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THE MEANING OF A DAY

Rob Seaman[*]

Springing from the day-and-night cadence of our calendars, civil timekeeping is time kept for a multitude of technical and cultural purposes. Just as the unit of Atomic Time is the SI-second, the natural unit of Universal Time is the *synodic day*. Atomic clocks can now keep time to better than a second in many millions of years. High precision does not imply an accurate clock, however. The day, the month, and the year are all tied to the quirky cadences of astronomical phenomena. It is the very precision of our clocks that creates challenges in synchronizing them to the varying and aperiodic rhythms of nature. However, time signals reach wherever computer networks take them and this now includes spacecraft to other worlds. Perhaps it is simply time to retire the 18th-century notion of Greenwich Mean Time? Rather, it is only Mean Solar Time based on the synodic day that can bring 21st-century coherence to timekeeping spanning our solar system. Whether on Earth, Mars or the Moons of Jupiter, the word *day* means the same thing.

INTRODUCTION

Hoover Dam was dedicated by President Franklin Delano Roosevelt on September 30, 1935. The dam, which at 6.6 million tons in weight and 221 meters in height is larger than the Great Pyramid, straddles the Nevada-Arizona border along the Colorado River. This event was captured by the sculptor Oskar J. W. Hansen in the memorial plaza on the Nevada side of the dam. Built on the site of the "safety island" that protected workers during pouring of the concrete for the dam, Hansen's artistic vision included not only a pair of monumental winged statues and a 140-foot tall flagpole, but the surface of the plaza itself is a terrazzo star map[†] as shown in Figure 1.

The map depicts the apparition of the stars, Moon and planets on the evening of the dedication. Hansen also conveys the anticipated permanence of the dam by showing the Earth's axial luni-solar precession. The map is an azimuthal stereographic projection of the circular clock face scribed by the precessing north pole. Sometimes called a Great Year, the roughly 26,000-year precessional period is represented for today's Polaris, as well as for Thuban, the pole star of the Pharoahs, and for the bright star Vega which will serve the purpose in about 12,000 years.

> *"I must down to the seas again, to the lonely sea and the sky, and all I ask is a tall ship and a star to steer her by"* – John Masefield

What can we say about that far-distant future world whose mariners will steer by a different star? Perhaps nothing more than that it will not closely resemble any predictions made by futur-

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[*] Data Engineer, National Optical Astronomy Observatory, 950 N Cherry Ave, Tucson, AZ 85719.

[†] http://longnow.org/membership/newsletters/02013-march-equinox/

ists or science fiction writers – and that each day will follow the last in a grand historical sequence of whole-numbered calendar dates that originated before the world of the ancient Egyptians and their pyramids.

The recent proposal[1] by Working Party 7A of the Radiocommunication Sector of the International Telecommunication Union (ITU) to cease the issuance of leap seconds appears on its surface to concern a technocratic detail of little interest beyond lab-coated scientists and engineers with pocket protectors. Rather, it is an implicit attempt to redefine the meaning of the word *day*, world wide and for all time. Each missed leap second (or equivalent adjustment) would cause 12:00 am to drift further away from midnight, secularly and in general monotonically. The operative word here is "attempt". Length of day on the Earth – and thus the meaning of *day* – is not a free parameter that can be adjusted to suit engineering preferences.

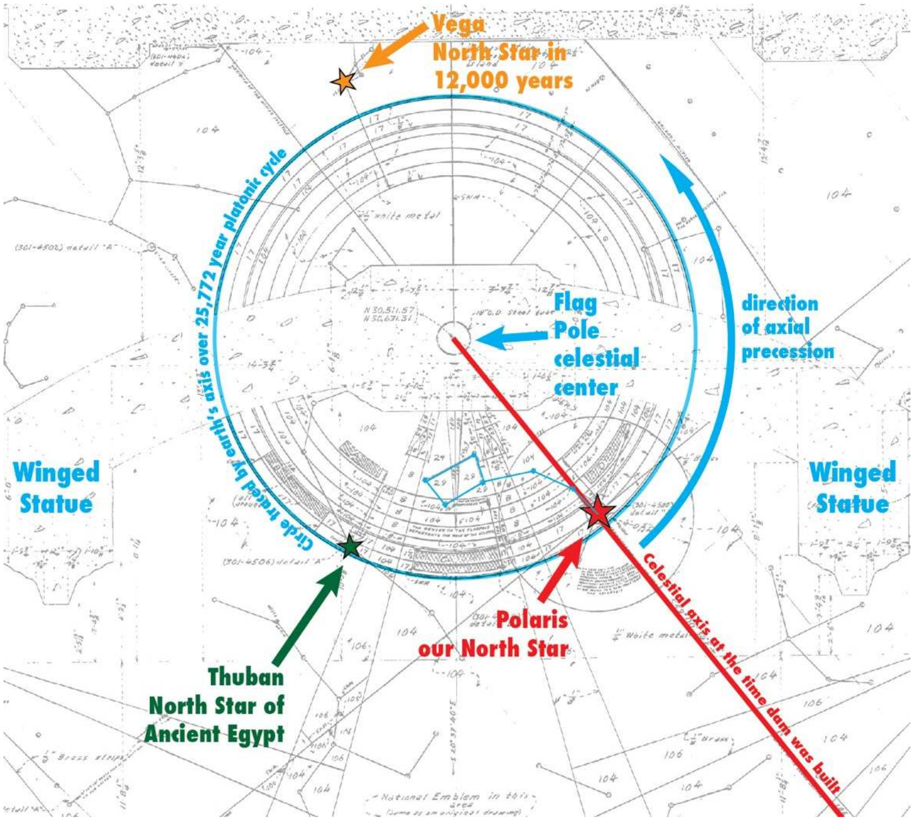

Figure 1. Blueprint for Oskar J. W. Hansen's "Safety Island" star map at Hoover Dam.[*] The Earth's precessional period is another clock based on Earth rotation.

That our days are marked by the rising of the Sun caused by the rotation of the Earth, and that our years are graced by seasons due to Earth's axial tilt and orbital motion, are facts so familiar that they are often taken for granted. It may seem strangely "natural" in our technological world to instead index our days to atomic clocks, quietly ticking in the laboratory. This is indeed what we have already done by layering Coordinated Universal Time (UTC) on top of International

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[*] Courtesy A. Rose, Long Now Foundation, San Francisco, CA.

Atomic Time (TAI). But the price for doing this is the leap second. It is precisely these small, rare adjustments that enable atomic clocks to keep the correct time.

The original chronometers were created to model the Earth's rotation by the keeping of Greenwich Mean Time (GMT).[2] It may be argued that since this invention was motivated by the need to navigate at sea, and since this need is now generally met by the Global Positioning System (GPS) and other technologies, that modern clocks need no longer be referenced to the Greenwich meridian. Rather, the nature of a chronometer remains to model an external physical phenomena, the Earth's rotation. This was stated clearly in the original UTC standard: *"GMT may be regarded as the general equivalent of UT."*[*]

Our diurnal clocks are not the only timepieces to result from the motions of the Earth. Since the Earth spins like a top, it also wobbles like a top. This produces the 25,771 year period of the *precession of the equinoxes* (and smaller amplitude wobbles of nutation and polar motion). In the precessional clock, each season is a second, one hour by Hansen's clock is a millennium.

CALENDAR DAYS

Entries in the Oxford English Dictionary are subdivided by their varied *significations*. For the noun *day*, the first three major divisions are *"I. the time of sunlight"*, *"II. as a period, natural division, or unit of time"*, and *"III. a specified or appointed day"*. These familiar categories circumscribe the possible meaning of the day in civil timekeeping. One need not belabor the connection of sunlight to daylight: *"Whether I retire to bed early or late, I rise with the sun,"* said Thomas Jefferson. A day is not just a period or even a unit of time – it is a natural division.[†] And each day is unique; every day occupies its appointed place in the calendar.

Calendars are cultural constructs that serve many technical and logistical, symbolic and religious purposes. They are implemented in a profusion around the world, and calendars have often changed throughout history. From the otherwise Eternal City of Rome, Julius Caesar and Pope Gregory XIII instituted calendar reforms sixteen centuries apart. These have not always gone smoothly.[3] Among the many differences between lunar and solar calendars, ecclesiastical and scientific calendars, calendars of many nations and cultures past and future, one might ask what is true of all of them?

What is true is that they are all constructed as aggregates of units of the day, the mean solar day. Even astronomers for most purposes schedule activities on solar calendars, not sidereal. Sometimes a day starts at midnight, sometimes at sunset, perhaps even at noon; the names of the days differ in different languages; the Sabbath varies. However, underneath the diversity lies unity in solar timekeeping, in the natural diurnal cadences of life on Earth – or for that matter on the Moon or Mars.

Sidereal Rotation Period

Everything in the universe is in constant motion. Moons orbit planets; planets orbit stars; stars circle the galaxy, and the Milky Way and Andromeda galaxies are due to collide four billion years from now. And everything rotates – fast or slow, clockwise or counter-clockwise.

> *"Every body continues in its state of rest or uniform motion in a straight line, except in so far as it doesn't."* – Sir Arthur Eddington[4]

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[*] https://www.itu.int/rec/R-REC-TF.460-4-198607-S/en

[†] As in the Doors lyrics: *"Day destroys the night, Night divides the day."*

That the universe is large and ancient and that we are young and small allows humans to refer to the “fixed stars” as a frame of reference. The sidereal rotation period of a planet, dwarf planet, moon or other body is its rotation measured with respect to the stars. Sometimes this is referred to as a *sidereal day*. This usage is incorrect, however. For example, the passage of one rotation period on Mercury is enough to turn blazing noon into predawn night. Mercury’s 3:2 spin-orbit resonance means that after one rotation relative to the stars the planet will have circled around behind the Sun and be coming out the other side.

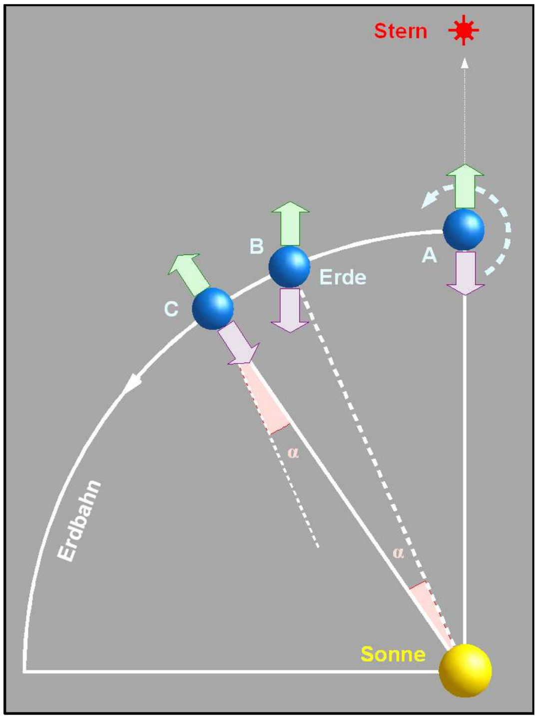

Figure 2. As Earth orbits the Sun, each day is longer than its rotation period relative to the stars, due to having to traverse an additional angle of arc ∠BC along the orbital path.[*]

Synodic Days

Figure 2 illustrates the difference between the sidereal rotation period of the Earth and the day on Earth – the solar day. As the Earth moves from point A to point B in its orbit, corresponding to one sidereal rotation period, a fixed star on the celestial sphere will return to the same position in the sky. The Earth must move a small additional arc along its orbit to point C, however, equivalent to rotating an additional angle ∠BC such that the Sun returns to its original position over head. This is the Earth’s *synodic day* or simply a “day”. In almost all instances of the usage of the word day, this solar day is what is meant.

> *“Day. n. II. As a period, natural division, or unit of time. 6. a. The time occupied by the earth in one revolution on its axis, in which the same terrestrial meridian returns to the sun…”* – the Oxford English Dictionary[†]

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[*] http://commons.wikimedia.org/wiki/File:Sidereal_and_Synodic_Day.png

[†] Copyright © 2013 Oxford University Press, OED correctly states that the meridian returns to the Sun

There is an even easier way to visualize the concept, however. The Earth rotates on its axis in the same direction as it orbits the Sun. As a result, as we lap the Sun during the course of a year one rotation of the Earth is “unwrapped”. Thus *the number of days in a year is exactly one fewer than the number of sidereal rotations per year*. Or inverting that, there is one more rotation than the number of days; Earth’s year is 365 and a quarter days, but it rotates 366 and a quarter turns per year relative to the stars.

Clocks Divide Days

Atomic time is a time scale constructed from the bottom up by accumulating the ticking of SI-seconds. Civil time, on the other hand, is built from the top down by subdividing the solar day. It is simply true that they are two different things. A pretence that they are one and the same cannot long be maintained, and will ultimately come at a greater cost than simply recognizing and planning for the distinction.

Discussions regarding timekeeping on Earth, especially about leap seconds and Coordinated Universal Time, are often politicized and narrowly focused. The International Telecommunication Union is a specialized agency of the United Nations.[*] An idiosyncratic chain of events has led the ITU’s Radiocommunication Sector[†] to claim oversight for UTC. UTC is our realization of Universal Time, and UT is the modern analogue of Greenwich Mean Time, because of an involved international history dating back more than a century.[‡]

Table 1. Length of day for the terrestrial planets and dwarf planets in the Solar System.[§] There is one fewer day/year than rotations/year for all. Retrograde rotators like Venus obey the same rule with the rotational period negative. Earth’s small 4 minute difference between solar day and sidereal rotation is atypical.

Object	**Year** *Earth-days*	**Rotational Period** *(d)*	**Rotations per Year**	**Days per Year**	**Day** *Earth-days*	**Difference**
Mercury	87.97	58.6467	1.5000	0.5000	175.94	117 d
Venus	224.70	-243.02	-0.9246	-1.9246	-116.75	-127 d
Earth	365.25	0.99727	366.25	365.25	1.00000	236 s
Moon	"	27.32	13.37	12.37	29.53	2.21 d
Mars	687	1.02596	669.6	668.6	1.02749	132 s
Ceres	1680	0.3781	4442.7	4441.7	0.3782	9 s
Pluto	90465	-6.387	-14164	-14165	-6.387	-39 s

SOLAR SYSTEM CENSUS

Earth, however, is not the only terrestrial world in our neighborhood. We share the solar system with at least twenty-four other worlds that have rocky/icy surfaces and are large enough to be gravitationally rounded.[**] Our clocks keep Earth-time because that is where we live, but the un-

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[*] http://www.un.org/Overview/uninbrief/institutions.shtml

[†] http://www.itu.int/en/ITU-R/

[‡] http://www.ucolick.org/~sla/leapsecs/timescales.html

[§] Data adapted from http://en.wikipedia.org/wiki/List_of_gravitationally_rounded_objects_of_the_Solar_System

[**] Many of these observations will also apply to smaller, potato or dumbbell shaped, moons or asteroids.

derlying physical nature of timekeeping on Earth is precisely the same as on other worlds. It is instructive to consider the differences and similarities.

Starting the grand tour with the four innermost terrestrial planets and the two dwarf planets inward of Pluto (see Table 1) one first notes that the Earth spins more rapidly than most. Our Moon is included in this table since it is bigger than either Pluto or Ceres, and indeed the Earth-Moon system is often considered a double planet. Mercury, Venus and the Moon rotate much more slowly than the Earth, and as a result the solar day on each is very different than their sidereal rotation periods – many days different instead of seconds. The sidereal period is simply a different concept than the "day" on each. However, the various time concepts are connected. The length of the year and the rotation period are required to figure out the length of the day. The lunar synodic day is also why there are twelve months in a year rather than thirteen.

Mercury is an interesting case due to its 3:2 spin-orbit resonance. The year on Mercury is longer than the rotational period, but the day is twice as long as the year. A patient astronaut standing in the blazing noon sunlight on Mercury will find that it is midnight a year later, and will have to wait two years until it is next noon again.

Venus, on the other hand, rotates retrograde. If our astronaut were in a spaceship "below" the south pole of the Sun looking "up", she would see almost all solar system objects orbiting and rotating clockwise. Venus would be one of the exceptions and would be seen to be rotating counter-clockwise (all planets orbit in the same direction). The rule still applies: there is one fewer day per year than rotations – just use a negative value for the retrograde rotational period.

Our Moon rotates synchronously, meaning it always keeps the same side facing Earth. In fact, all large moons are synchronous rotators in the solar system, and most small ones as well. A month on the Earth, from one new moon to the next, is literally a (synodic) day on the Moon.

A day on Mars is just a bit longer than on Earth. Thanks to the Mars rover missions it is also a place where humans use clocks on a daily basis.[*] The drivers and pit crews for the rovers are on Earth, but schedule their daily shifts using a combination of solar time on Mars as well as Earth.

Ceres was the first asteroid discovered and contains a third of the mass of the asteroid belt. Referred to as a planet when first discovered on New Years Day in 1801, it is now classed as a dwarf planet. When Pluto was demoted, Ceres was promoted. It rotates rapidly and the difference between the synodic day and sidereal rotation period is just 9 seconds.

Table 2. Length of day for the Galilean moons of Jupiter.

Object	**Year** *Earth-days*	**Rotational Period *(d)***	**Rotations per Year**	**Days per Year**	**Day** *Earth-days*	**Difference**
Io	4332.59	1.76914	2448.98	2447.98	1.76986	62 s
Europa	"	3.55118	1220.04	1219.04	3.55409	251 s
Ganymede	"	7.15455	605.57	604.57	7.16638	1022 s
Callisto	"	16.68902	259.61	258.61	16.75355	5575 s

Pluto will be discussed below with its largest moon, Charon. Its very long year also results in a small synodic–sidereal difference even though its day is six times as long as on Earth. Distant

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[*] http://www.giss.nasa.gov/tools/mars24/

from the Sun, daylight on Pluto is still about equivalent to 250 times the illumination of the full moon on Earth. Dwarf planets further out than Pluto will be correspondingly more dimly lit and one might have to ask at some point whether the concept of day applies at all.

Jupiter is more than twice as large as all the other planets combined. As a result of its size (and also its proximity to the asteroid belt) Jupiter has many moons. Galileo discovered the first four large moons that will be discussed here. The Galilean moons form a remarkable mini-solar system of their own, see Table 2. The first thing to note is that the synodic–sidereal difference grows larger for the outermost moons, opposite to the general trend for the planets. For the planets the year grows much longer in the outer solar system. Thus there are many more rotations per year, and the fractional difference is less. The differing sidereal-synodic behavior is due to two effects. Firstly, moons share the same year as the planets they orbit. The second effect is due to the moons all rotating synchronously in their planetary orbit, that is, they all keep the same face toward Jupiter. Because the outer moons orbit more slowly they also rotate more slowly, and since the year is the same for both Jupiter and its moons there are fewer days per year for the outer moons. Thus the synodic–sidereal difference grows the further from a planet you go.

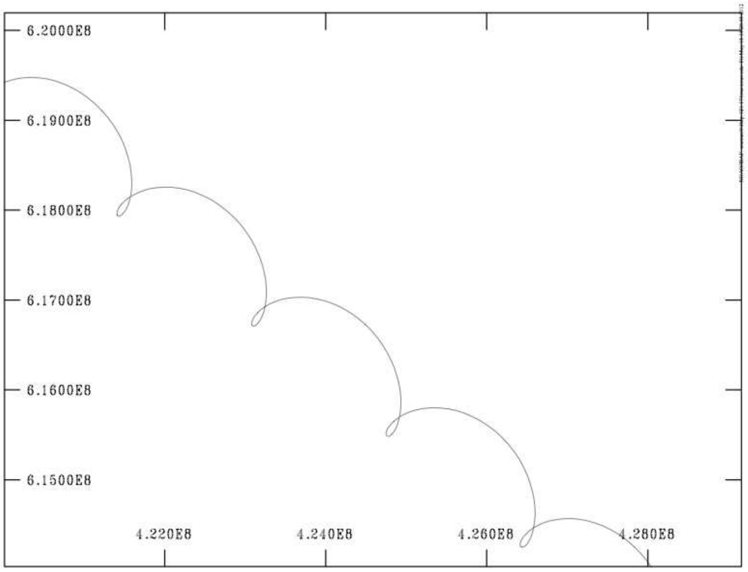

Figure 3. Cartesian plot of the Sun-centered orbit of Io.[*]
For comparison, the orbit of the Earth's Moon is everywhere concave toward the Sun.

Orbit of Io

Another dynamical issue is remarkable in not having any effect at all on the length of the day. Since the gas giant planets are far from the Sun, they move more slowly in their orbits. Because they are also very massive, their moons revolve around the gas giants very quickly. For example, Io orbits about as far from the center of Jupiter as the Moon orbits Earth, yet Io takes less than two days to orbit Jupiter while the Moon takes 27 days orbiting Earth. Combining these two effects an inner moon can actually move backwards in its own sun-centered orbit. Io provides the

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[*] Io ephemerides from NASA/JPL HORIZONS, http://ssd.jpl.nasa.gov/horizons.cgi

best example for the major moons, see Figure 3. While from a Jupiter-centered perspective Io's orbit is nearly circular and its speed constant, from a fixed Cartesian vantage point (data from NASA/JPL), Io makes swooping loop-de-loops that alternate direct and retrograde orbital motions (and hence coming to a momentary stop in between).

Remarkably, this psychedelic motion has no effect on the length of day on Io, or on the resulting solar clock model. Since Io orbits quickly, it also rotates quickly, and this dominates the length of day calculation.

Laplace Resonance

The solar system is home to some remarkable gravitational resonances. We have already discussed Mercury's 3:2 spin-orbit coupling. The three innermost of the Galilean moons, Io, Europa and Ganymede, are tied together in the 4:2:1 *Laplace Resonance*, see Equation 1. In the simplest case of synchronously rotating moons locked in an integer-valued orbital resonance, a solar clock on one world would tell time, halved or doubled, on the others.

$$\Phi_L = \lambda_{\mathrm{Io}} - 3 \cdot \lambda_{\mathrm{Eu}} + 2 \cdot \lambda_{\mathrm{Ga}} = 180^\circ$$

Equation 1. The mean longitudes of Io, Europa and Ganymede are phase locked.

For the Galilean moons, the behavior is a bit more complex than Figure 4 suggests since the point of perijove is precessing at the same time. This can be seen in the not quite integral ratios from Table 2. The functional complexity just serves to emphasize the point that the underlying conceptual model is the synodic day derived from sidereal rotation. No other choice of clock behavior could be used to solve the problem for all three worlds simultaneously. Indeed, Equation 1 is an identity when solved using either solar time or sidereal time.

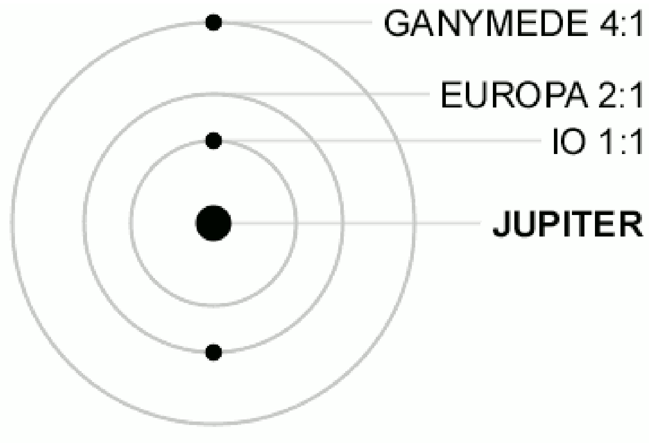

Figure 4. The innermost three Galilean moons are in the 4:2:1 Laplace Resonance.[*] For each orbit of Ganymede, Europa circles Jupiter twice and Io orbits four times. (The moons can never all be on the same side of Jupiter.) The entire mini-solar system is also precessing about Jupiter at the same time.

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[*] Original is an orbital animation, http://en.wikipedia.org/wiki/File:Galilean_moon_Laplace_resonance_animation.gif

The outer solar system

In addition to its remarkable system of rings, Saturn has several large moons, including Titan, home to hydrocarbon lakes and an atmosphere thicker than Earth's. Table 3 again shows the effect of Saturn's moons' synodic–sidereal differences growing with increasing distance from the planet. The difference for the most distant of these moons, Iapetus, is more than one-half of an Earth day. We will return to the tiny "shepherd" moon Daphnis later.

Table 3. Length of day for the major moons of Saturn.

Object	**Year** *Earth-days*	**Rotational Period** *(d)*	**Rotations per Year**	**Days per Year**	**Day** *Earth-days*	**Difference**
Daphnis	*10759.22*	*0.59408*	*18110.73*	*18109.73*	*0.59411*	*3* s
Mimas	"	0.94242	11416.59	11415.59	0.94250	7 s
Enceladus	"	1.37022	7852.18	7851.18	1.37039	15 s
Tethys	"	1.88780	5699.34	5698.34	1.88813	29 s
Dione	"	2.73692	3931.14	3930.14	2.73762	60 s
Rhea	"	4.51821	2381.30	2380.30	4.52011	164 s
Titan	"	15.945	674.77	673.77	15.969	2071 s
Iapetus	"	79.322	135.64	134.64	79.911	50890 s

Table 4 has the remaining grab bag of moons from the outer parts of the solar system. Uranus orbits on its side, rolling along its orbit during some parts of its long year. During these periods its moons are illuminated from above their north or south poles. Two decades after such a polar alignment the moons will be illuminated near their equators. (Uranus's year is 84 Earth-years.) At intermediate times the polar regions will see long periods of dawn and dusk with the Sun circling the horizon as at Earth's poles. During these apparitions length of day becomes something of a metaphysical concept – which again emphasizes that day is a concept directly connected to the diurnal rising and setting of the Sun.

Table 4. Length of day for the major moons of Uranus, Neptune, and Pluto.

Object	**Year** *Earth-days*	**Rotational Period** *(d)*	**Rotations per Year**	**Days per Year**	**Day** *Earth-days*	**Difference**
Miranda	30685.4	1.414	21701	21700	1.414	6 s
Ariel	"	2.52	12177	12176	2.52	18 s
Umbriel	"	4.144	7405	7404	4.145	48 s
Titania	"	8.706	3525	3524	8.708	213 s
Oberon	"	13.46	2280	2279	13.47	510 s
Triton	60189	-5.877	-10241	-10242	-5.876	-50 s
Charon	90465	-6.387	-14164	-14165	6.387	-39 s
Pluto	*90465*	*-6.387*	*-14164*	*-14165*	*6.387*	*-39* s

Neptune's moon, Triton, rotates retrograde as does Pluto's moon, Charon. In fact, Pluto and Charon are tidally locked. Charon hangs motionless in the sky above Pluto, eight times the angular size of Earth’s moon[*] and cycling through its phases every six days. As on Earth, eclipses can occur when Charon’s orbital plane around Pluto crosses the orbital plane around the Sun. A year on Pluto is 250 years, so this occurs about every 125 years. This last happened in the late 1980s when every orbit generated both eclipses and their inverse transits; the next series of such eclipses will occur in the early 22nd century. Pluto has several smaller moons which similarly eclipse.

A remarkable thing about the Pluto / Charon system is that a timepiece on Pluto would tell time on Charon and *vice versa* (perhaps shifted by local equivalent of time zones). The Plutonian ITU-R would have to coordinate standards-making activities with the Charonians.

THE LENGTH OF THE DAY

The argument in the current paper is that the natural unit of time in civil timekeeping is the mean solar day – on Earth just as on other terrestrial worlds. For roughly spherical planets and moons that are either direct or retrograde rotators, with solid crusts and circling single stars, it is simply a fact that there is one fewer day per year than sidereal rotations. The planet laps the Sun and one rotation is unwrapped as a result.

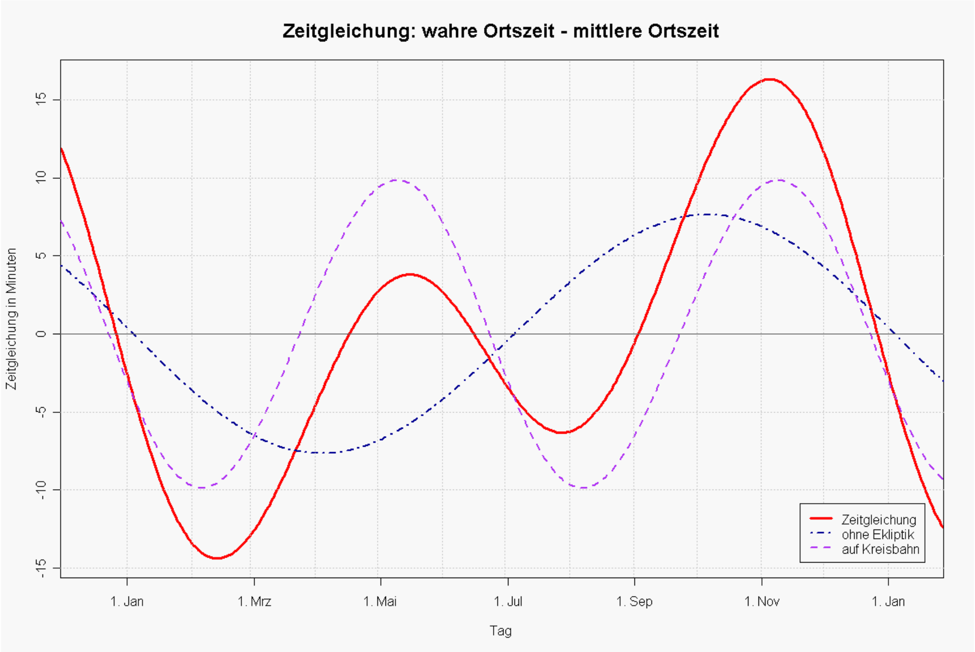

Figure 5. Apparent Solar Time is derived from the underlying Mean Solar Time by the application of the *equation of time* which combines two effects. The non-circular eccentricity of the Earth's orbit creates an annual variation, plus a seasonal variation due to the tilt of the Earth’s axis.[†]

The conceptual model on which civil timekeeping, Universal Time, and hence UTC have been built is precisely the mean solar day as originally defined through Greenwich Mean Time. Indeed, this model was how the SI-second was originally calibrated – not just as any conveniently small atom of time, but as that unit that best mimicked the fraction 1/86,400 of a mean solar day. That the rotation of the Earth is secularly slowing over the millennia due to lunar tides is a charming

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[*] In Charon’s sky, Pluto is about 15 times the visual size of our Moon.

[†] *Zeitgleichung* = Equation of Time, http://commons.wikimedia.org/wiki/File:Zeitgleichung.png

complication and is the natural result of dynamical astronomy. That length-of-day varies periodically and aperiodically with daily, seasonal, annual, decadal, and longer terms reflects profound aspects of geophysics. The complex effects do not, however, invalidate the conceptual model, but rather the opposite. Real-world timekeeping is required to be conformant to real-world physics and geophysics. Pragmatic implementations of timekeeping systems must remain flexible so as to implement close approximations to the synodic conceptual model, even as length-of-day changes at millisecond or larger scales.

Because calendars are built up out of the integral bricks of the synodic day, and since civil clocks subdivide those synodic days into traditional sexigesimal fractions of 24 hours, 60 minutes, and 60 seconds, we are motivated to look at length of day with fresh eyes. One place to start is the *equation of time* which appears in tabular or graphical form[*] on sundials and globes. Figure 5 shows the equation of time relating the Earth's underlying mean solar day to the derived apparent time. The title, "Zeitgleichung: wahre Ortszeit – mittlere Ortszeit" may be translated as "Equation of time: true time – mean time". Sometimes such plots refer to the true Sun versus a "fictitious" mean Sun. This latter phrasing engenders confusion – it is hard to argue with the Sun we see in the sky, but does the "true Sun" correspond to the "true time"? Figures and tables of the equation of time will often use phrasing like "sundial fast/slow" or even "sun fast" or "sun slow", and when reading the dial the instructions will be to add or subtract the day's value for the equation of time to correct the sundial and get the genuine (mean solar) time. Just as a separate correction is made for the sundial's longitude to recover standard time. Occasionally a plot of the equation of time will indeed refer to your watch being fast or slow, but this just emphasizes the incoherence of the conceptual model.

The German language key to Figure 5 may help in untangling the concepts. The three functions plotted are 1) the *Zeitgleichung*, the equation of time itself, 2) *ohne Ekliptik*, or "without the ecliptic", and 3) *auf Kreisbahn*, or "on orbit". This terminology is inverted from how it is usually described. The dot-dashed *ohne Ekliptic* line is the annual contribution (one cycle per year) due to the non-circular eccentricity of the Earth's elliptical orbit. And the dashed *auf Kreisbahn* line is the seasonal contribution (two cycles per year) due to the tilt of the Earth's axis relative to the ecliptic (the Earth's orbital plane). In each case the creator of this plot has chosen instead to describe each effect as if removing a complication layered on top of the underlying concept of mean solar time and the synodic day. The original (English) caption emphasizes this point of view: *"the equation of time; also: the equation of time without the ecliptic and as if the earth was turning around the sun on a circle."*

The equation of time has been generally introduced into discussions about civil timekeeping as a rhetorical argument to diminish the centrality of mean solar time – that somehow local apparent solar time is more "real" than standard time. Rather, the equation is a completely neutral agent that reversibly relates one time scale to the other. Given sundial time one can compute clock time, but equivalently, given clock time one can predict what the sundial will read.

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[*] The two dimensional version is the *analemma,* http://www.analemma.com

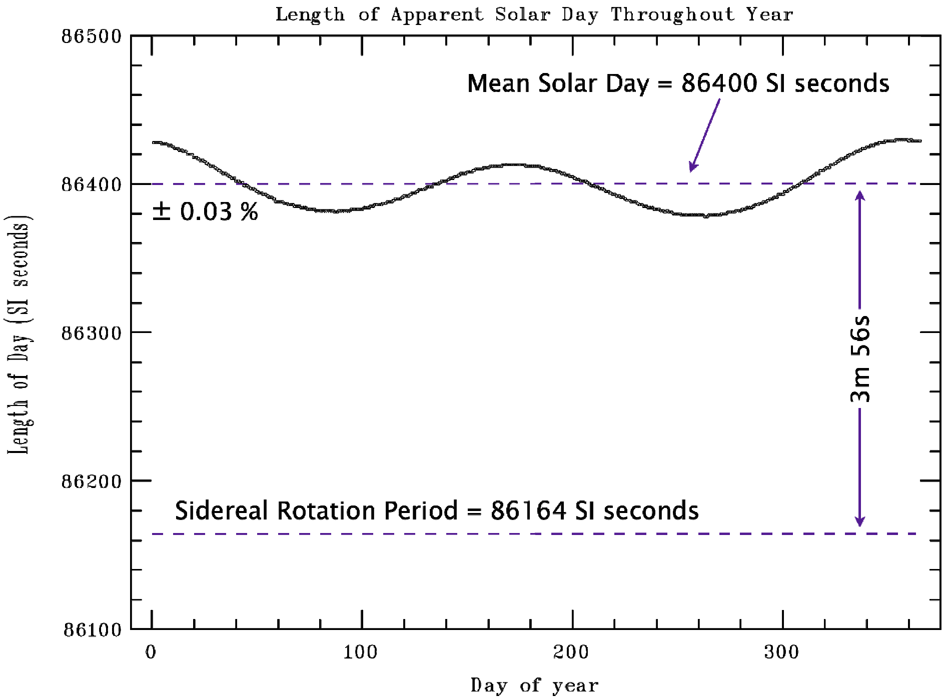

Figure 6. Rather than the relatively large excursions of the Equation of Time, the underlying length of the solar day varies by only a tiny fraction of a percent over a year.

There is a larger problem with the equation of time, however. It is not a direct reflection of the length of the day. The deviations of +/- 15 minutes[*] in Figure 5 are the result of the annual accumulation of small daily deltas as shown in Figure 6.

The actual length of day on Earth (from midnight to the next midnight) varies by only about plus-or-minus one-half minute, a 0.03% effect. And rather than mean solar time representing an esoteric average of highly divergent values dependent on large effects due to the Earth's axial and orbital eccentricities, it is the daily apparent day length that diverges by only small amounts from the underlying mean solar day.

INTERNATIONAL SYSTEM OF UNITS

An international standard represents a living consensus between institutions and individuals around the world. One of the most valued and fundamental standards is the International Standard of Units (*Système International d'Unités* or *SI*).[†] Standards must also be responsive to the physical nature of things, as demonstrated in a possible revision of the base SI units currently under consideration.[5] Each of the seven base units (kilogram, meter, second, kelvin, ampere, candela, mole) would be responsive to a corresponding measured physical constant as a fundamental reference, but also to others of the units and constants as shown in Figure 7. Each of these units was

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[*] +/- 15 minutes on Earth – the larger eccentricity of Mars' orbit makes the width of Mars' analemma / equation of time ~ +/- 45 minutes. If we lived on Mars it would be harder to confuse daily varying apparent phenomena with the underlying synodic mechanism and the resulting mean solar time.

[†] SI is overseen by various national and international organizations under the umbrella of the *Bureau International des Poids et Mesures* (BIPM), http://www.bipm.org/en/si/

originally defined by reference to physical artifacts or phenomena accessible at the time of their creation. For example, the meter, originally conceived as one ten-millionth of the distance from the Earth's equator to the pole, was implemented by two lines incised in a platinum-iridium meter bar. Similarly the international prototype kilogram is a physical weight, also of platinum-iridium.

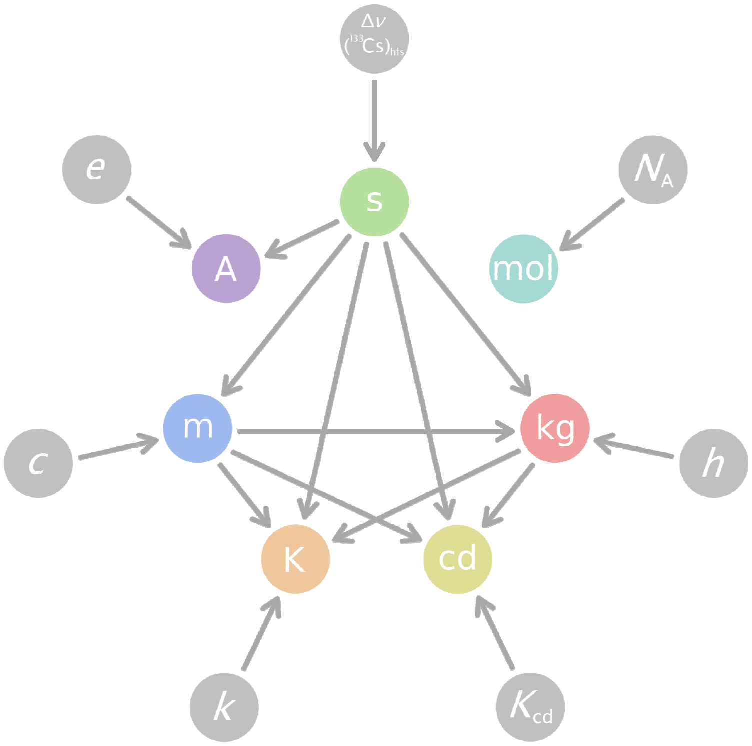

Figure 7. The International System of Units (SI) is derived from the measured values of physical constants. All of the base units (other than the mole) depend on the value of the second. The second is derived solely from a frequency standard.[*]

While the implementation of each of these base SI units has evolved over time, the underlying physical nature of each has been carefully managed to remain unchanged. Length remains length, and derived civil and scientific quantities do not drift or jump about. For instance the meter was first redefined in terms of so many wavelengths of a red emission line of Krypton-86, and later as the distance traversed at the speed of light through a vacuum in a tiny fraction of a second (that is, of an SI-second). Each redefinition realizes the same fundamental concept.

The proposed new SI continues this trend. Only two of the seven base units remain fundamental. The mole, currently set by the number of atoms in 0.12 kg of carbon-12, becomes defined in terms of specifying a precise value for Avogadro's number. The second, on the other hand, would realize an interesting inversion of its definition while remaining precisely the same. The current definition is: *"The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom."*[†] While under the new SI this becomes: *"The second, symbol s, is the unit of time; its magni-*

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[*] http://en.wikipedia.org/wiki/File:Relations_between_new_SI_units_definitions.png

[†] Quotes from http://nvlpubs.nist.gov/nistpubs/jres/116/6/V116.N06.A01.pdf

tude is set by fixing the numerical value of the ground state hyperfine splitting frequency of the cesium 133 atom to be equal to exactly 9 192 631 770 when it is expressed in the SI unit s–1, which is equal to Hz." Instead of periods of cesium-133, it becomes the corresponding frequency.

Perhaps this is a matter of semantics, but semantics matter in definitions of international standards. All the SI base units other than the mole are referenced to the second, and the second is defined in terms of a frequency, as a reciprocal of number expressed in terms of the unit of frequency, the hertz. It is unfortunate that the SI definition of the second and the traditional definition as the sexigesimal fraction of synodic day reuse the same word for different concepts. Science and society are, however, full of such instances.

CONCLUSION

The word day has many fascinating shades of meanings as demonstrated by its lengthy entry in the Oxford English Dictionary. The cultural importance of the word is shown by its frequent usage. The meaning of a day is as varied as humanity ourselves. Underneath the rich semantics is a common signification as "a period, natural division, or unit of time".

Words related to time are among the most frequently used in the language. This is particularly true of words expressing units of time, as shown in Table 5, showing rankings and frequency of usage in the 450 million word Corpus of American English. It is not surprising that an issue that has been generally discussed as whether or not to eliminate leap seconds should focus on units of seconds. In reality, however, seconds are far less frequently discussed than other units – by a ratio of less than 1:7 compared to the day. Words related to calendars (day, month, and year) are used more than four times as frequently as words related to clocks (hours, minutes, and seconds). The discrepancy in the ranking is even more striking when limited to nouns. *Day* is the 5th most frequently used noun, *second* is 257th. *Time* and *year* are number one and number two, respectively, indicating the importance of civil times and dates to the general public.

Table 5. Frequency of Word usage in the 450 million word Corpus of Contemporary American English.[*]

Rank (in top 5000 words)	**Time-related Word**	**Frequency** ***(out of 450 million)***
54	Year	769254
90	Day	432773
188	Week	199268
237	Month	162685
273	Hour	138955
309	Minute	126660
606	Century	65667
683	Second	56022
731	Decade	53727

A robust standard is applicable both broadly and under novel circumstances. This is achieved by deriving the standard from a physically correct model. In the case of civil timekeeping, that model is the synodic day. This is distinct from the frequency and duration standard derived from

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[*] http://www.wordfrequency.info/free.asp?s=y

atomic clocks. Few circumstances are as novel as timekeeping on Daphnis, a shepherd moon orbiting within Saturn's rings. See Figure 8 for an appreciation of the dramatic circumstances of this little moon – even discounting the vision of Saturn itself off the top of the picture. The orbit of Daphnis is slightly inclined to the plane of the rings and it plunges above and below the rings twice each 14 hour day. Its gravity raises waves that lead the moonlet in the inward ring and trail due to differential rotation in the outward ring. With all this going on the length of the day is yet governed by the same expression: Daphnis has one fewer day per year than sidereal rotations.

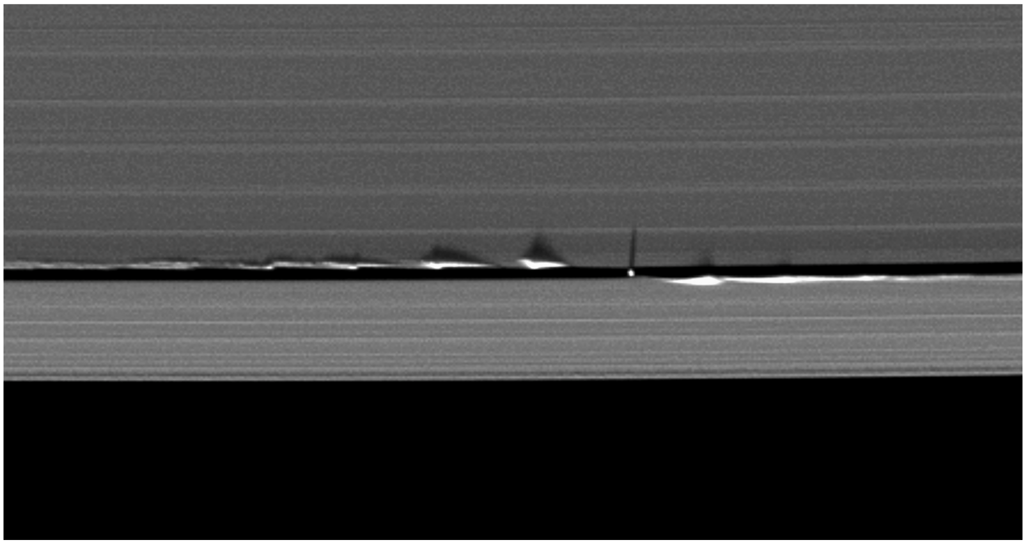

Figure 8. Daphnis is the *shepherd moon* that orbits within the Keeler gap in Saturn's rings.*[*] For scale the Keeler gap is about the width of the English Channel, and the waves Daphnis's gravity raises in the rings on either side are up to about 10 times the height of the white cliffs of Dover.

To sum up: the Earth spins like a top – this is the sidereal rotation period. There is one fewer day per year than sidereal rotations – thus the mean solar day is about 4 minutes longer than the sidereal rotation period. The Earth's orbit is slightly elliptical and its axis is somewhat tilted so apparent day length varies throughout the year by a few seconds either way from the mean (synodic) day. This variation is orders of magnitude larger than the periodic and secular geophysical effects that engender the need for leap seconds. That difference in scale does not mean that leap seconds (or equivalent intercalary adjustments) are negligible, but rather that these are completely different phenomena that must be modeled correctly.

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