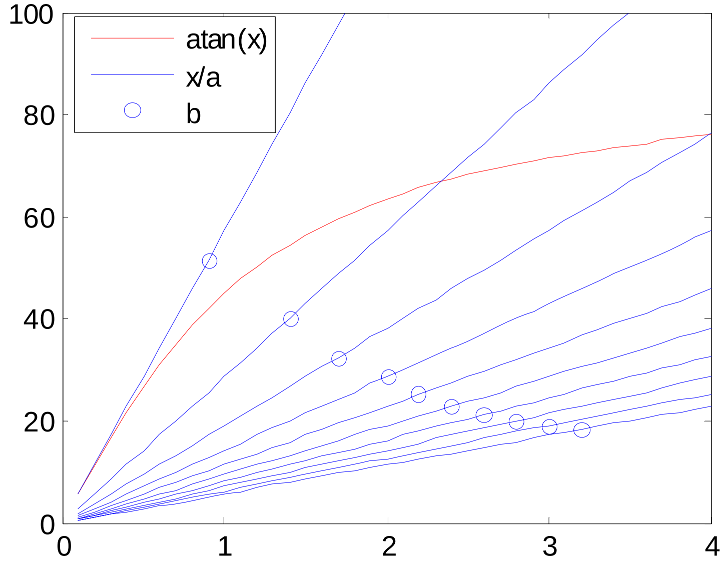
Stability Analysis

The linearized system is stable if

(i) $G_i R_u w \leq \frac{S}{a\tau}$ (ii) $G_i R_u w^2 > \frac{PC}{b\sqrt{b^2+1}}$

(iii) $G_d w \leq \frac{SN}{aC\tau}$ (iv) $G_d w^2 > \frac{PN}{b\sqrt{b^2+1}}$

where a $\geq$ 1 and b/a + arctan(b) = π / 2



a bigger → slower response

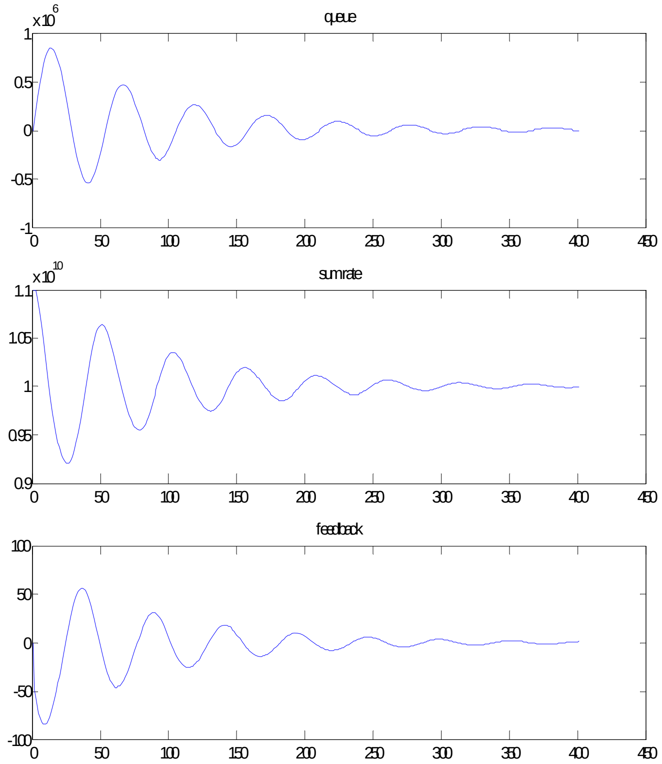
→ b bigger → N can be bigger

Sufficient condition

(i) and (ii) corresponds to the source equation Fb>0

(iii) and (iv) corresponds to the source equation Fb<0

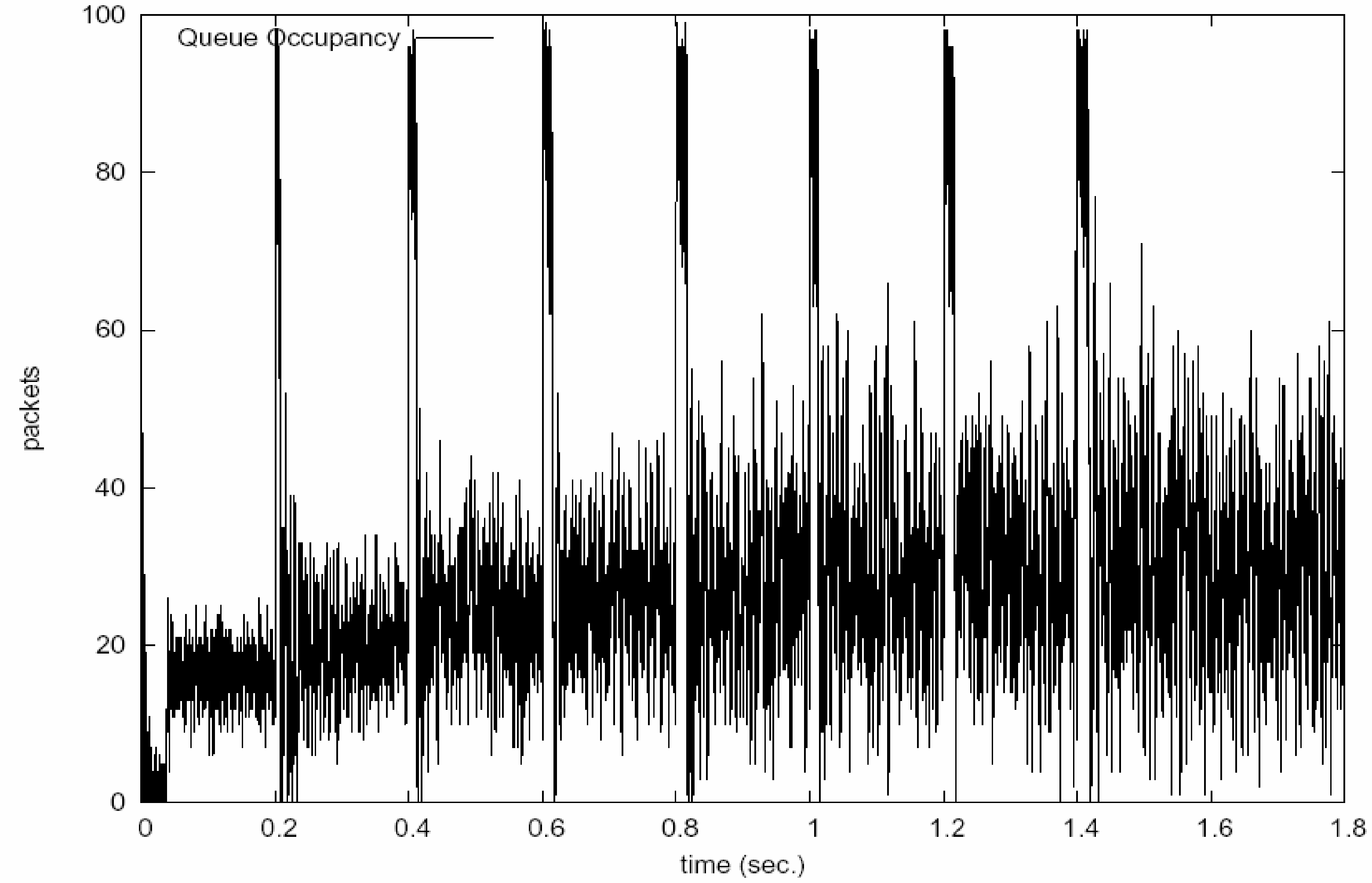
We show that these conditions are <u>sufficient</u> for the stability of the switching system.



Scenario

- Every 0.2 s, 50 new long-lived flows inserted
- Starting rate: 100 Mbps
- $q_{eq} = 16$
- Buffer size = 100 x 1500 Bytes
- P= 0.01
- $G_i = 4$, $Ru = 1e^6$, $w = 2$, $G_d = 1/128$
  obtained with $a = 5$ and $b = 2.2$

Stability



Queue Occupancy
100
80
60
40
20
0
packets
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
time (sec.)

Self-increase

Self-increase: RP may gently increase its sending rate in various ways (see below), even when there are no BCN signals from its CP.

This is a good idea for several reasons:

- It is fail-safe (messages may be lost)
- Gently probe for extra bandwidth
- V.useful for fairness, as we shall see

Let's consider 3 types of self-increase

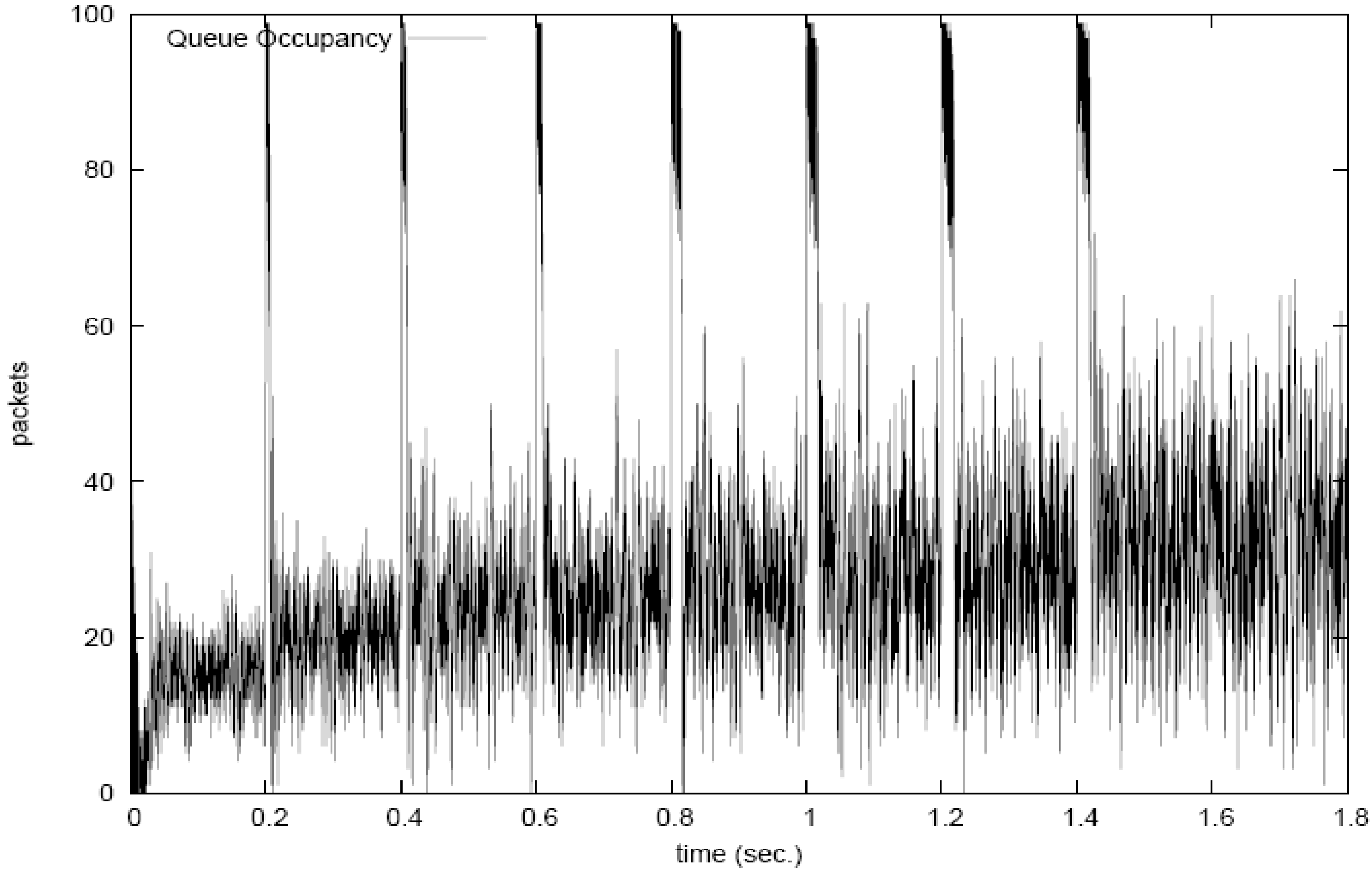
1. At a fixed rate of A bps
2. At a rate AxR bps, where R is the current sending rate
3. At a rate A/(# of negative feedback signals)

Type 2 brings a bounded amount of extra work, *regardless* of the number of sources

Type 3 similar to type 1, but fairer

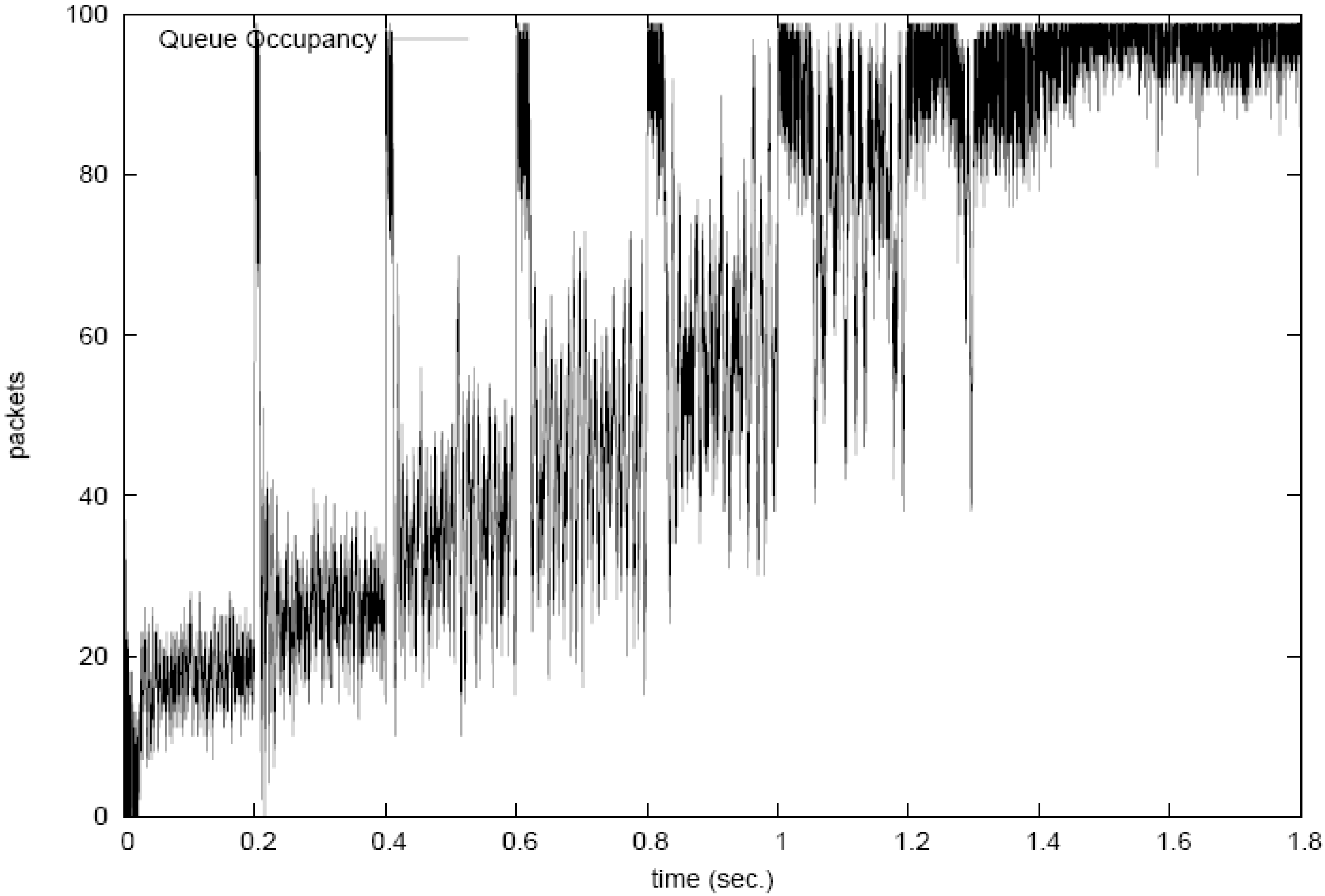
Self-increase: stability

Type 1: Gentle increase of 10 Mbps/s



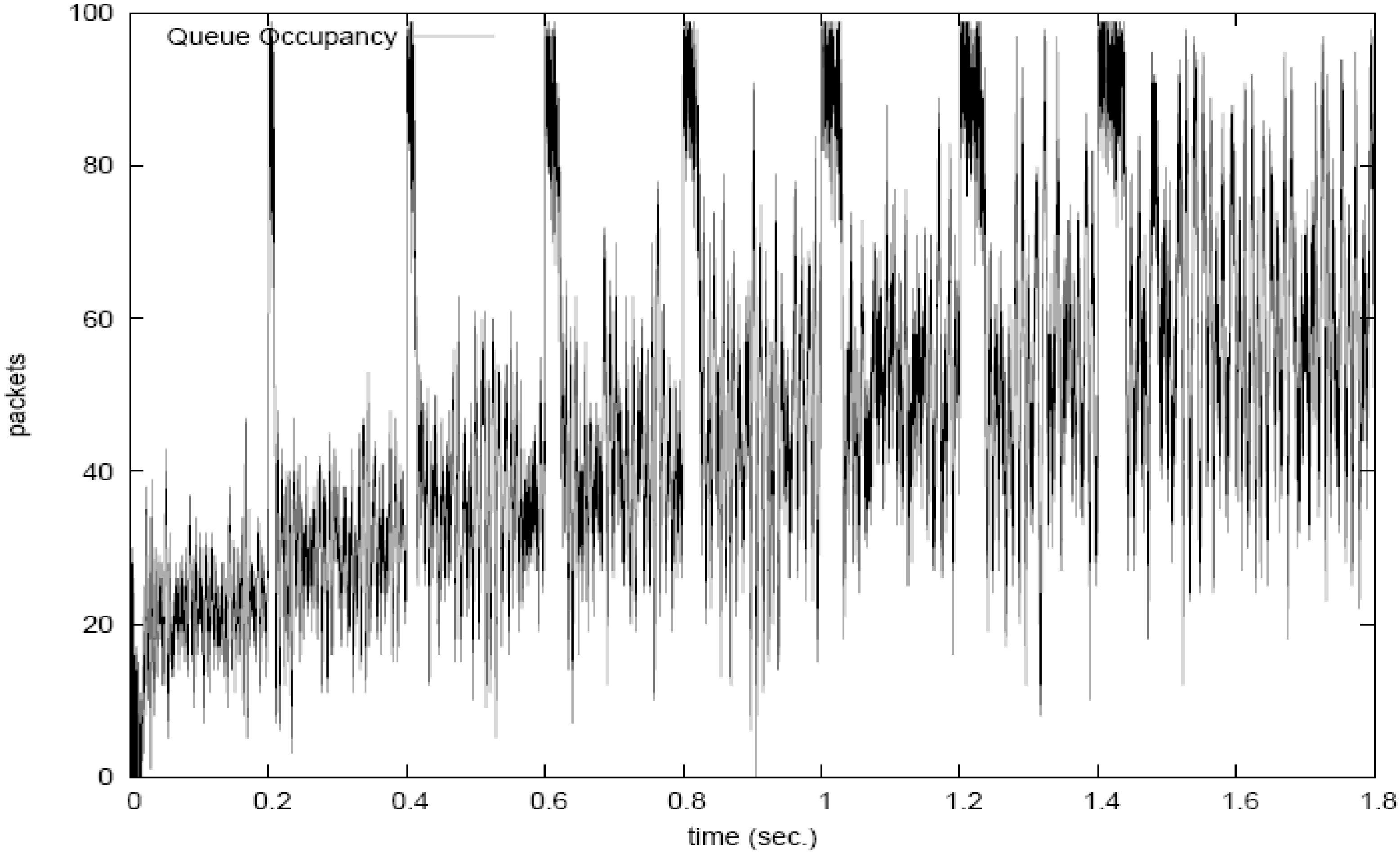
Self-increase: stability

Type 1: Aggressive increase of 500 Mbps/s



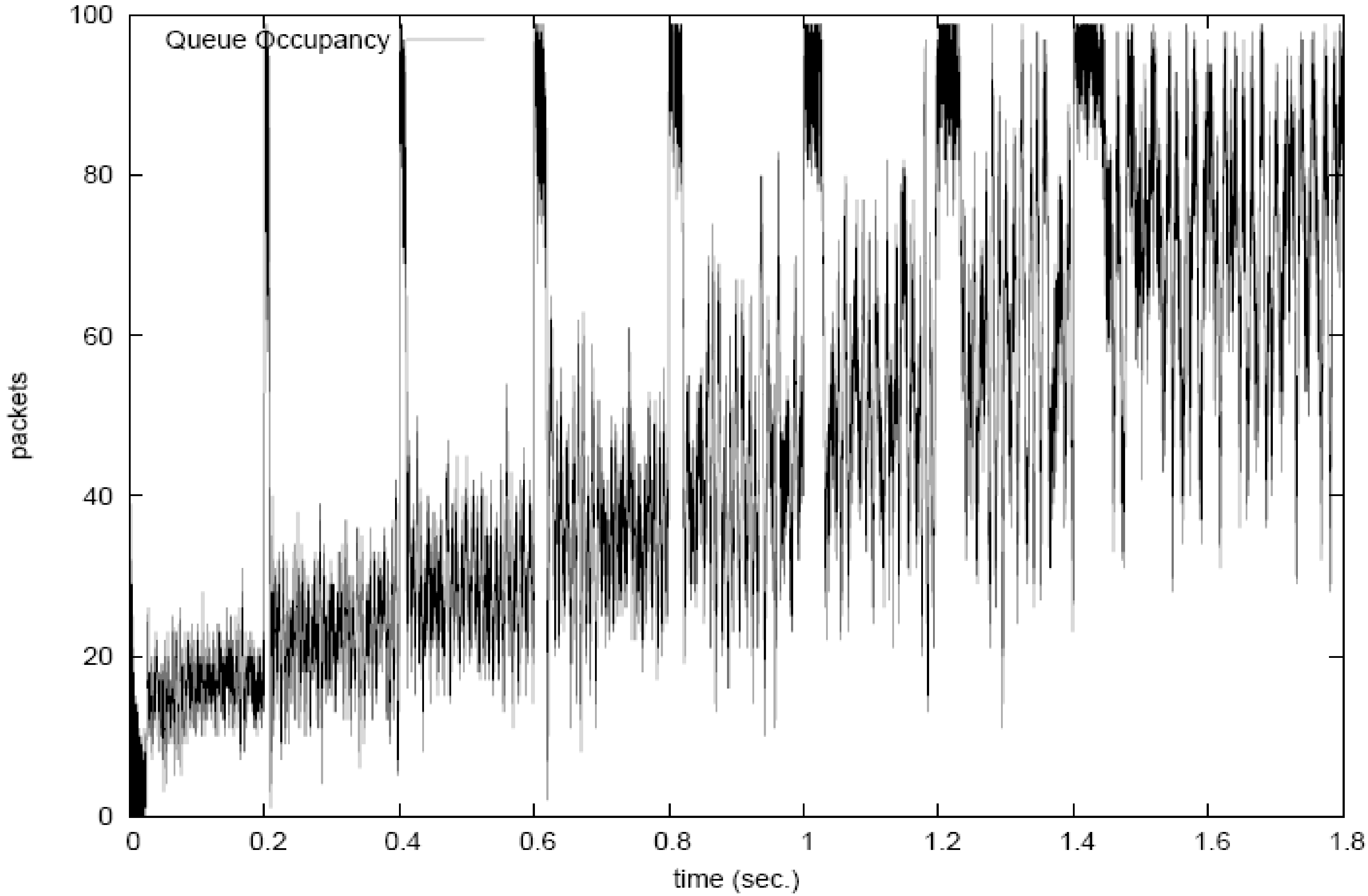
Self-increase: stability

Type 2: Aggressive increase factor = 10/s



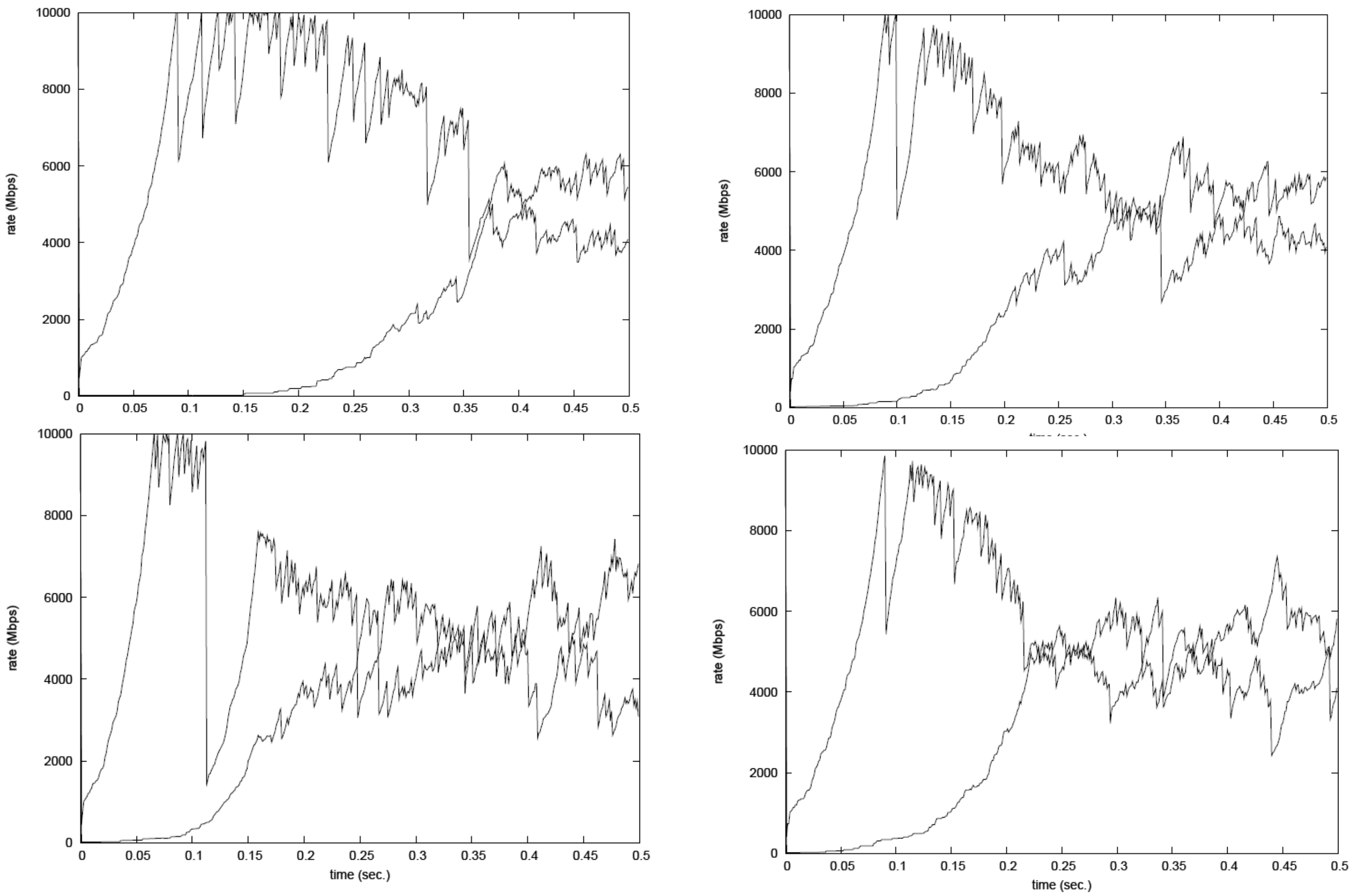
Self-increase: stability

Type 3: Aggressive increase of 500 Mbps/s



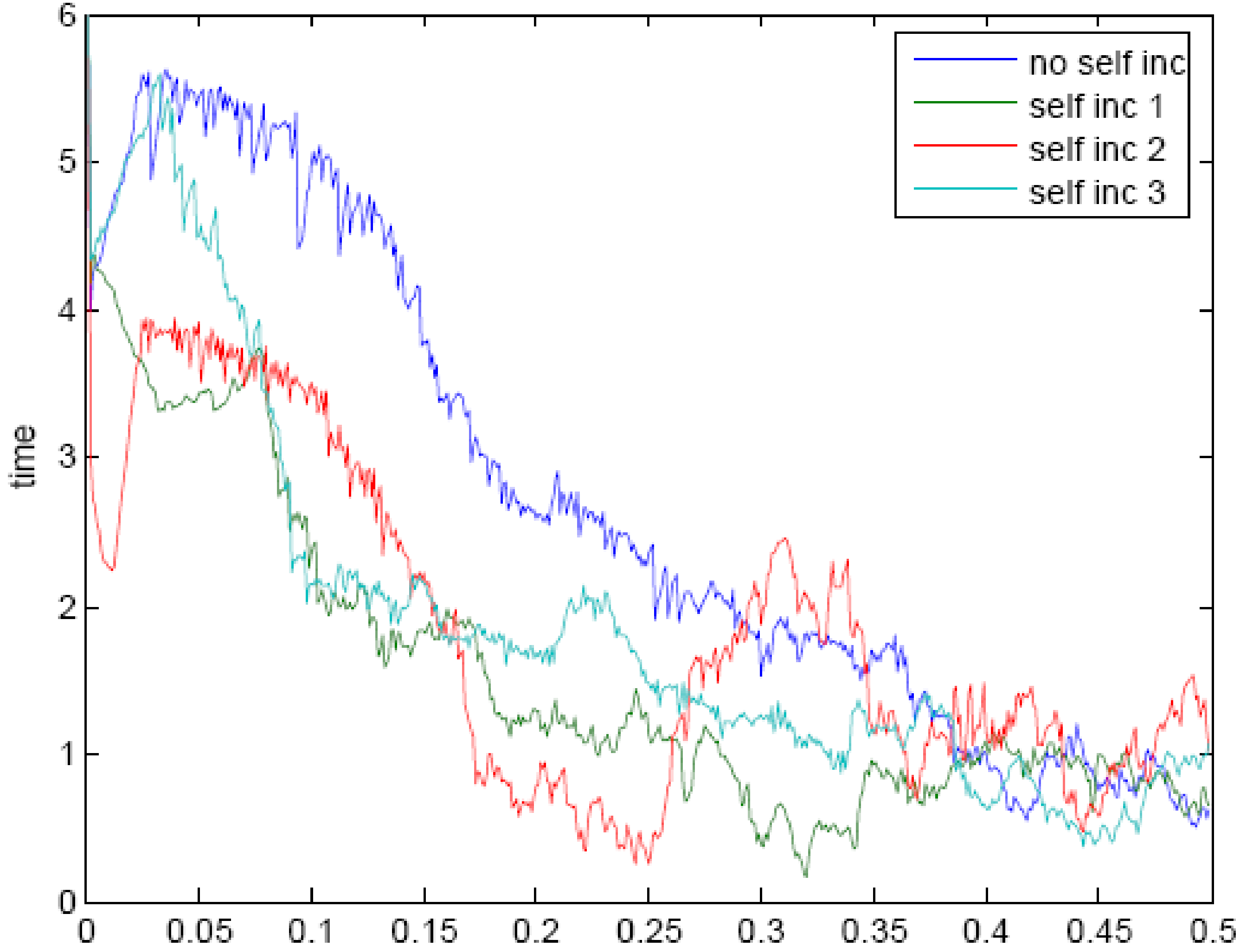
Fairness

- Self-increase helps improve fairness properties



Fairness (unfairness index)

$$\sum_i \frac{|x_i - \bar{x}|}{\bar{x}}$$



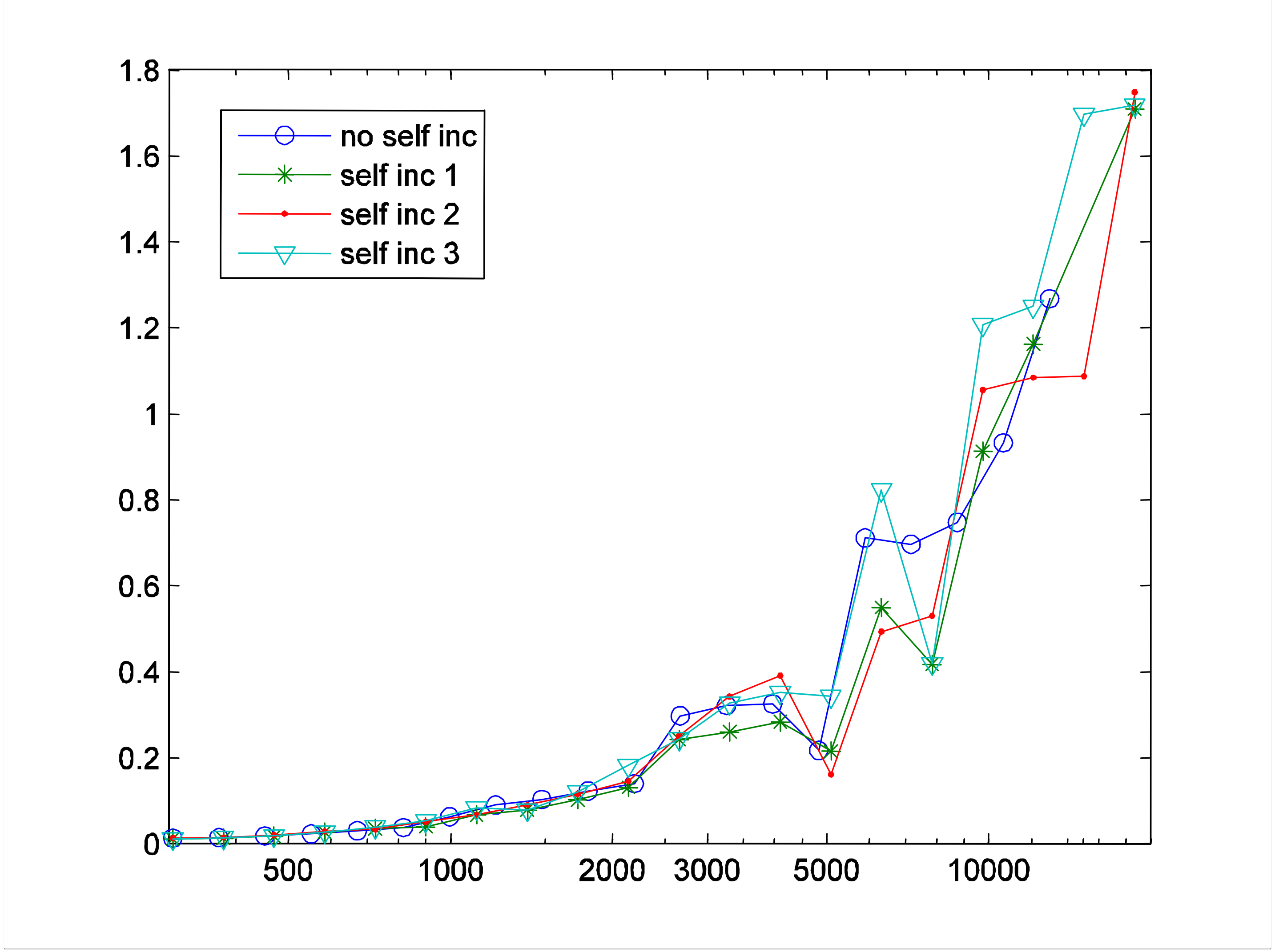
Fairness → Flow Competion Time

- Plots of fairness properties for infinitely long-lived flows are not very informative.
- We realize that fairness has its implications in scenarios with flows arriving and departing
- Fairness can be translated into: For flows within a size range, the completion times are similar
- Lack of fairness is hence reflected by the large variance in completion times
- Simulations shows that self-increase helps reduce the variance in completion time, and does not hurt the average

Scenario

- Flow size distribution ~ Pareto 1.8
- Mean flow size 1 MB
- Arrival rate Poisson 1125 flow/sec
- 9 Gbps average traffic
- Starting rate 1Gbps

Average completion time



Normalized standard deviation