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COMPUTATION ERRORS IN LOOK ANGLE AND RANGE DUE TO REDEFINITION OF UTC*

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With the decision on whether or not to discontinue leap seconds scheduled for January 2012, it is important to develop tools to evaluate the error that can be expected in operational software that tracks space objects from the ground or ground objects from space. These tools focus on the error that would occur in software that uses Coordinated Universal Time (UTC) as an approximation to Universal Time 1 (UT1). These error evaluation tools input the difference (in seconds) between UT1 and redefined UTC and a user-specified altitude for space objects. From these inputs, one of the tools plots a grid of look angles in polar coordinates thus generating a "sky plot" as seen from a particular ground location. This tool shows the true position in the sky when one uses UT1 to compute Earth orientation and the biased position in the sky when one uses redefined UTC. The two positions are connected by an arc from the true position to the biased position. This arc is the path the biased position would take as one gradually increases the separation between UT1 and redefined UTC. The color of the arc changes according to the bias in range. This tool also outputs the true and biased values for range, azimuth angle and elevation angle at all grid points. This, and related tools provide the user a sense of the adverse operational impacts as the biased position deviates more and more from the true position.

INTRODUCTION

The time scale that forms the basis for civil time is known as Coordinated Universal Time (UTC). UTC is typically shifted by an integer number of hours to produce local civil time for various time zones worldwide. UTC is tied to the orientation of Earth about its axis relative to the sun. This orientation is defined in terms of the angle measured eastward from the point on the equator opposite the sun's mean position to the Greenwich meridian. When measured in hours of arc (1 hour = 15°), this angle serves as a "time" known today as Universal Time 1 (UT1).

Strictly speaking, UT1 is not a true time scale because it slows down in an irregular fashion as the tidal forces on Earth slow Earth's rotation rate. On average, Earth's slowing rotation rate causes the solar day to increase by about 1.4 milliseconds per century. The irregular slowdown is due primarily to the changing mass distribution within Earth and its atmosphere and oceans. Despite this variability, Universal Time has been used for centuries as the basis for civil time. Be-

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cause UT1 is primarily an Earth orientation angle, it is used to compute the position of space objects (e.g. satellites and celestial objects) relative to observation points on Earth, as well as the position of Earth objects relative to points in space. To compute Earth's orientation in an inertial coordinate frame, UT1 is converted from an angle relative to the *mean sun*[*] to an angle measured eastward from the vernal equinox point, known as *right ascension*. The right ascension of the Greenwich meridian θ_G in degrees is often given by the polynomial:

$$\theta_G = 280.46061837 + 360.98564736629 \times d + \left[\frac{d}{1854436}\right]^2 - \left[\frac{d}{12355622}\right]^3 \tag{1}$$

where d is UT1 measured in days elapsed since January 1st in the year 2000 at 12:00, an epoch referred to as J2000.0. Figure 1 shows the geometry associated with this formula at a time when Earth has an orientation of 30°. The θ_G angle is essential for computing the inertial coordinates of space objects relative to a rotating Earth as well as the Earth-fixed coordinates of objects on Earth relative to an inertial coordinate frame in space.

Unlike UT1, atomic time is a true time in the strictest sense. It does not vary with Earth's slowing rotation rate. It is based on a solar day with exactly 86,400 System International (SI) seconds. The SI second is defined as, "the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom."[1] This number of periods was chosen to be as close as possible to one second of Ephemeris Time (ET), now known as Terrestrial Time (TT). The ET second was defined as 1/31,556,925.9747 the length of the tropical year for the year 1900.[2] Today, the TT second is defined to be identical to the atomic SI second. There are three major atomic times in common use today, each differing by a constant offset in seconds: Terrestrial Time (TT), International Atomic Time (TAI) and GPS Time (GPST). These three atomic times progress at a steady rate and never gain on each other. On the other hand, centuries ago, Universal Time (UT1) was significantly faster than atomic time, but today is generally slower.

Aesop's "tortoise and the hare" fable is used in Figure 2 to illustrate the relationship between the steady atomic-time "tortoises" and the erratic universal time "hare." As the centuries progress, the different times advance upward as shown on the left-hand-side of the figure. When referenced to the progression of the atomic time "tortoises", as shown on the right-hand-side of the figure, the universal time "hare" catches up and surpasses the atomic times and then falls behind. The progress of UT1 and the three atomic times over the last four centuries is shown in Figure 3, adapted from Figure 1 in Nelson *et al*.[2]

Coordinated Universal Time (UTC) is a "hybrid" time scale, having characteristics of both UT1 and atomic time. It was established 1 January 1972 as the new "civil time" replacing Greenwich Mean Time (GMT). UTC is kept within ±0.9 second of UT1 through periodic leap second adjustments. This way, UTC is always an integer number of seconds off from International Atomic Time (TAI). Figure 4 was adapted from a download off the Earth Orientation Center web page.[†] It shows the relationship between UT1 and UTC since 1972. UTC was 10 seconds behind TAI in 1972 and today is 34 seconds behind. GPS Time is a constant 19 seconds behind TAI. Since UTC is kept within ±0.9 second of UT1, UTC may be used to approximate Earth's

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[*] The term *mean sun* is no longer used in current realizations of the motion of Earth around the sun, but is still a useful concept for heuristic purposes.

[†] http:/hpiers.obspm.fr/eop-pc/earthor/uts/leapsecond.html

orientation within ±13.5 arc seconds which is equivalent to ±418 meters of error on Earth's equator at the surface. Many space tracking applications can tolerate these errors and use this approximation.

Despite UTC's usefulness as both a time and as an approximate Earth orientation angle, several commercial systems needing precise time have difficulties introducing leap seconds whenever they occur. Consequently, a group of international stakeholders has been advocating discontinuance of leap second adjustments. However, they do not want to eliminate leap seconds from only their own internal system time, but want the whole word to transition to a "civil atomic time" so they can remain consistent with a standard worldwide time scale.

The International Telecommunication Union - Radiocommunication Sector (ITU-R) has been sponsoring a series of international meetings to decide whether or not leap seconds should be discontinued with a decision expected at the upcoming ITU Radiocommunication Assembly in January 2012. If the delegates vote to discontinue leap seconds, discontinuance could go into effect as early as 1 January 2018, resulting in a civil time no longer tied to Earth's orientation. The Assistant Secretary of Defense for Networks and Information Integration issued a policy letter (29 Jun 09) to the US State Department supporting the discontinuance of leap seconds, but asked that leap seconds be discontinued no earlier than 1 January 2019 to "allow sufficient time for necessary system modifications to be accomplished." The "necessary system modifications" are likely to be upgrades to software and firmware used to track space objects from the ground or to track ground objects from space. The source of potential software problems lies in the fact that the difference between UT1 and a new "civil atomic time" without leap seconds will likely grow indefinitely, a situation that some space tracking software has not been designed to handle, as discussed by Seago and Storz.[3]

ERROR DISPLAY TOOLS

This section describes two tools used to display the error in look angle and range one can expect from space tracking software that uses UTC as an approximation for UT1. Both of these tools were written in MATLAB®* and were developed at Headquarters Air Force Space Command. Both tools make use of standard coordinate transformations as described in P.R. Escobal.[4]

DUT1 Sky Plot Error Tool

This tool (*DUT1-Skyplot*) was developed to investigate the slow degradation with time (after leap second discontinuance) in space tracking algorithms that use UTC to approximate UT1. This tool plots the error in elevation angle, azimuth angle, and range as a function of direction in the sky, latitude of the ground site, altitude of the space object, and the value for DUT1 (= UT1-UTC). The length of the arc represents error in look angle (both elevation and azimuth). The color of the arc represents the range error. Deep red indicates computed ranges more than 10 km greater than truth; deep blue indicates computed ranges more than 10 km less than truth. In between is a rainbow color scheme with yellow-green indicating a zero range error.

Figure 5, Figure 6, and Figure 7 show the error in look angle and range due to an unapplied DUT1 value of 30 seconds for space objects at an altitude of 100 km. Notice that at both the equator and at 60° N latitude, the arcs are aligned along great circles spanning from the eastern horizon to the western horizon. The look angle error is largest at the equator as shown in Figure 5

* MATLAB® is a high-level language and interactive environment that enables one to perform computationally intensive tasks. MATLAB was developed by The MathWorks, Inc.

and is about 8° at the zenith. For applications that use UTC to approximate UT1, this is the kind of look angle error one can expect 30 to 50 years after leap seconds are discontinued. Since the error is proportional to the cosine of the latitude, it falls to about 4° at 60° N latitude at the zenith (Figure 6), and is zero at the north pole (Figure 7). The very small look angle arcs in Figure 7 are actually parallel to lines of constant elevation angle. This is revealed in Figure 8 where the DUT1 was artificially increased to 900 seconds for visual effect. Notice in Figure 8 that the arcs appear to spin around the zenith which corresponds to the celestial ephemeris pole (CEP) at this latitude.

Figure 9, Figure 10, and Figure 11 reveal how the error arcs transition from an east-west orientation to an orientation circling the celestial ephemeris pole as the altitude of the space objects increase. A fixed latitude of 60° N was chosen to demonstrate this. Notice in Figure 6 that, for space objects at an altitude of 100 km, the orientation of the arcs is almost purely in an east-west direction and each arc is nearly aligned with a great circle reaching from the east cardinal point to the west cardinal point on the horizon. The arcs do not appear to spin around the celestial ephemeris pole. This is due to the proximity of objects at 100 km altitude. They are so close to the observer that they behave like the corners of ceiling tiles in a building as an observer walks a few steps toward the east. However, as one increases the altitude of the space objects, there is a gradual transition to the geometry where the arcs appear to spin around the celestial ephemeris pole. Figure 9 shows the behavior of the error arcs for space objects at an altitude of 1000 km. This is the altitude where the arcs begin to spin around the north cardinal direction, thus appearing to be dots in that direction. Figure 10 shows the behavior for objects at an altitude of 10,000 km. Notice that the arcs are spinning around a "false" celestial pole that has a lower elevation angle than the true celestial ephemeris pole. Figure 11 shows the behavior for objects at an altitude of 1,000,000 km. At this altitude, the space objects behave like stars with look angle error arcs spinning around the celestial ephemeris pole. The value of DUT1 had to be increased with altitude for visual effect; otherwise the arcs would be too small to display.

DUT1 Cardinal Direction Error Tool

This tool (*DUT1-Cardinal*) was also developed to investigate the degradation in accuracy of space tracking algorithms that use UTC to approximate UT1. The tool plots the error in elevation angle, azimuth angle and range as a function of satellite altitude in km, and the value for DUT1 in seconds. The plots in Figures 12 through 16 display the error in 5 cardinal directions.

Figure 12 shows how the elevation angle error varies with space object altitude when an observer on the equator is looking toward the zenith. The zenith direction at the equator produces the largest look angle errors. The seven curves in Figure 12 correspond to a DUT1 of 0, 5, 10, 15, 20, 25 and 30 sec. Notice that for a DUT1 of 30 seconds, the elevation angle error is over 8°. This would take the computed look angle outside the field of view for many sensors.

Figure 13 and Figure 14 show how the azimuth angle error varies with space object altitude when an observer on the equator is looking north or south (respectively). The seven curves in Figures 13 and 14 also correspond to a DUT1 of 0, 5, 10, 15, 20, 25 and 30 sec. Notice that Figure 13 is the mirror image of Figure 14.

Figure 15 and Figure 16 show the range error when an observer on the equator looks east or west (respectively). The seven curves in Figure 15 and Figure 16 also correspond to a DUT1 of 0, 5, 10, 15, 20, 25 and 30 sec. Notice that the range error does not vary with altitude.

Figure 17 though 21 are analogous to the previous five figures. The only difference is that these plots were generated for latitude 60° N instead of the equator. Since the errors vary with the cosine of the latitude, they are roughly half the error values at the equator. However, the north and south cardinal points on the horizon exhibit a marked asymmetry. Notice that the curves in

Figure 18 for the north cardinal point converge near 1000 km altitude. This is related to the transition of the error arcs from an east-west orientation to an orientation circling the celestial ephemeris pole as the altitude increases. In particular, this convergence point occurs at the altitude where the "false celestial pole" is at the north cardinal point as was shown in Figure 9. The altitude where these curves converge varies with latitude. At high latitudes, this point occurs at a lower altitude than at low latitudes. For the southern hemisphere, Figure 19 (south cardinal direction) would exhibit this convergence point instead of Figure 18 (north cardinal direction). Figures 20 and 21 look very much like Figure 15 and Figure 16 (respectively), but the magnitude of the error is about half as much due to the dependence on cosine of the latitude.

CONCLUSION

Discontinuing leap seconds would fundamentally change the way civil time is defined. Civil time would no longer be kept close to (within 1 second of) UT1, it would no longer be tied to the orientation of Earth relative to the sun, and it would resemble another atomic time. The Space Community needs to develop a plan for upgrading operational software in case leap second discontinuance goes into effect. These error evaluation tools are important for assessing the potential operational impacts to space tracking software. These tools could also play a part in upgrading operational software, especially if the upgrades are made at the back end of legacy algorithms, after erroneous look angles and ranges have already been computed.

ACKNOWLEDGMENTS

I would like to acknowledge Mr. Bill Guilfolye and 1Lt Lacey Castaneda, both with Air Force Space Command, Analyses and Assessments Division for help in coding the MATLAB tools.

APPENDIX: FIGURES

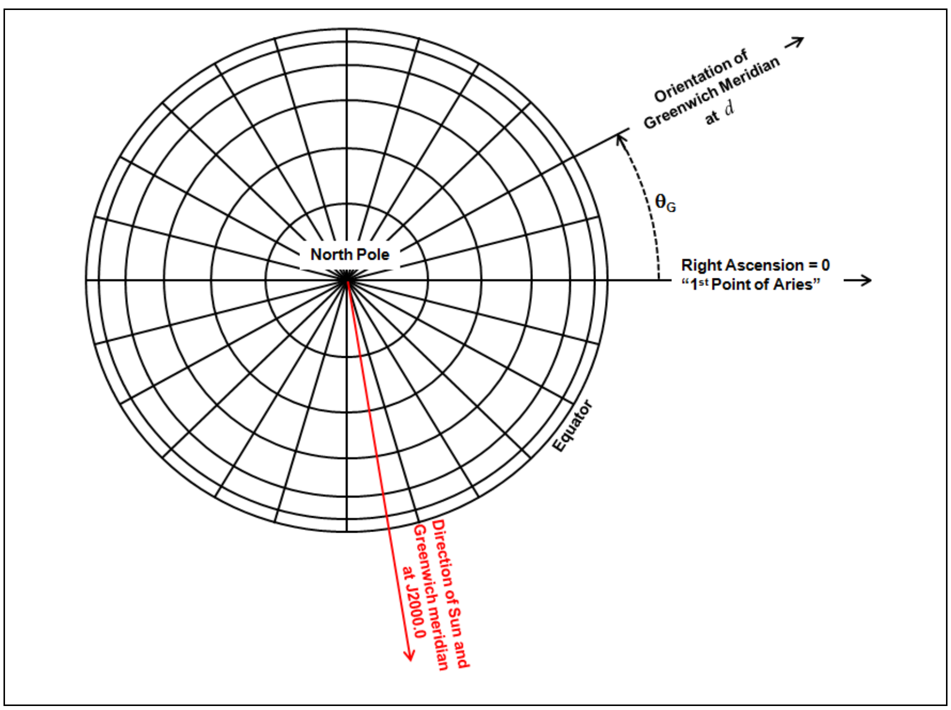

Figure 1. Diagram of the Greenwich meridian relative to the "First point of Aries".

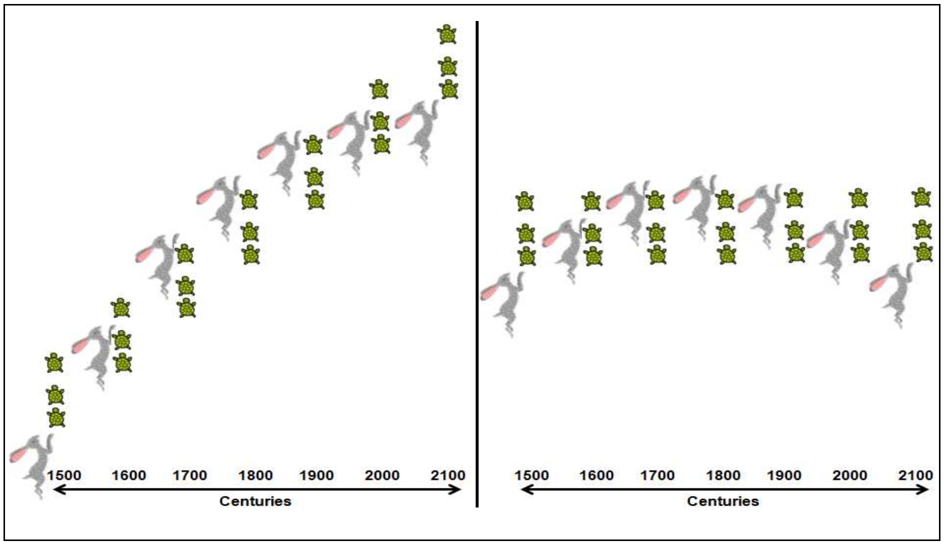

Figure 2. The "tortoise and the hare" analogy for atomic times and universal time.

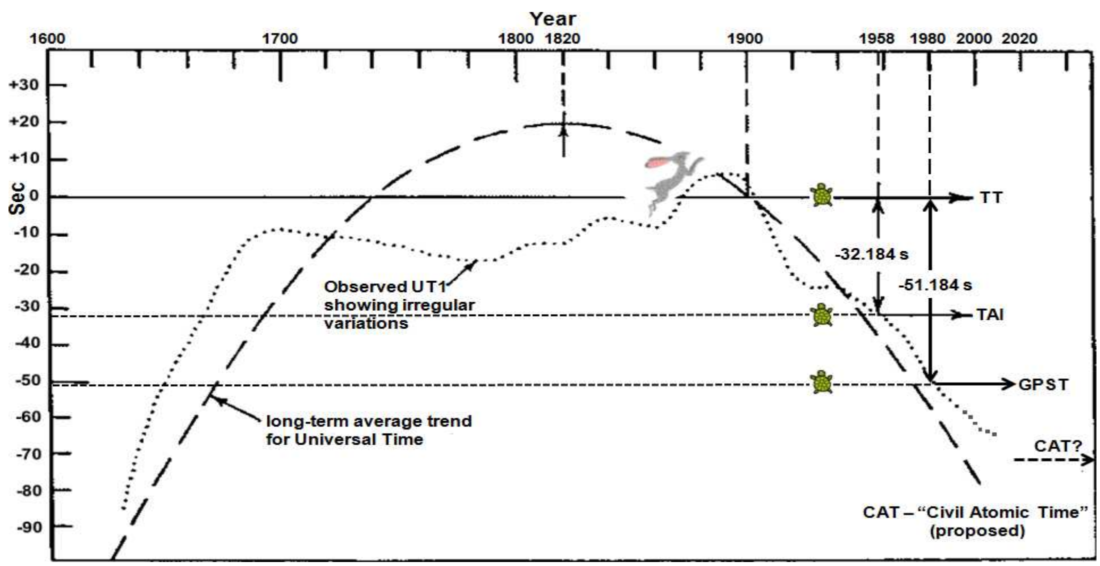

Figure 3. UT1 and atomic times over the last four centuries.

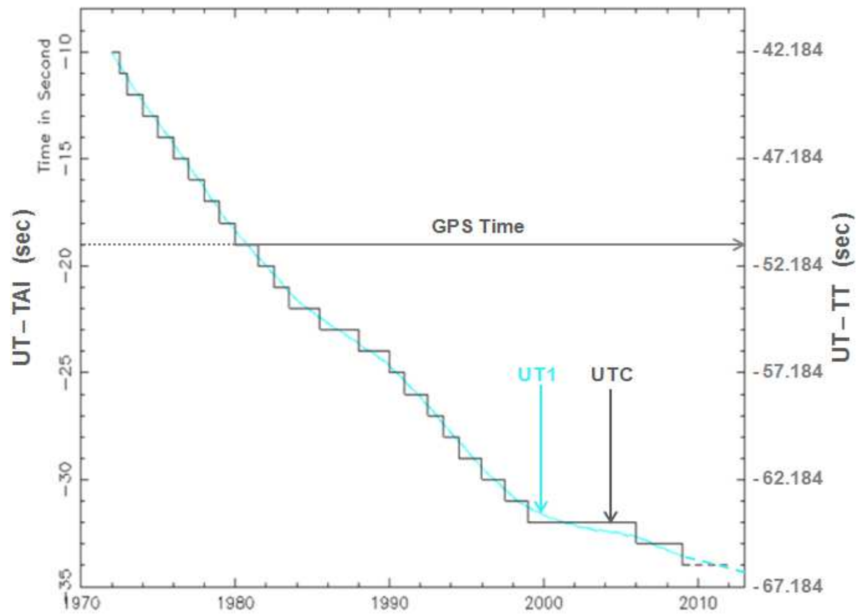

Figure 4. UT1 and UTC since 1972.

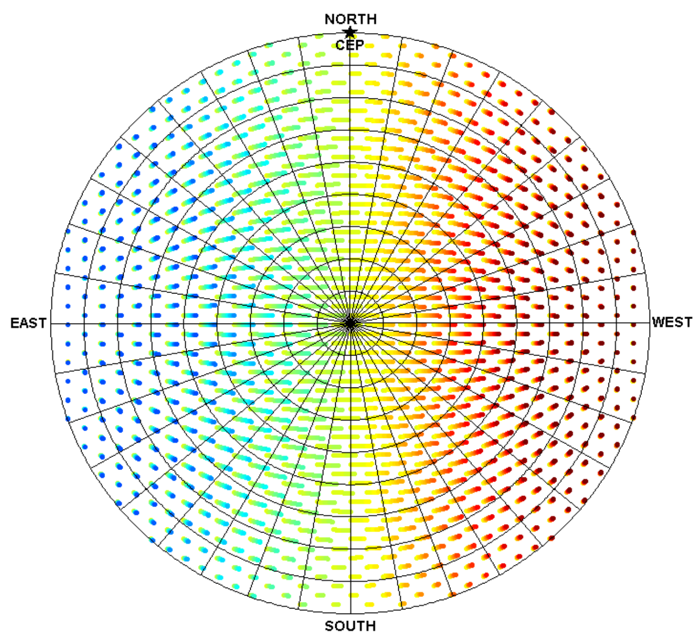

Figure 5. Sky Plot at Equator for space objects at 100 km altitude (DUT1 = 30 sec)

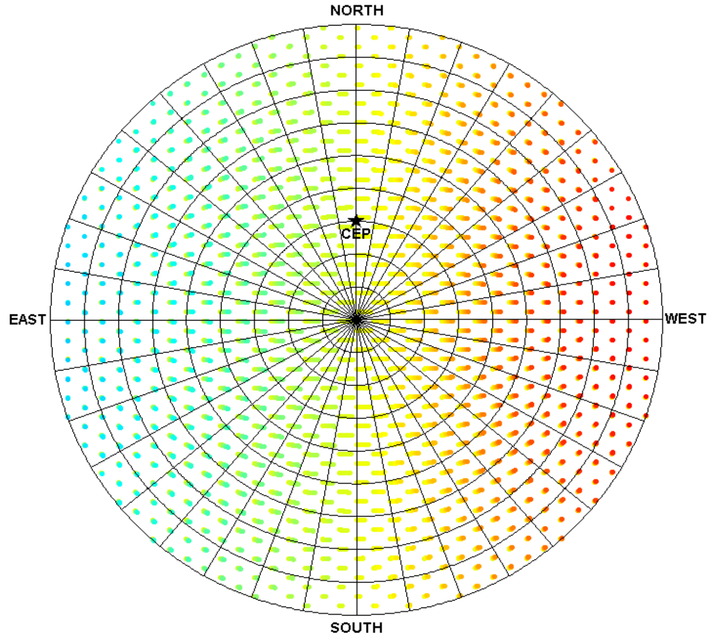

Figure 6. Sky Plot at 60° N latitude for space objects at 100 km altitude (DUT1 = 30 sec)

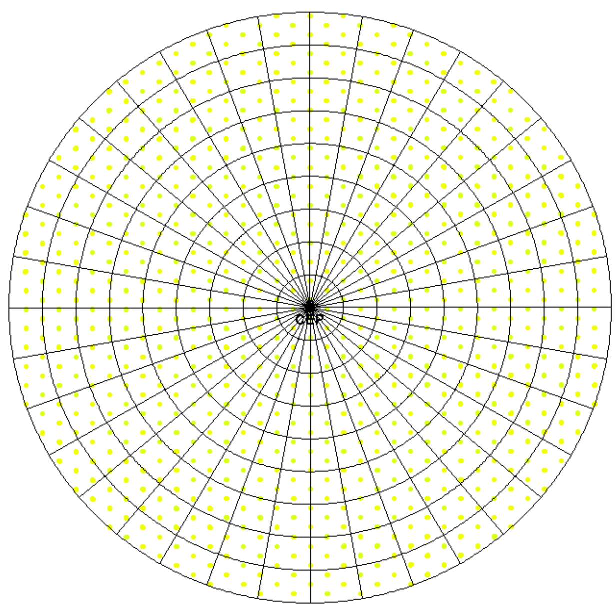

Figure 7. Sky Plot at 90° N latitude for space objects at 100 km altitude (DUT1 = 30 sec)

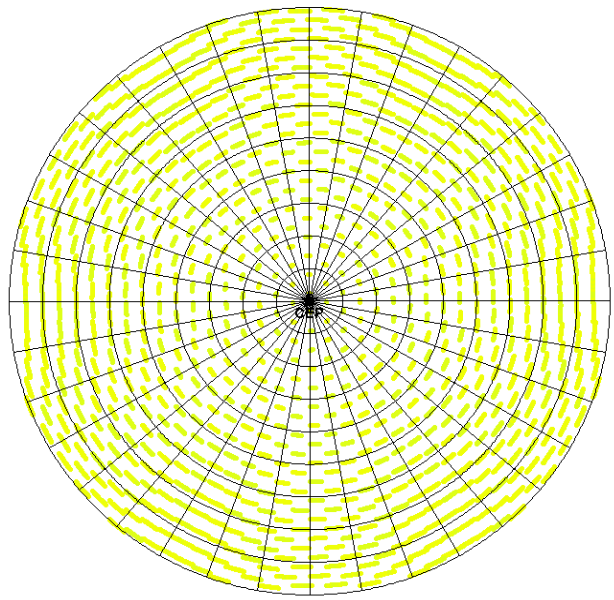

Figure 8. Sky Plot at 90° N latitude for space objects at 100 km altitude (DUT1 = 900 sec)

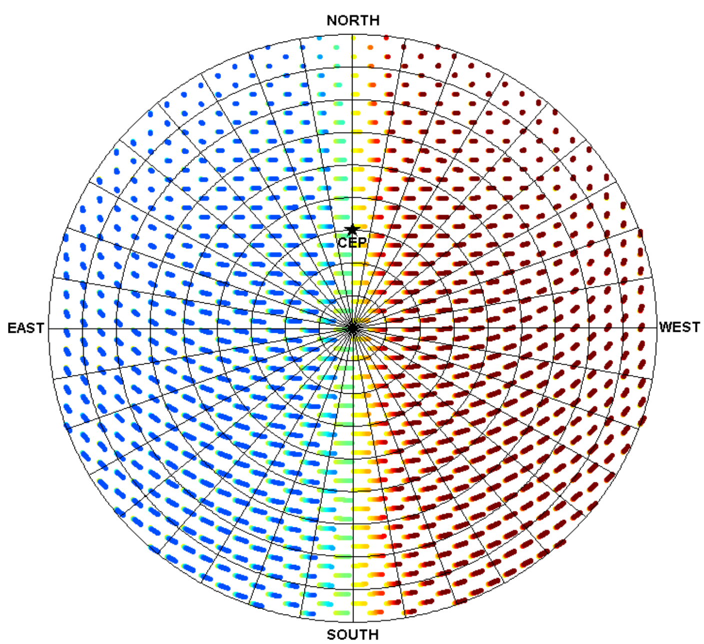

Figure 9. Sky Plot at 60° N latitude for space objects at 1000 km altitude (DUT1 = 300 sec)

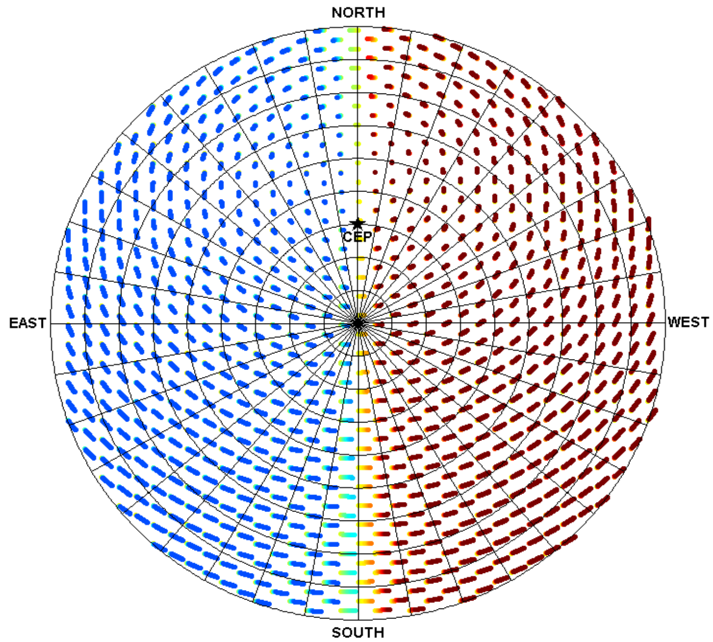

Figure 10. Sky Plot at 60° N latitude for space objects at 10,000 km altitude (DUT1 = 600 sec)

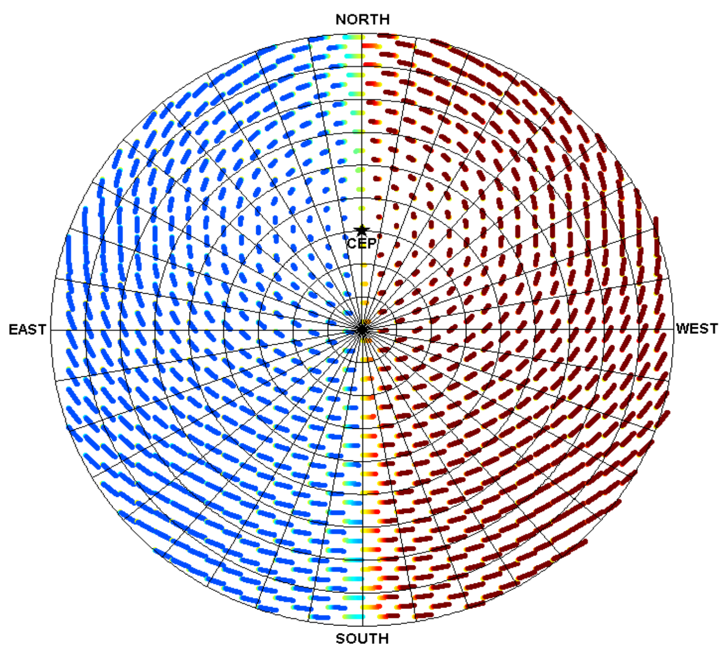

Figure 11. Sky Plot at 60° N latitude for space objects at 1,000,000 km altitude (DUT1 = 900 sec)

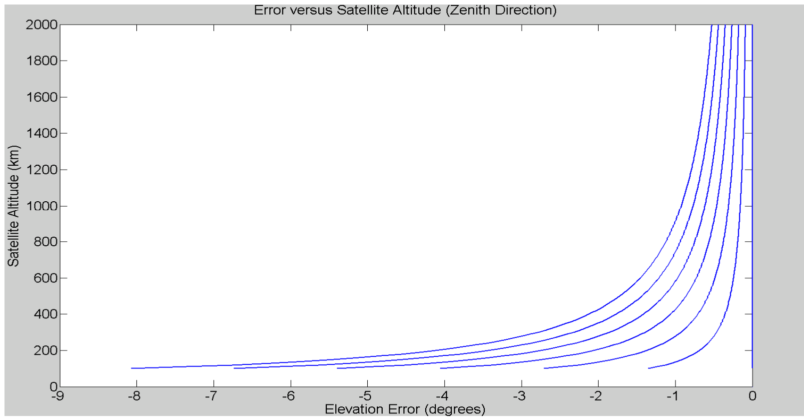

Figure 12. Elevation angle error at the equator when looking toward zenith

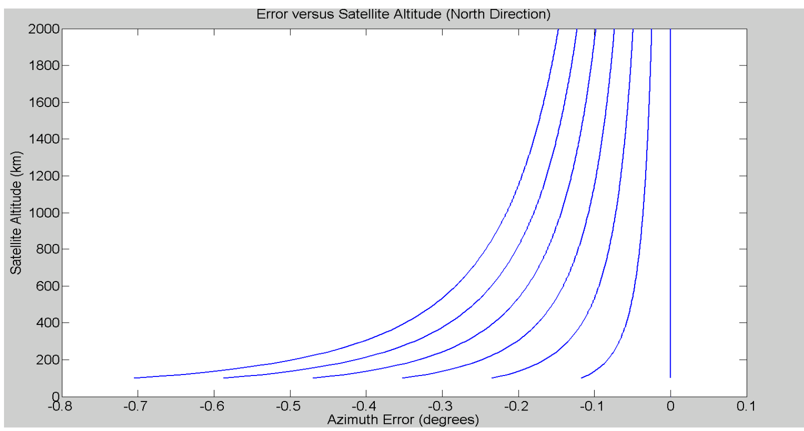

Figure 13. Azimuth angle error at the equator when looking north

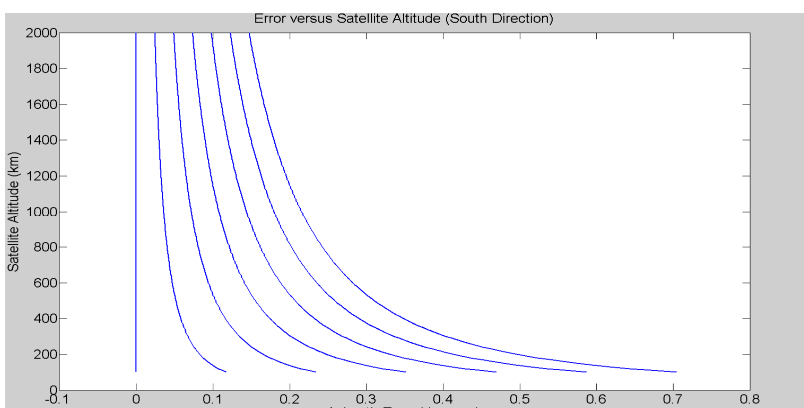

Figure 14. Azimuth angle error at the equator when looking south

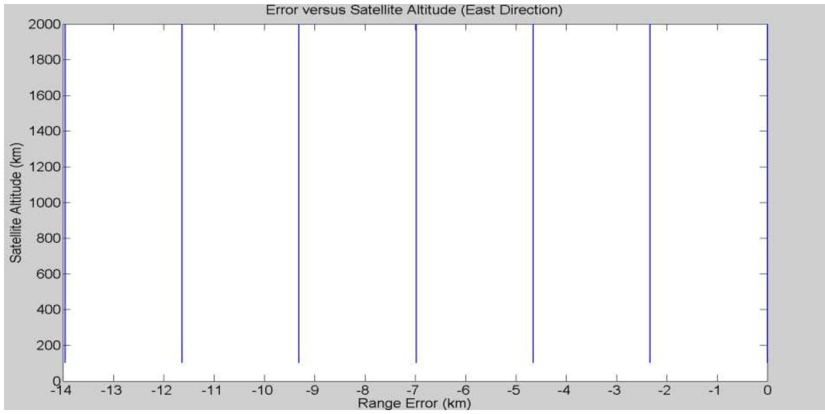

Figure 15. Range error at equator when looking east

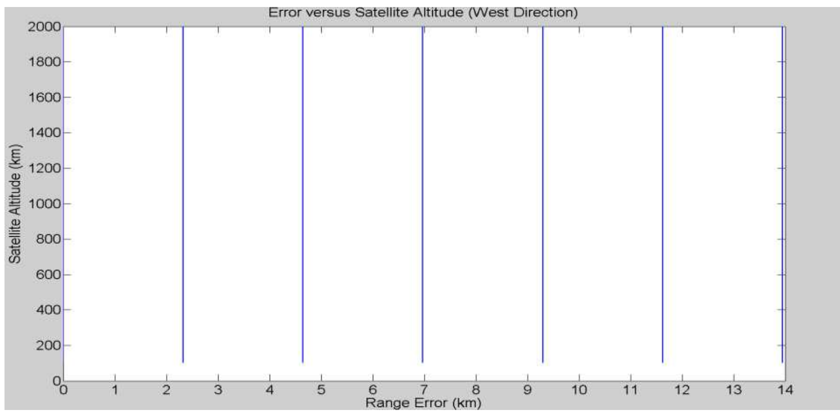

Figure 16. Range error at equator when looking west

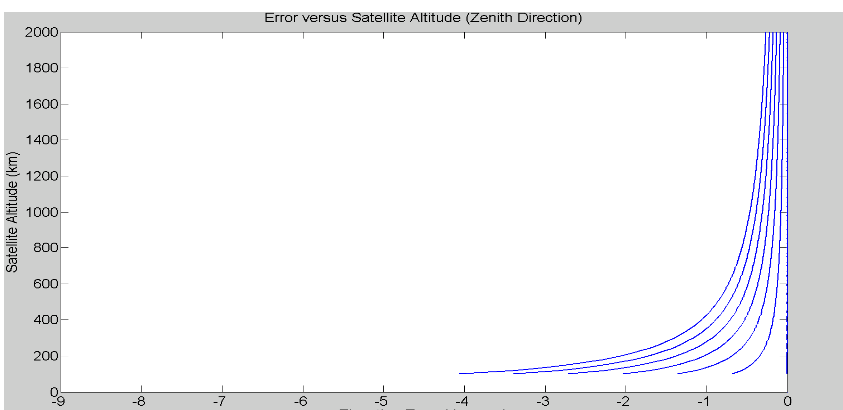

Figure 17. Elevation angle error at 60° N latitude when looking toward zenith

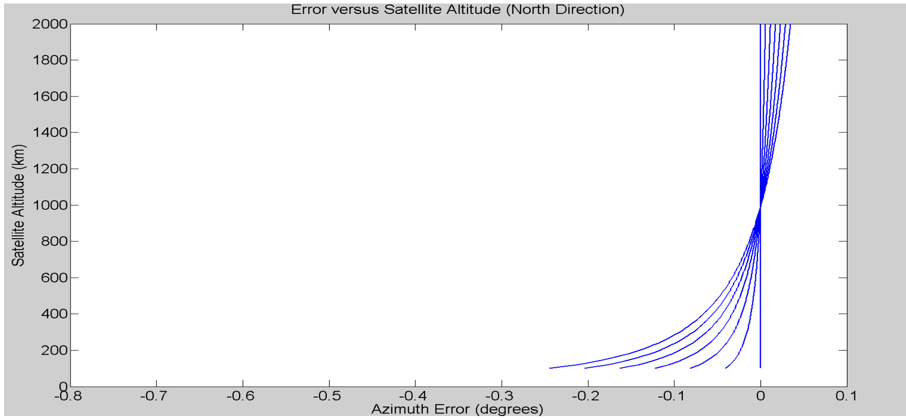

Figure 18. Azimuth angle error at 60° N latitude when looking north

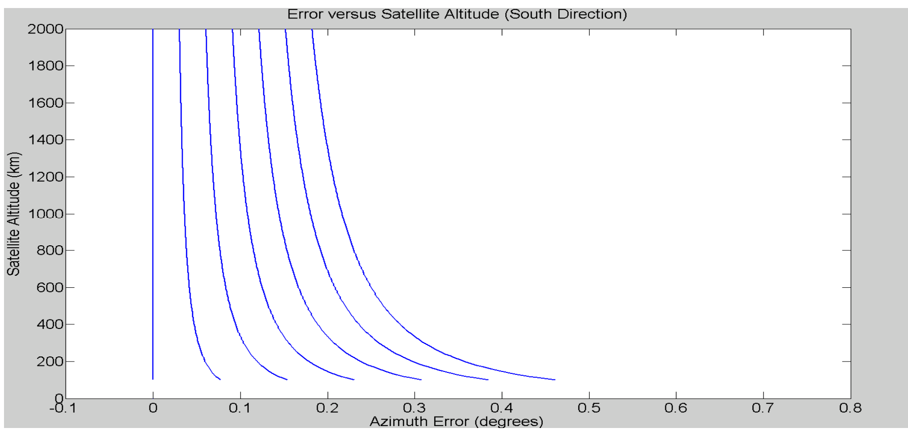

Figure 19. Azimuth angle error at 60° N latitude when looking south

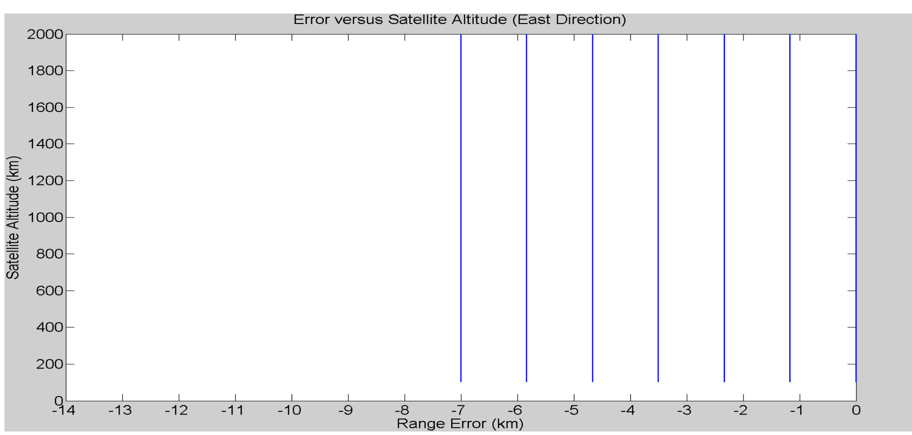

Figure 20. Range error at 60° N latitude when looking east

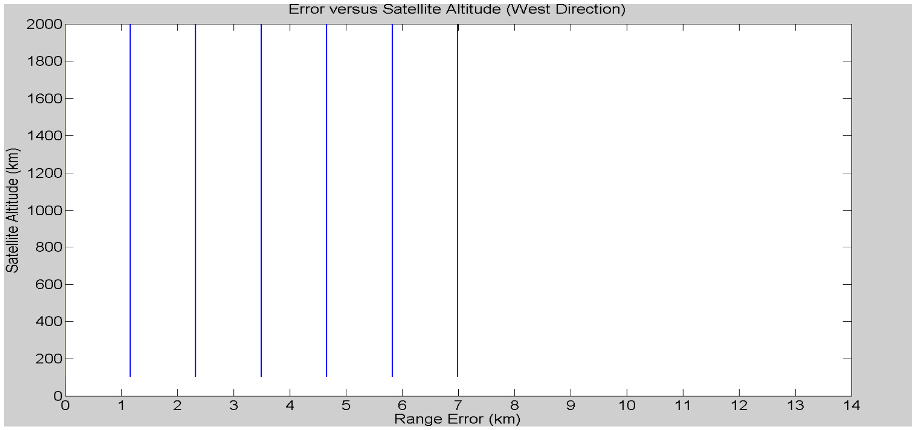

Figure 21. Range error at 60° N latitude when looking west

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