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THE CONSEQUENCES OF DECOUPLING UTC ON SUNDIALS

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Severing the link between the rotation of the Earth and time-signal broadcasts will require a fourth correction to convert between solar time and standard legal time in the long term. For sundials, this represents an additional complexity, both for people using these instruments as educational tools, and for those who design them. Knowing the precision that can be expected of sundials, how long will it take for the decoupling of UTC to become noticeable?

INTRODUCTION

Though sundials may seem old-fashioned today in the time of the GPS, they are nevertheless part of our scientific and cultural inheritance. The first astronomical constants like the latitude, the ecliptic obliqueness or the inequality of the seasons were determined during Antiquity with a simple gnomon. The study of the gnomon's shadow trajectory represented a powerful tool in mathematical research on conics.[1] Considered as objects of prestige during the Greco-Roman period, their accuracy improved. The works of Vitruve (*De Architectura*, book IX) and especially of Ptolemy (Treaty of *De l'analemme*) greatly improved the mathematics of sundial layout. Under the influence of Arabo-Persian astronomers, sundials began to indicate the hours of prayers, while retaining their aesthetic qualities.

Let us highlight two important points: firstly, from antiquity to the end of the Middle Ages, the hour is the twelfth part of the interval of time between the rising of the Sun and its sunset.[2] The duration of this hour, called "temporary", varies according to the date and place; this hour is equal to 60 uniform minutes only at the equinoxes. The second point is related to the equation of time. Thanks to Ptolemy, one knows at least since the IInd century of our era that solar time doesn't flow uniformly because of the obliqueness of the ecliptic and the non-uniform speed of the Sun along the ecliptic. The equation of time was thus known to Greek astronomers and their astronomical observations (made in apparent solar time) were corrected to mean solar time.[3]

The arrival of the clock industry during the XIIIth century partly stimulated the development of sundials in the West. These sundials became references for the adjustment of clocks and pendulums until the XIXth century.

FORESEEABLE IMPACT ON ACCURACY

In Europe during the XVIth and XVIIth centuries, the first indoor meridians followed the reform of the Gregorian calendar for the determination of the tropical year length or the decrease of

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the obliqueness of the ecliptic. It is estimated that the accuracy of a meridian such as the one Cassini built in 1680 at the Paris Observatory, is about a few seconds of time.

Two crucial factors diminish the accuracy of a sundial: firstly and most importantly there is the effect of the gnomon penumbra (or spot of light), and secondly there is refraction.[4] Let us discard the decrease of the ecliptic obliqueness, which affects only the reading of the diurnal arcs indicating the date. In general, a meridian is much more accurate than a simple ornamental sundial for which an accuracy of about one minute of time is acceptable, even if many sundials made during the last decades are accurate to 10 or 20 seconds of time and even better.

The idea of incorporating the equation of time into the calculation of a meridian, which allows a direct reading of the local mean solar time, is due to the astronomer Grandjean de Fouchy in 1730.[5] Sundials with a figure of eight (analemma) became increasingly numerous during the XVIIIth and XIXth centuries. The acceptance of the international Greenwich meridian in 1884, the unification of the hour in each country at the end of the XIXth century, not to mention the advance of Universal Time (UT) for economic reasons, lead the meridian and sundial users to introduce two additional corrections: one in longitude and one due to advance of the legal time with respect to UT.

It is obvious that much of the time, the public is not aware of all these relations that link these different times scales, and that sundials are generally considered as old-fashioned and inaccurate; it even happens that standard sundials' gnomon are manually twisted by their owner in order to indicate the hour given by the clock.

Hence, since almost a century, to convert the solar time read on a sundial to legal time, one has been taking three corrections into account: the equation of time, the longitude of the place and the advance with respect to Universal Time. Of course, these corrections can be partly or entirely integrated on a sundial, in order to make a direct reading. What does the decoupling of UTC entail for a sundial? If one calls ΔE the difference between UTC-without the leap seconds and UT1, then in the long term this fourth correction that is ΔE is going to be necessary in order to always be able to compare the apparent solar time with the Sun:

$$\begin{gathered}\text{Legal time at the clock} = \\ \text{apparent solar time} + \text{equation of time} + \text{longitude} + (1\text{h or }2\text{h}) + \Delta E\end{gathered} \tag{1}$$

Let us be realistic: except for specific sundials (either very accurate meridians or exceptional sundials), only an accuracy of about one minute of time is expected from these instruments. Some time will elapse before ΔE reaches this order of magnitude (approximately 140 years if only the secular deceleration of Earth rotation is taken into account).

However, let us consider some particular cases. The drawback of sundials indicating the mean solar time is that the equation of time slowly varies, causing an error which increases over centuries. Over a century, the maximum variation of the equation of time is a little more than 20 seconds, which is rather low (the main factor of variation of the equation of time is the displacement of the perigee).[6] Assuming that the only source of error is the variation of the equation of time, the error after a century on a sundial indicating the mean solar time will thus amount to 20 seconds in absolute value. This is perfectly acceptable for most sundials featuring an analemma. But obviously, the difference between UTC-without-leap-seconds and UT1 has to be added to this variation of the equation of time. Let us note that, for the past 40 years, 23 leap seconds were added for the difference between UT1 and UTC to remain less than 0.9 second of time. Thus, the drift due to the Earth's rotation is more important than the variation of the equation of time: the sum of the two can predictably reach 1 minute of time within much less than a century!

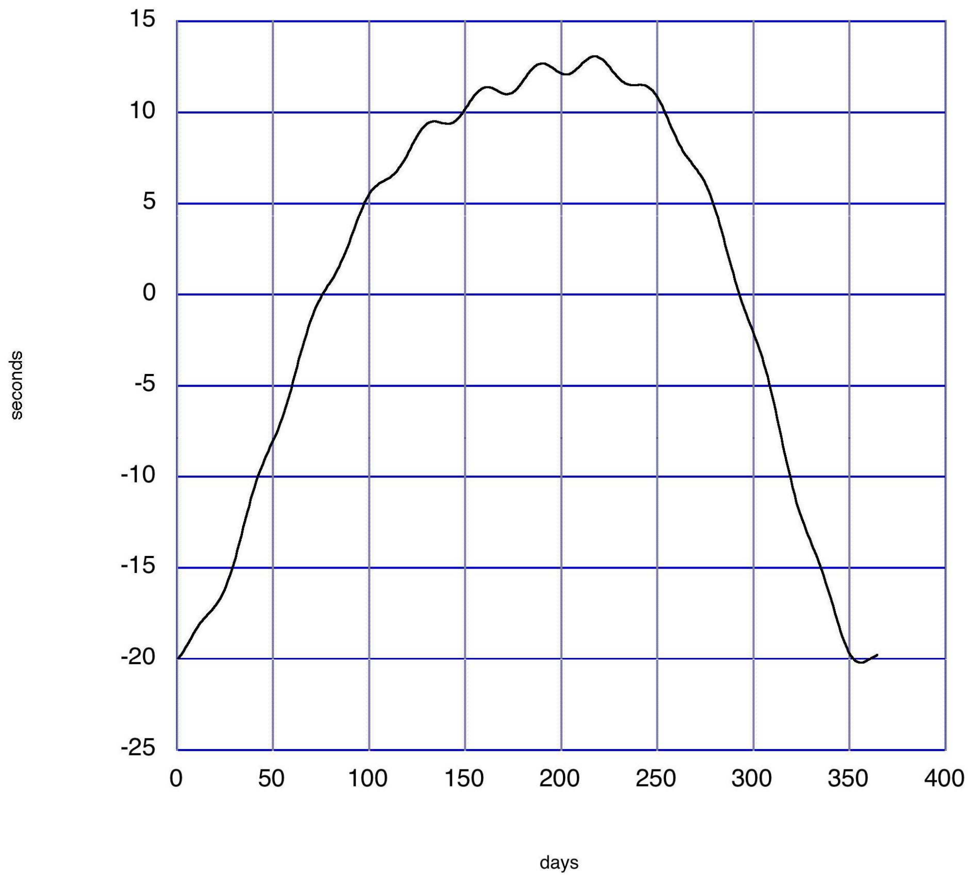

Figure 1. Variation of the Equation of Time between 2001 and 2101.

With regard to the accurate analemma of midday Universal Time, one will no longer be able to observe the perfect coincidence between the shadowing of this figure and the "beep" of the speaking clock: there will be an increasing offset over time. Let us underline that the ΔE correction is not foreseeable a long time in advance, contrary to the equation of time. Thus, it is going to be impossible for the sundials' calculation to take it into account: does it mean that sundials indicating the mean solar time with an analemma are to disappear? It depends on the desired accuracy.

Let us note that sundial manufacturers use the Sun to determine the astronomical orientation of walls and to trace meridians on the ground. It will be necessary to take the ΔE term into account since all these operations are based on Universal Time. Neglecting these few seconds would lead to annoying consequences: for instance, considering that the azimuth of the Sun varies most quickly close to the summer solstice, a 1-second error in the determination of the time of passage on the meridian line at 30° of latitude would lead to an error in azimuth greater than 0° 02 '.

CONCLUSION

From a pedagogical point of view, the decoupling from UTC can be seen as an additional complexity that is not easy to explain. Until now, sundials have been wonderful teaching objects, usable as early as elementary school to explain the seasons or the inequality of the days; and then later at college, Kepler's laws; without forgetting that they are also a useful practical application of trigonometry.[7] But experience shows that the most difficult point to understand, for the pupils or the general public, is the conversion from solar time to legal time, in particular the equation of time. So, if it becomes necessary to explain an additional correction that corresponds to the difference between a uniform scale of time and the solar time, sundials could be considered as complex objects indeed!

However, it seems unlikely that the decoupling of UTC will prevent the general public or astronomers from enjoying sundials, because their interest goes well beyond the problem of the conversion of time, which is known only by a very few. But sundials are almost the last link between the measure of time and the Sun, and it would be somewhat afflicting to note the ending of a link going back to Antiquity.

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